

DATA MINING DATA EXPLORATION AND STATISTICS

Exploratory data analysis

Basic Statistics

Exploratory analysis of data – What does my data look like?

- **Summary statistics**: numbers that summarize properties of the data
- Summarized properties include **frequency**, **location** and **spread**
 - Examples: location - mean
spread - standard deviation
- Most summary statistics can be calculated in a single pass through the data
- Computing **data statistics** is one of the first steps in understanding our data

Frequency and Mode

- The **frequency** of an attribute value is the percentage of time the value occurs in the data set
 - For example, given the attribute 'gender' and a representative population of people, the gender 'female' occurs about 50% of the time.
- The **mode** of an attribute is the most frequent attribute value
- The notions of frequency and mode are typically used with categorical data or discrete numerical data
- We can visualize the data frequencies using a **value histogram**
- Frequency and frequency histogram are the empirical analogue of probability and probability distribution

Example

<i>Tid</i>	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	10000K	Yes
6	No	NULL	60K	No
7	Yes	Divorced	220K	NULL
8	No	Single	85K	Yes
9	No	Married	90K	No
10	No	Single	90K	No

Marital Status

Single	Married	Divorced	NULL
4	3	2	1

Mode: Single

Example

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10	No	Single	90K	No

Marital Status

Single	Married	Divorced	NULL
40%	30%	20%	10%

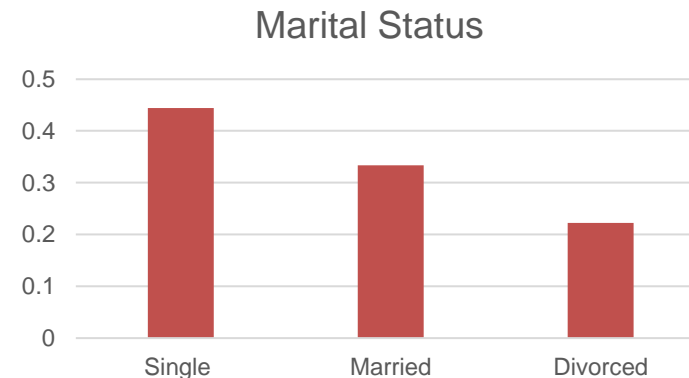
Example

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7	Yes	Divorced	220K	NULL
8	No	Single	85K	Yes
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10	No	Single	90K	No

We can choose to ignore NULL values

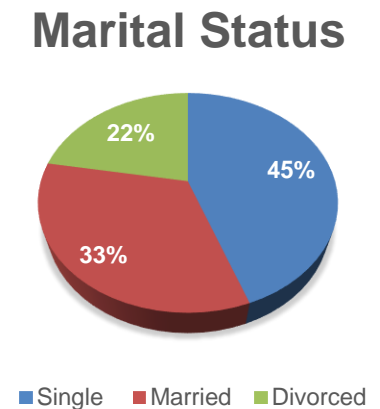
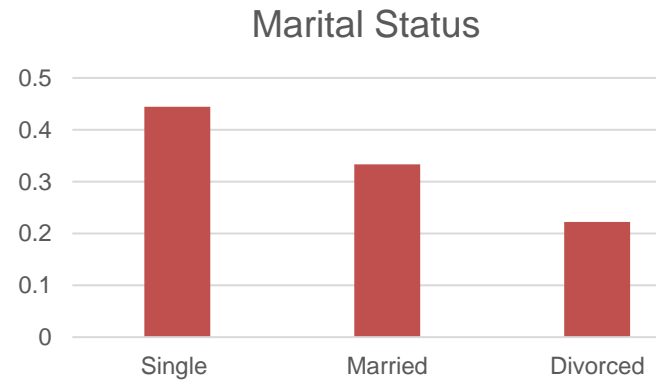
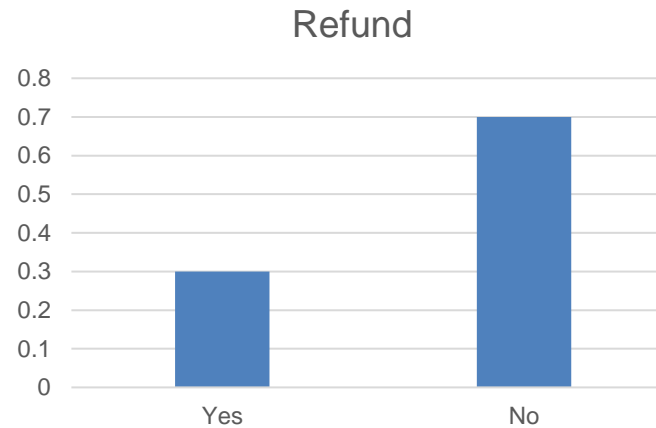
Marital Status

Single	Married	Divorced
45%	33%	22%



Data histograms

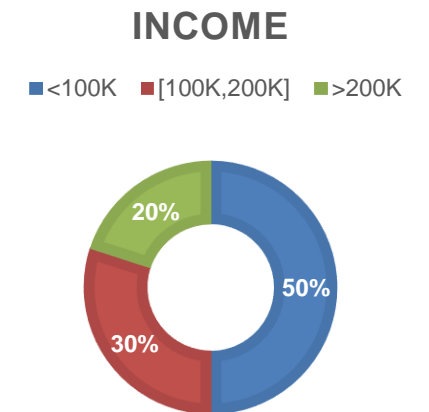
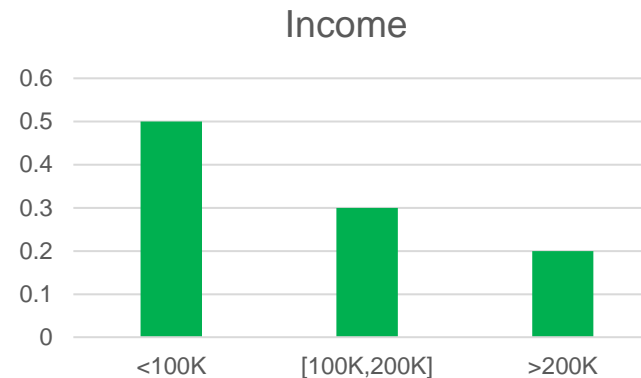
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Data histograms

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For real numerical values we use **binning** to create the histogram



In most plotting libraries, we specify the number of bins and the method creates an equiwidth histogram

Data histograms

- When we have discrete values that have a small range we can treat them as

Percentiles

- For continuous data, the notion of a percentile is more useful.

Given an ordinal or continuous attribute x and a number p between 0 and 100, the p^{th} percentile is a value x_p of x such that $p\%$ of the observed values of x are less or equal than x_p .

- For instance, the 80th percentile is the value $x_{80\%}$ that is greater or equal than 80% of all the values of x we have in our data.
- The percentiles are the empirical analogue of the cumulative probability function

Example

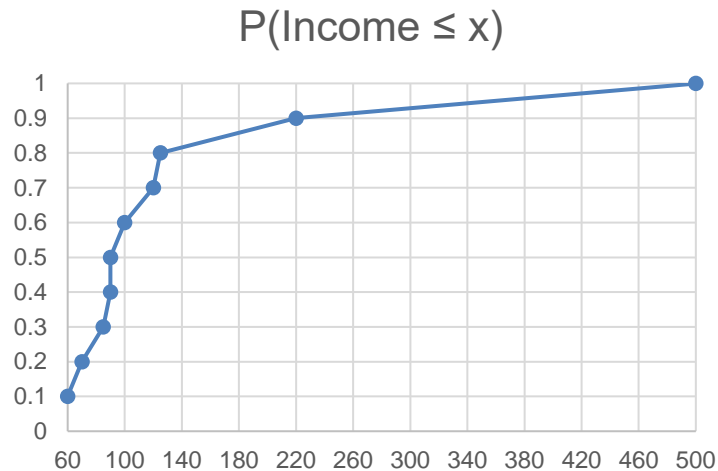
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	Taxable Income
1	10000K
2	220K
3	125K
4	120K
5	100K
6	90K
7	90K
8	85K
9	70K
10	60K

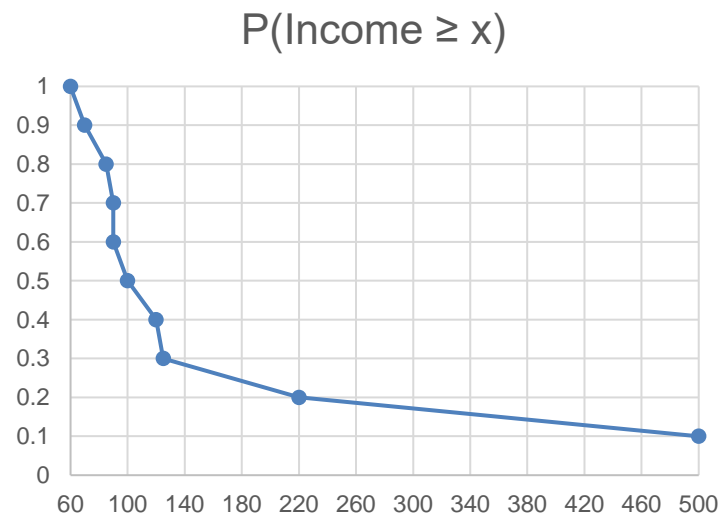
$$x_{80\%} = 125K$$

Plotting the cumulative distribution

	Taxable Income
1	500K
2	220K
3	125K
4	120K
5	100K
6	90K
7	90K
8	85K
9	70K
10	60K



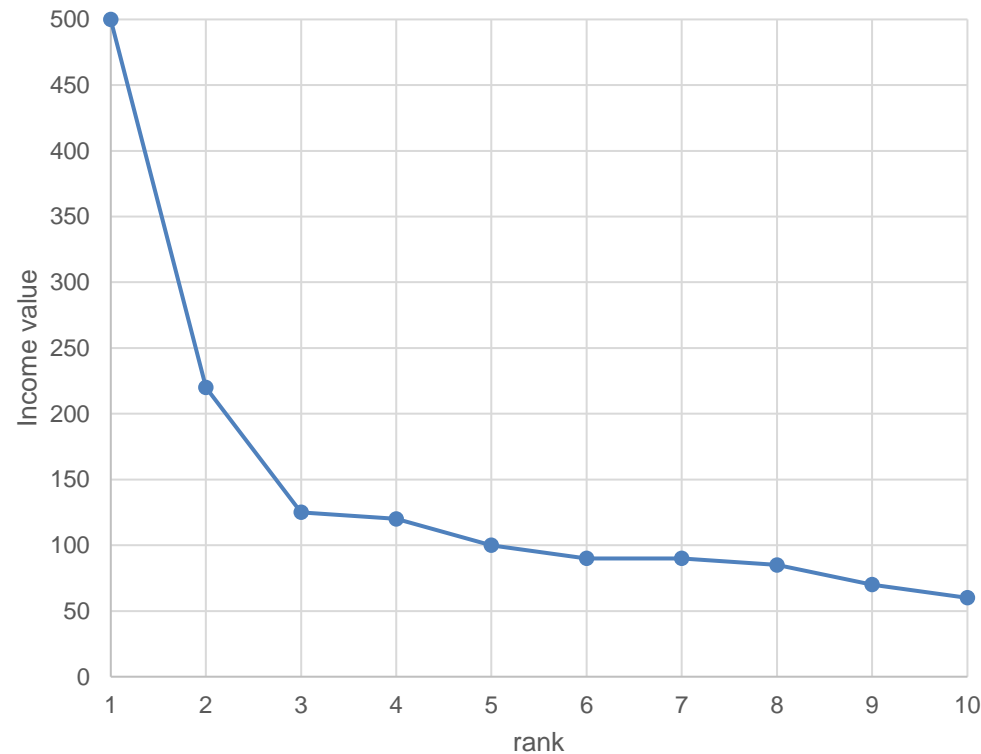
Plotting the fraction of entries that have value **less or equal to x**, for all possible values x of income in the data



Plotting the fraction of entries that have value **greater or equal to x**, for all possible values x of income in the data

Rank-Value plot

	Taxable Income
1	500K
2	220K
3	125K
4	120K
5	100K
6	90K
7	90K
8	85K
9	70K
10	60K



Plotting the values of the income (y-axis) against their rank (x-axis)

The rank of a value is its order when all values are sorted in decreasing order

Frequency-count plots

- In some cases, we have to put some more work
- Example: market-basked data

Id	Basket contents
1	milk, coffee
2	milk, coffee, sugar
3	milk, coffee, sugar, cookies
4	milk, tea, bread, butter, jam
5	milk, bread, butter, honey
6	milk, cream, honey, flour, eggs
7	milk, coffee, eggs, bacon
8	milk
9	milk, coffee, sugar, eggs, bacon, bread
10	eggs, bacon, bread

How do we describe this data?

Frequency-count plots

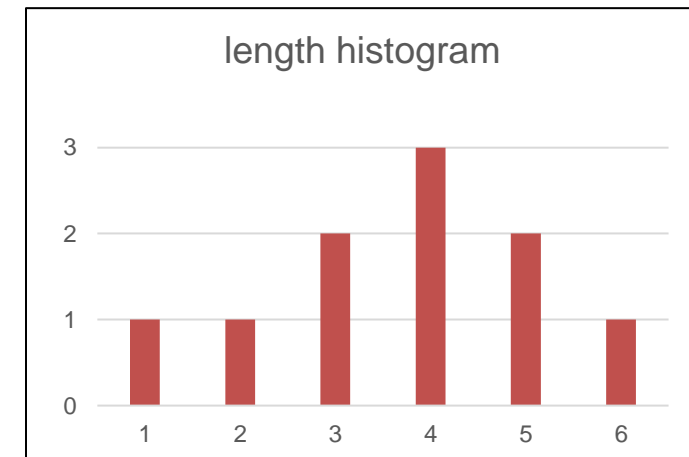
- Example: market-basked data

Id	Basket contents
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2	milk, coffee, sugar
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5	milk, bread, butter, honey
6	milk, cream, honey, flour, eggs
7	milk, coffee, eggs, bacon
8	milk
9	milk, coffee, sugar, eggs, bacon, bread
10	eggs, bacon, bread

Basket length

Id	length
1	2
2	3
3	4
4	5
5	4
6	5
7	4
8	1
9	6
10	3

length	count
1	1
2	1
3	2
4	3
5	2
6	1



Frequency-count plots

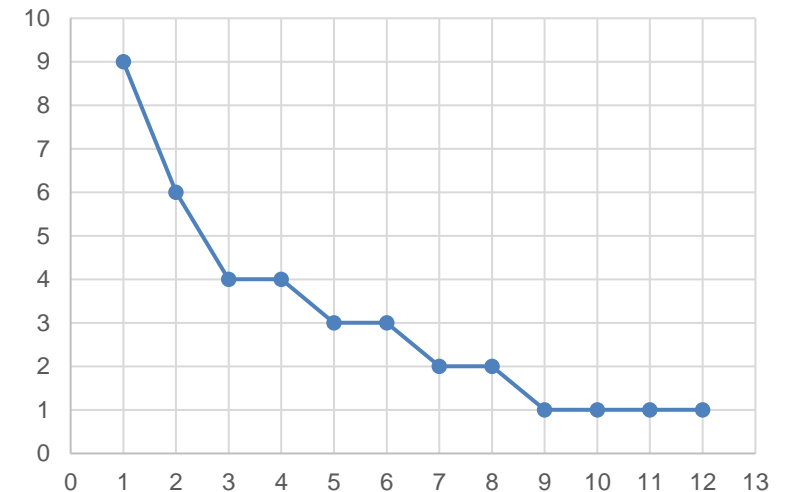
- Example: market-basked data

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7	milk, coffee, eggs, bacon
8	milk
9	milk, coffee, sugar, eggs, bacon, bread
10	eggs, bacon, bread

Item counts

Item	count
milk	9
coffee	6
eggs	4
bread	4
sugar	3
bacon	3
butter	2
honey	2
cookies	1
tea	1
jam	1
cream	1

value-rank plot



Frequency-count plots

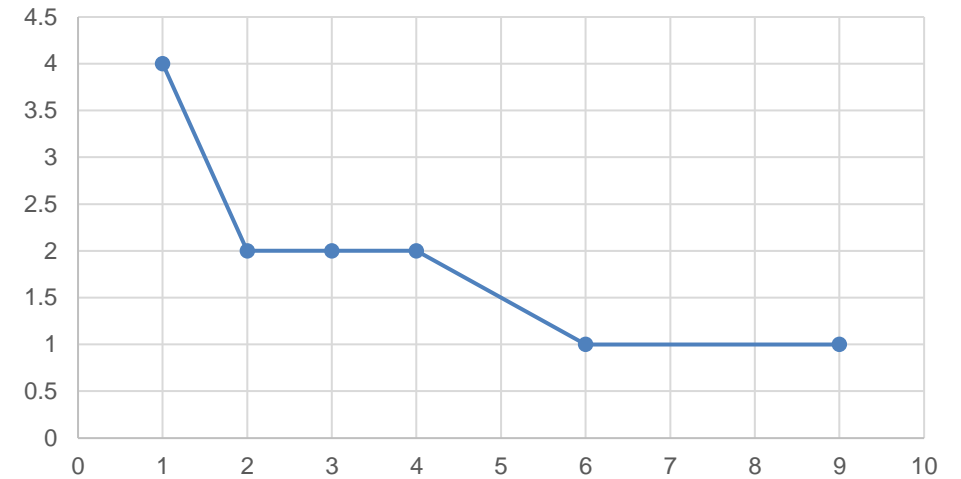
- Example: market-basked data

Item	count
milk	9
coffee	6
eggs	4
bread	4
sugar	3
bacon	3
butter	2
honey	2
cookies	1
tea	1
jam	1
cream	1

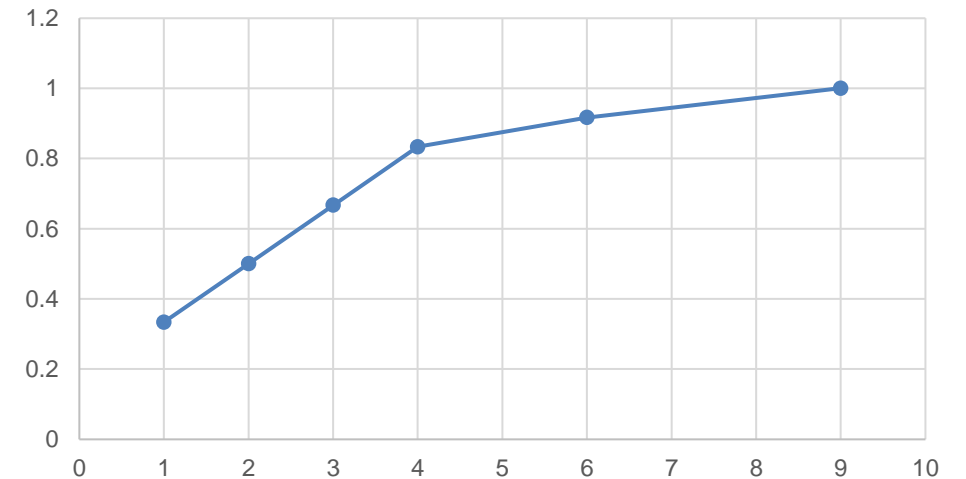
Count histogram

Count	Frequency
1	4
2	2
3	2
4	2
6	1
9	1

Count frequency



Cummulative Distribution



Measures of Location: Mean and Median

- The **mean** is the most common measure of the location of a set of points.

$$\text{mean}(x) = \bar{x} = \frac{1}{m} \sum_{i=1}^m x_i$$

- However, the mean is very sensitive to outliers.
- Thus, the **median** is also commonly used.

$$\text{median}(x) = \begin{cases} x_{(r+1)} & \text{if } m \text{ is odd, i.e., } m = 2r + 1 \\ \frac{1}{2}(x_{(r)} + x_{(r+1)}) & \text{if } m \text{ is even, i.e., } m = 2r \end{cases}$$

- Or the **trimmed mean**: the mean after removing min and max values

Example

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8	No	Single	85K	Yes
9	No	Married	90K	No
10	No	Single	90K	No

Mean: 1090K

Trimmed mean (remove min, max): 105K

Median: $(90+100)/2 = 95K$

Measures of Spread: Range and Variance

- **Range** is the difference between the **max** and **min**
- The **variance** or **standard deviation** is the most common measure of the spread of a set of points.

$$var(x) = \frac{1}{m-1} \sum_{i=1}^m (x - \bar{x})^2$$

$$\sigma(x) = \sqrt{var(x)}$$

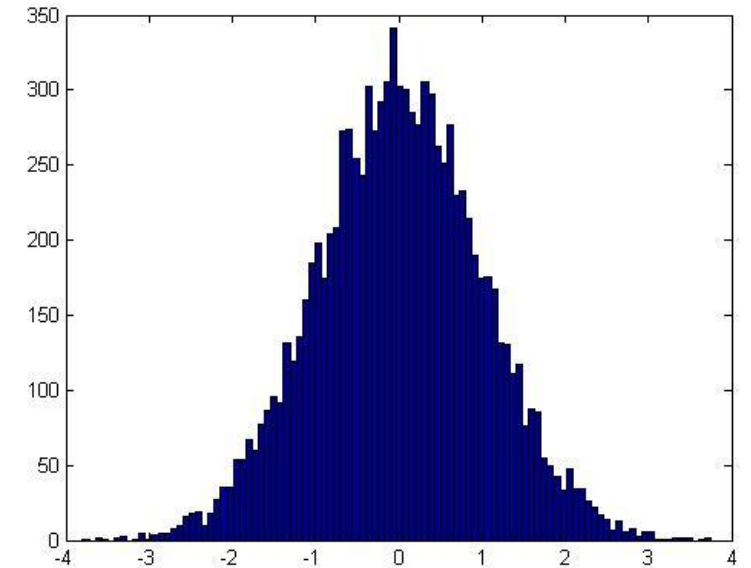
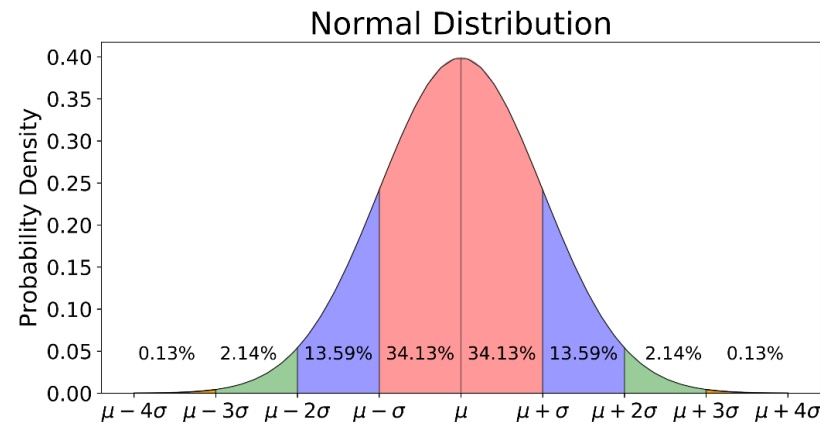
m or $m - 1$?

When computing the **sample variance** $m-1$ is used

For large data it does not make much difference

Normal Distribution

- $$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

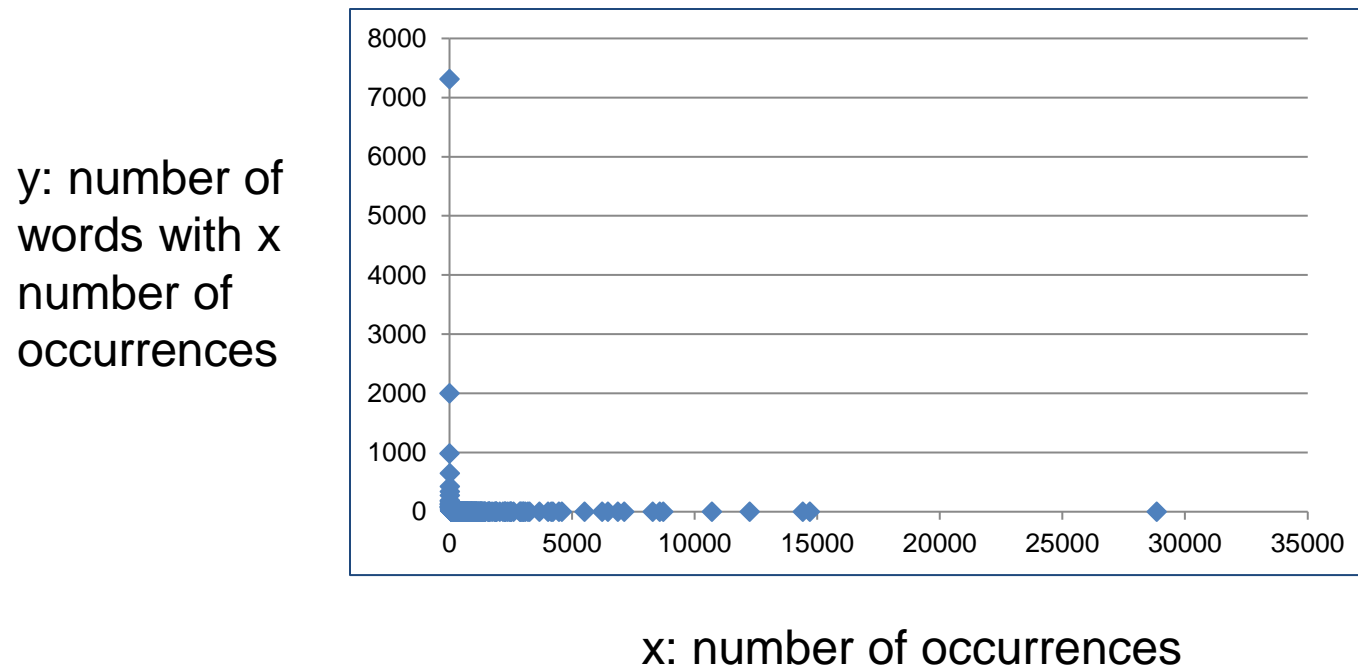


This is a value **histogram**

- An important distribution that characterizes many quantities and has a central role in probabilities and statistics.
- Appears also in the **central limit theorem**: the distribution of the sum of IID random variables.
- Fully characterized by the **mean** μ and standard **deviation** σ

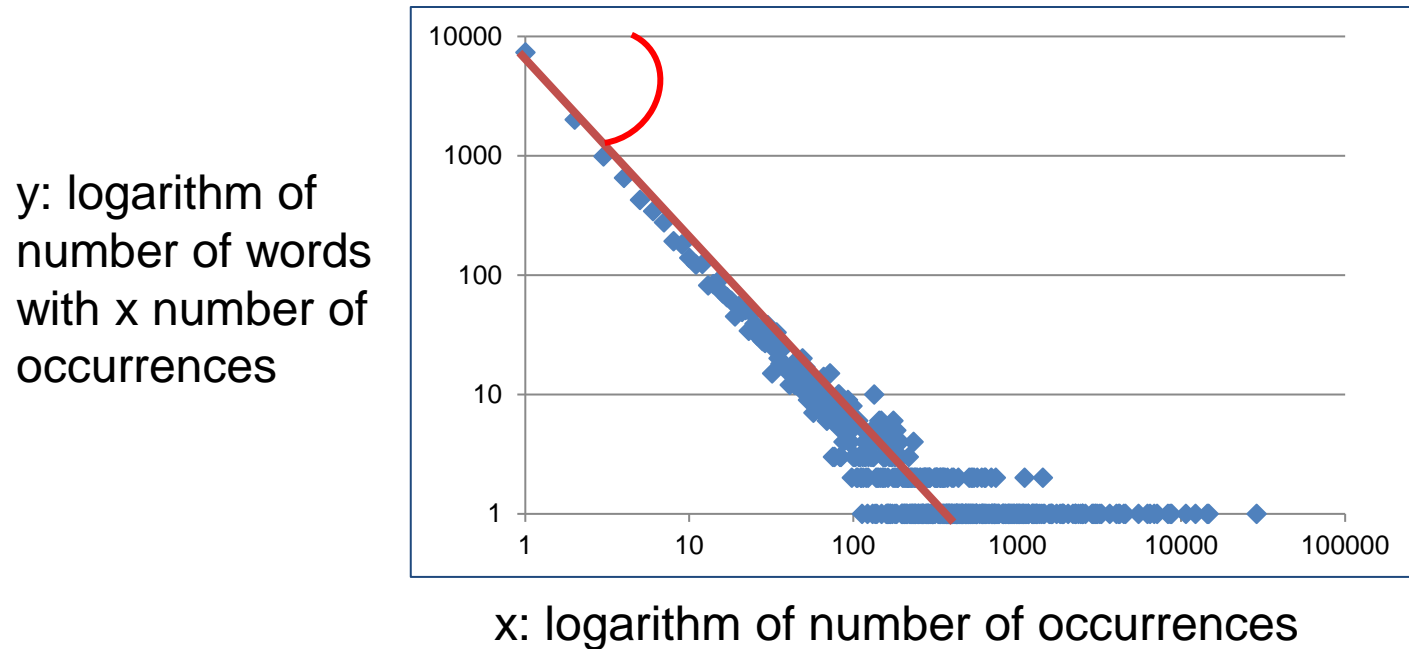
Not everything is normally distributed

- Plot of number of words with x number of occurrences



Power-law distribution

- We can understand the distribution of words if we take the **log-log** plot



Linear relationship in the log-log space

$$\log p(x = k) = -a \log k$$

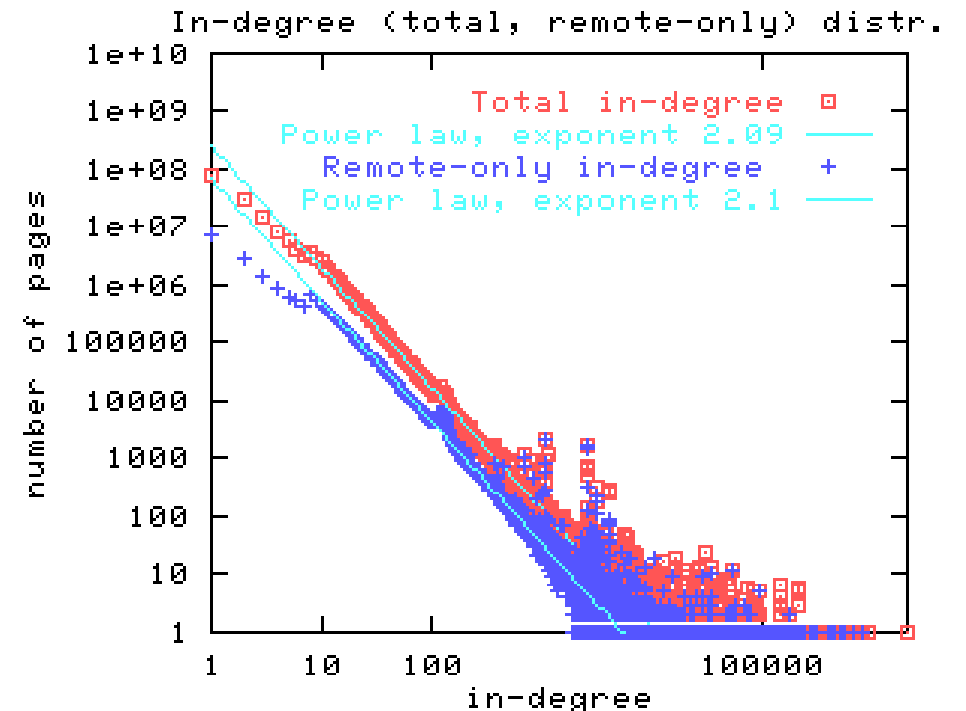
Power-law distribution:

$$p(k) = k^{-a}$$

The **slope** of the line gives us the exponent **α**

Power-laws are everywhere

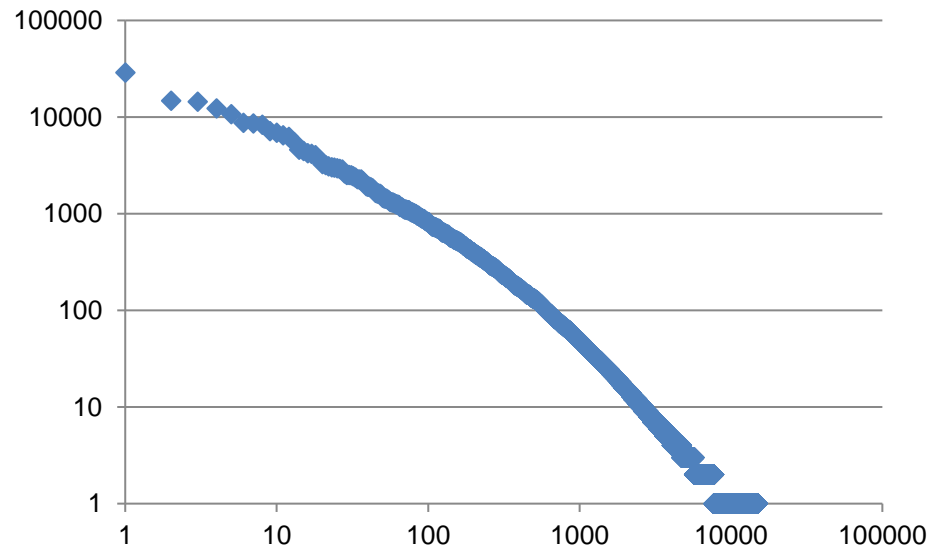
- Incoming and outgoing links of web pages, number of friends in social networks, number of occurrences of words, file sizes, city sizes, income distribution, popularity of products and movies
 - Signature of human activity?
 - A mechanism that explains everything?
 - Rich get richer process
- Related distribution: log-normal
 - Taking the log of the values gives a normal distribution



Zipf's law

- Power laws can be detected also by a linear relationship in the log-log space for the **rank-frequency** plot

y: number of occurrences of the r-th most frequent word



r: rank of word according to frequency (1st, 2nd ...)

Zipf distribution:

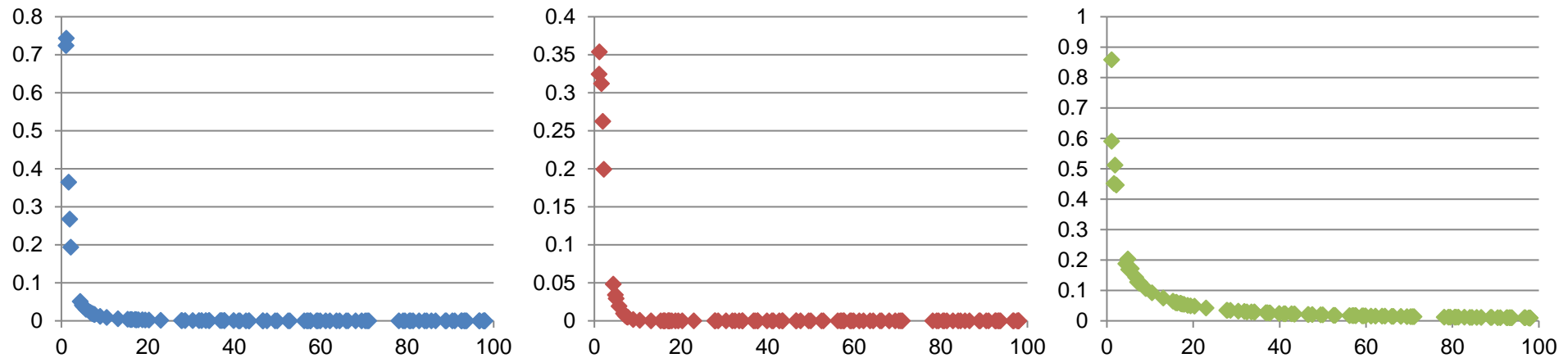
$$f(r) = r^{-\beta}$$

- $f(r)$: Frequency of the **r-th** most frequent word

$$\log f(r) = -\beta \log r$$

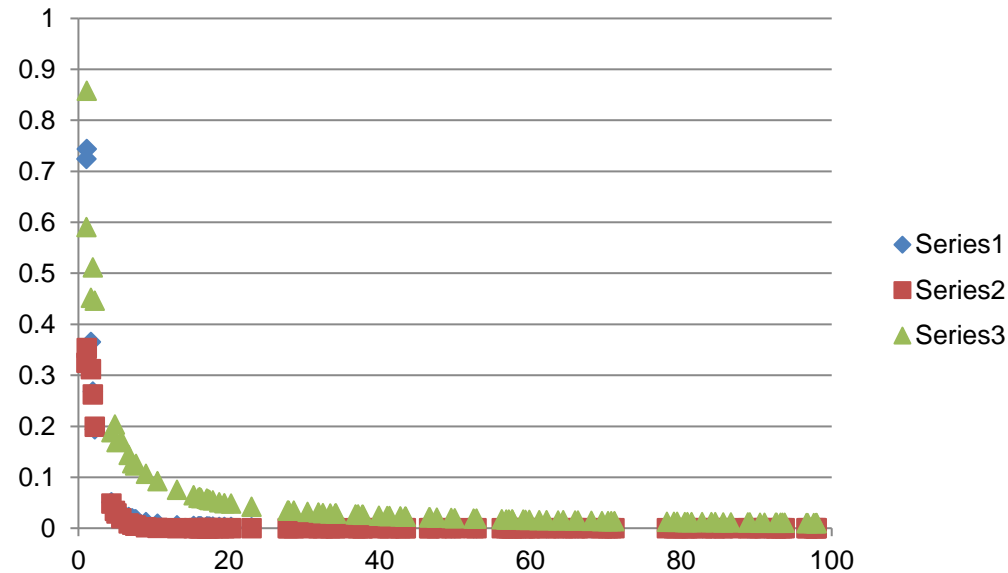
The importance of correct representation

- Consider the following three plots which are histograms of values. What do you observe? What can you tell of the underlying function?



The importance of correct representation

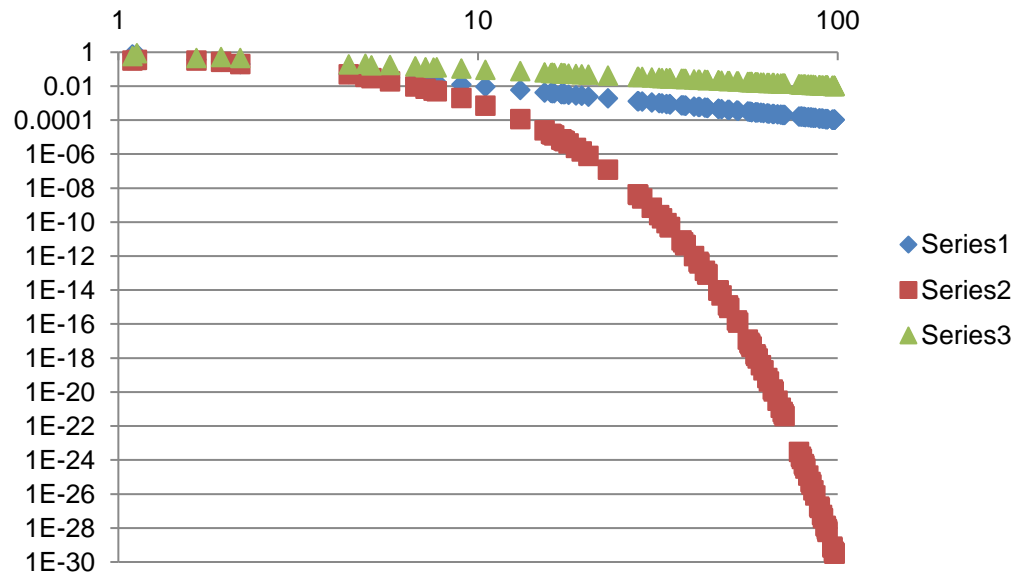
- Putting all three plots together makes it clearer to see the differences



- Green falls more slowly. Blue and Red seem more or less the same

The importance of correct representation

- Making the plot in log-log space makes the differences more clear



Linear relationship in log-log
means polynomial in linear-linear
The slope in the log-log is the
exponent of the polynomial

Exponential relationship remains
exponential in log-log

- Green and Blue form straight lines. Red drops exponentially.

- $y = \frac{1}{2x+\epsilon}$

$$\log y \approx -\log x + c$$

- $y = \frac{1}{x^2+\epsilon}$

$$\log y \approx -2 \log x + c$$

- $y = 2^{-x} + \epsilon$

$$\log y \approx -x + c = -10^{\log x} + c$$

Attribute relationships

- In many cases it is interesting to look at two attributes together to understand if they are **correlated**
 - E.g., how does your marital status relate with tax cheating?
 - E.g., Does refund correlate with average income?
 - Is there a relationship between years of study and income?
- How do we measure and visualize these relationships?

Correlating categorical attributes

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1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
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8	No	Single	85K	Yes
9	No	Married	90K	No
10	No	Single	90K	No

Confusion or Contingency Matrix

	No	Yes
Single	2	1
Married	4	0
Divorced	2	1

Correlating categorical attributes

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7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	90K	No
10	No	Single	90K	No

Joint Distribution Matrix

	No	Yes
Single	0.2	0.1
Married	0.4	0.0
Divorced	0.2	0.1

Confusion Matrix

	No	Yes
Single	2	1
Married	4	0
Divorced	2	1

	No	Yes
Single	0.2	0.1
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It can also be represented as a heatmap

Correlating categorical attributes

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Joint Distribution Matrix

	No	Yes	
Single	0.2	0.1	0.3
Married	0.4	0.0	0.4
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	0.8	0.2	1

Marginal
distribution
for Marital
Status

Marginal distribution
for Cheat

Correlating categorical attributes

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How do we know if there are interesting correlations?

Joint Distribution Matrix P

	No	Yes	
Single	0.2	0.1	0.3
Married	0.4	0.0	0.4
Divorced	0.2	0.1	0.3
	0.8	0.2	1

Independence Matrix E

	No	Yes	
Single	0.24	0.06	0.3
Married	0.32	0.08	0.4
Divorced	0.24	0.06	0.3
	0.8	0.2	1

Compare the values P_{xy} with E_{xy}

The product of the two marginal values $0.3 \cdot 0.8$

Correlating categorical attributes

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Independence Matrix E

	No	Yes	
Single	0.24	0.06	0.3
Married	0.32	0.08	0.4
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	0.8	0.2	1

We can compare specific pairs of values:

- If $P(x, y) > E(x, y)$ there is **positive correlation** (e.g, Married, No)
- If $P(x, y) < E(x, y)$ there is **negative correlation** (e.g., Single, No)
- Otherwise there is no correlation

The quantity $\frac{P(x,y)}{E(x,y)} = \frac{P(x,y)}{P(x)P(y)}$ is called **Lift**, or **Pointwise Mutual Information**

Correlating categorical attributes

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	No	Yes	
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Married	0.32	0.08	0.4
Divorced	0.24	0.06	0.3
	0.8	0.2	1

Or compare the two attributes:

Pearson χ^2 Independence Test Statistic:

$$U = N \sum_x \sum_y \frac{(P_{xy} - E_{xy})^2}{E_{xy}}$$

We want this to be large. But how large is large enough?

Hypothesis testing

- How important is the statistic value U that we computed?
- Formulate a **null hypothesis H_0** :
 - H_0 = the two attributes are independent
- Compute the distribution of the statistic U in the case that H_0 is true
 - In this case we can show that the statistic U follows a χ^2 distribution
- For the statistic value $U = \theta$ we observe in our data, compute the probability $P(U \geq \theta)$ **under the null hypothesis**
 - For most distributions there are tables that give these numbers for our data
- This is the **p-value** of our experiment:

The p-value is the probability under H_0 (independence) of observing a value of the test statistic the same as, or more extreme than the one that was actually observed

- We want it to be small (ideally ≤ 0.01 , ≤ 0.05 is good , ≤ 0.1 is ok)
 - This means that the observed value is interesting and we can **reject the null hypothesis**

Hypothesis Testing and p-values – A simple example

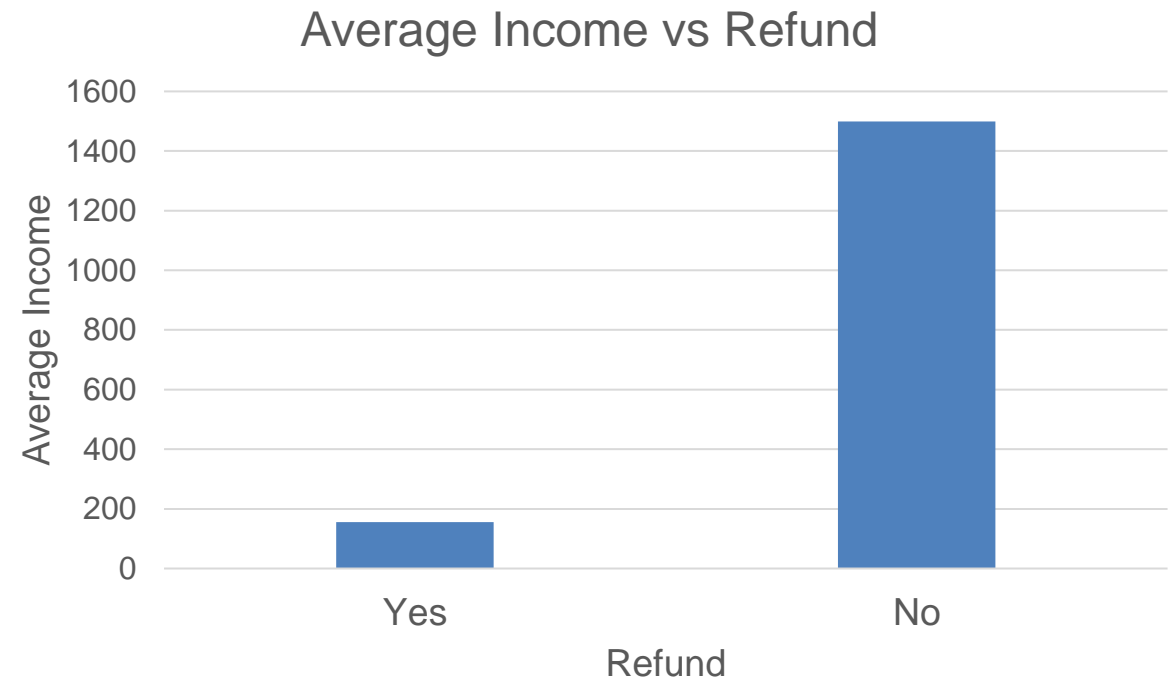
- A coin is tossed 20 times, and we get 16 heads.
- Hypothesis H_1 = “The coin is not fair”
- Null Hypothesis H_0 = “The coin is fair” (probability 50% for head)
- p-value: What is the probability of getting a number of heads that is the same or more extreme than 16?
 - One-sided p-value: $\Pr(H \geq 16) = 0.0059$
 - Two-sided p-value: $\Pr(H \geq 16) + \Pr(H \leq 4) = 0.0118$
- With significance level $\alpha = 0.05$ we can conclude that we can reject the null hypothesis

P-values

- The p-value tells us the probability that the value we observe could appear in data generated under the null hypothesis.
 - The null hypothesis proposes a (random) model for the data generation
 - The p-value answers the question: “If the null hypothesis model was correct how likely would it be to observe the value we observe”?
- Be careful!
 - A p-value ϕ does not mean that the null hypothesis is correct with probability ϕ
 - A high p-value (e.g., 90%) does not mean that the null hypothesis is true, it only means that the data is consistent with the model of the null hypothesis
 - A p-value ϕ does not mean that our hypothesis is correct with probability $1 - \phi$
 - A p-value of 3% does not mean that our hypothesis is correct with probability 97%
 - It only means that the data is not consistent with the null hypothesis random model

Categorical and numerical attributes

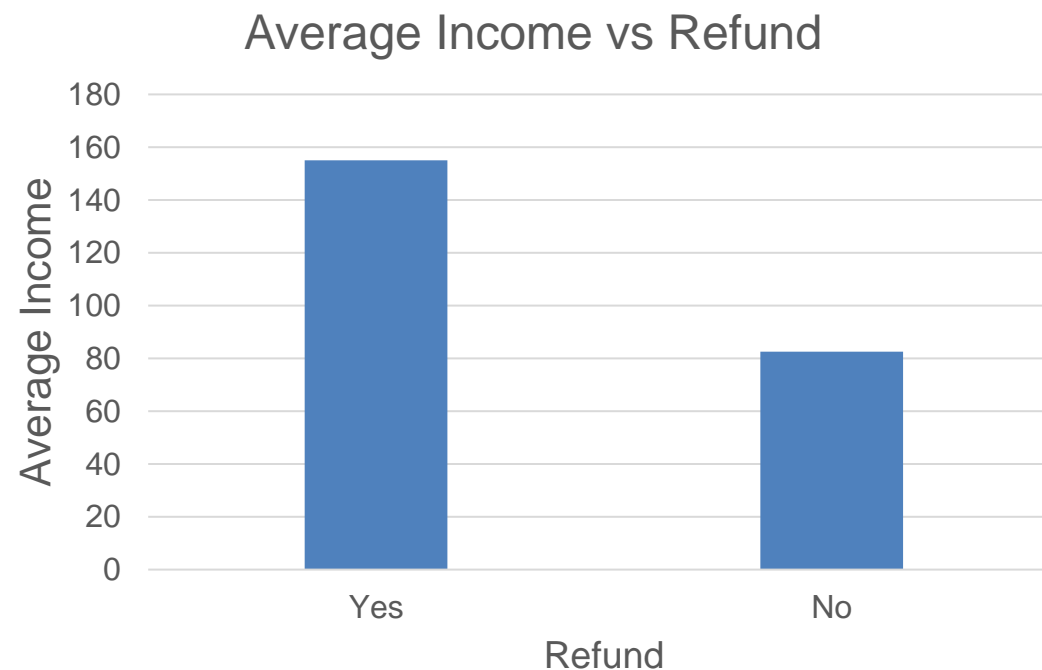
<i>Tid</i>	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	10000K	Yes
6	No	NULL	60K	No
7	Yes	Divorced	220K	NULL
8	No	Single	85K	Yes
9	No	Married	90K	No
10	No	Single	90K	No



Categorical and numerical attributes

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1	Yes	Single	125K	No
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7	Yes	Divorced	220K	NULL
8	No	Single	85K	Yes
9	No	Married	90K	No
10	No	Single	90K	No

After removing the outlier value

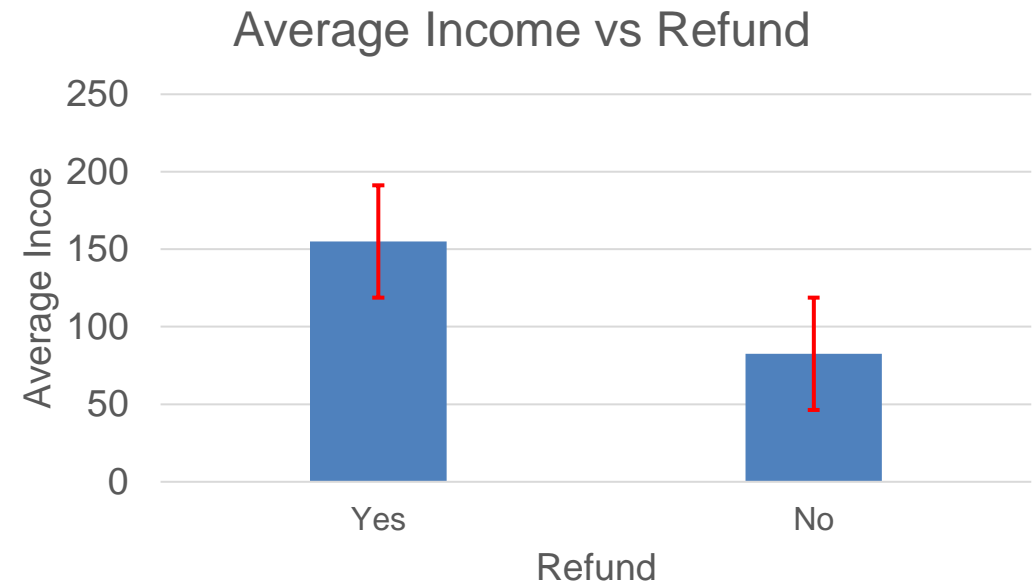


How informative are the means?

Categorical and numerical attributes

Compute **error bars**

<i>Tid</i>	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
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6	No	NULL	60K	No
7	Yes	Divorced	220K	NULL
8	No	Single	85K	Yes
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10	No	Single	90K	No



Error bars give a measure of the variability of the mean

Error bars

- Error bars may be:

- The range
- The standard deviation
- The standard error
- The 95% confidence interval



Descriptive error bars: They tell us something about the underlying distribution of the data



Inferential error bars: They tell us something about the quality of the estimation of the mean

- Inferential error bars get more informative the more data we collect.

- We should always specify what the error bars mean in a plot.

Standard Error (of the Mean)

- The **Standard Error (SE)** is usually defined for the mean of a sample of values X (it is also known as SEM – Standard Error of the Mean) and it is a measure of **the deviation of the sample mean from the true mean**.
- It is defined as:

$$se = \frac{\hat{\sigma}(X)}{\sqrt{n}}$$

where $\hat{\sigma}(X)$ = empirical standard deviation.

- As the sample size grows the SE is reduced (we have a better estimation of the mean)
- Computation follows from the fact that

$$se = \hat{\sigma}(\hat{\mu}), \hat{\mu} = \frac{1}{n} \sum_i X_i$$

- We assume that X_i are **independent samples** of the random variable X that come from the **same distribution**. We use the fact that:

$$Var\left(\sum_i \alpha_i X_i\right) = \sum_i \alpha_i^2 Var(X_i)$$

Confidence interval

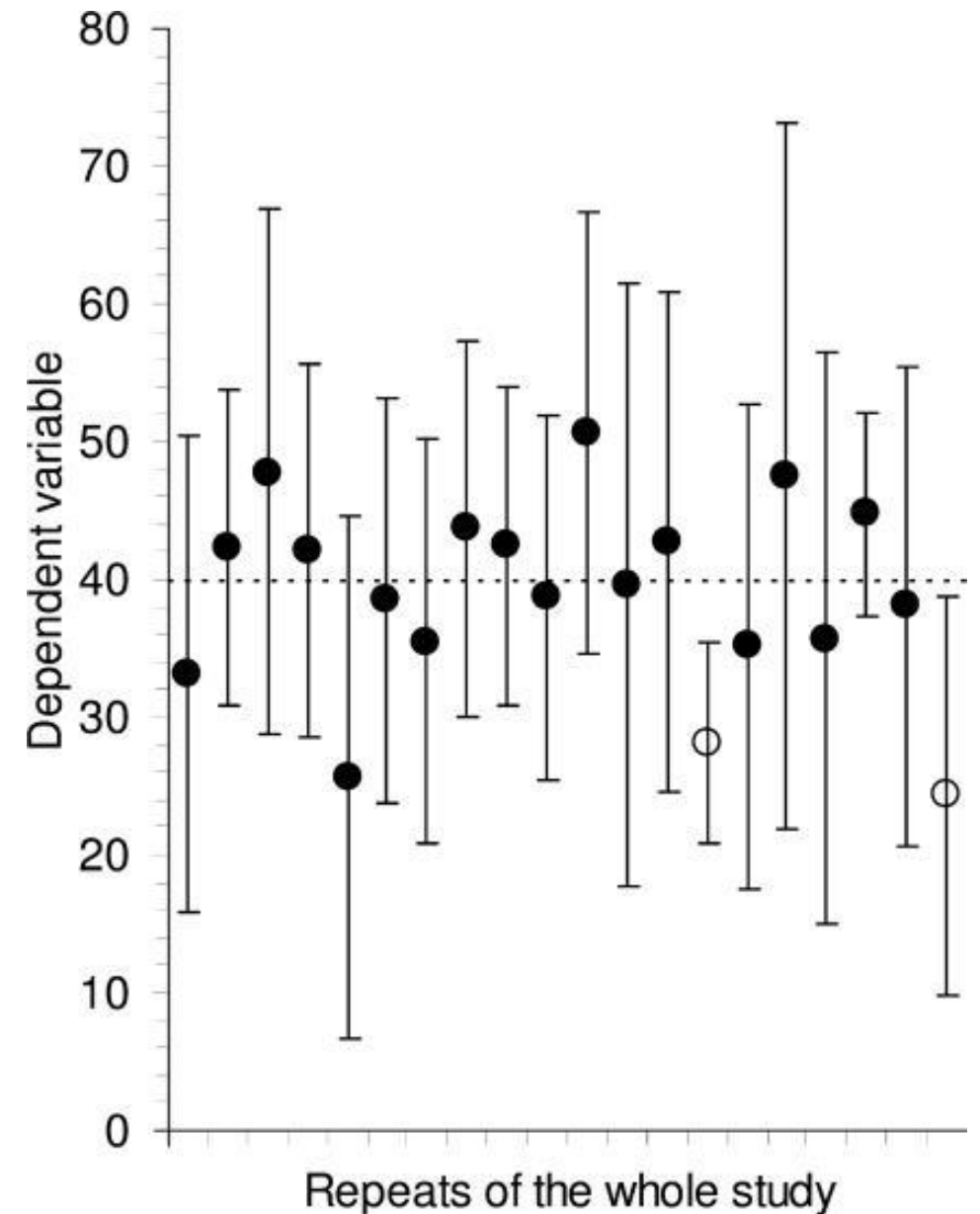
- We want to estimate the average income μ which is a fixed value.
- We have a sample of the population and the measurements $\{X_i\}$ of incomes and we estimate the average income as:

$$\hat{\mu} = \frac{1}{n} \sum_i X_i$$

- The **p -confidence interval** of the value μ is an interval of values C_n such that
$$P(\mu \in C_n) \geq p$$
 - We usually ask for the **95% confidence interval**
- **Important:** The probability is taken over the many different samples of the population
 - Different samples will generate different confidence intervals
 - There is a 95% chance that each of these intervals contains the true mean μ
 - It is incorrect to say that this is the probability that μ belongs to the interval
- The value $\hat{\mu}$ follows a **normal distribution** for large n . For normal distributions the 95% confidence interval (for large enough n) is:
$$(\hat{\mu} - 2se, \hat{\mu} + 2se)$$

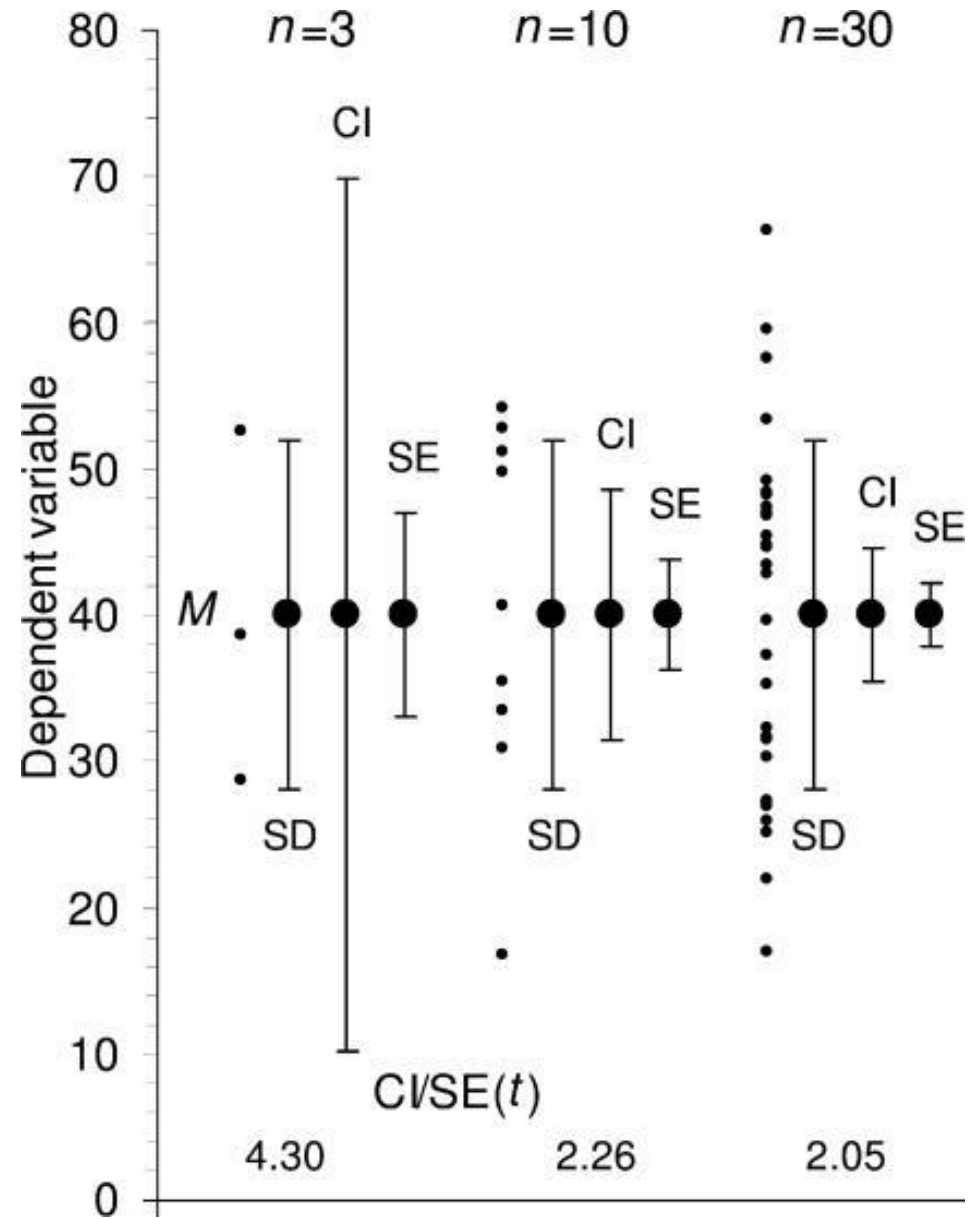
Example

- If we obtain an estimate of the mean for 20 different population samples, we will obtain 20 different 95%-confidence intervals.
- We expect that 1/20 of these intervals will not contain the true mean (the dotted line)



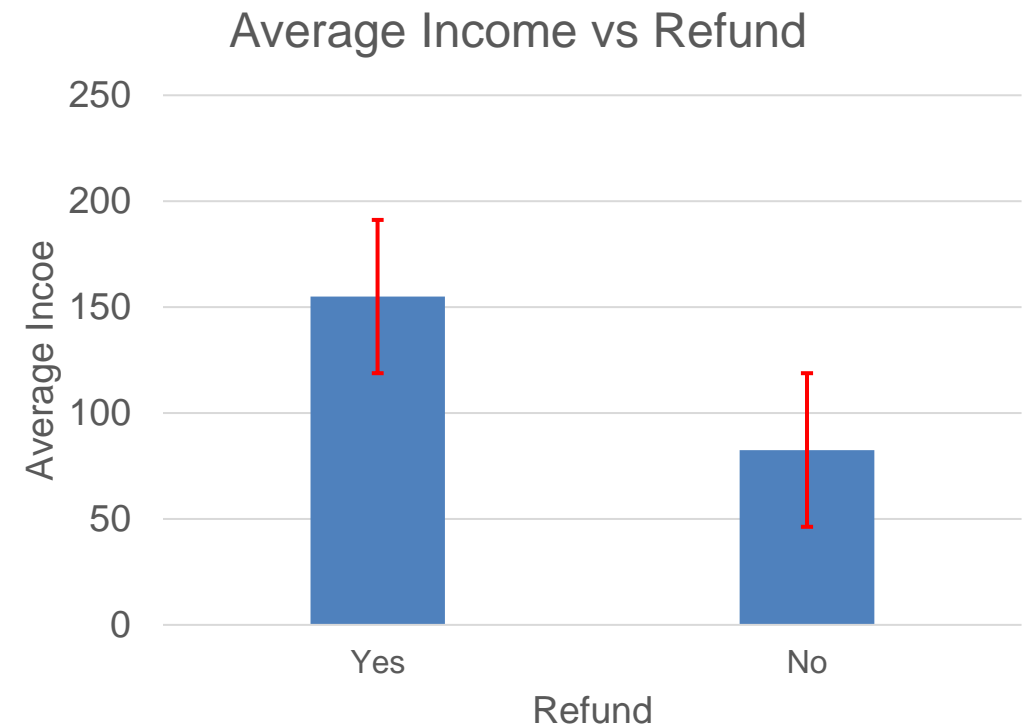
Error bars example

- The different error bars and how they change as the sample size increases
- Out of the four different error bars, the confidence interval is probably the most informative.



Statistical significance

- Given the means of two populations an important question is whether the difference we observe is **statistically significant**
- Statistical significance is estimated by computing a **p-value** with respect to a **null hypothesis**
- The value is compared to a **significance level α** which is usually set to 0.05 (or 0.01)



Statistical significance via error bar overlap

- It is **not always** safe to declare that there is statistical significance when error bars do not overlap
 - We may have statistically significant differences when there is overlap, or no statistical significance when there is no overlap
- We can say that there is statistically significant difference of means when sample sizes are comparable and the 95%-confidence intervals do not overlap
- There are a little more complex rules for the standard error.

Statistical tests

- Statistical tests measure specific values and determine their statistical significance
 - For example measure the importance of the difference between the means (e.g., average grade) of two populations (e.g., students in cities vs students in rural areas).
- The magnitude of the value that is measured is also called the **effect size**
- The statistical significance of this value is measured with respect to a **null hypothesis**
 - For example: the difference of the means is zero
- The statistical test assumes a random **model** for the underlying data
 - For example, the data are generated by a Gaussian distribution
- The statistical test produces a **p-value** for the statistical significance of the values we observe

Statistical tests – The Student t-test

- The **Student t-test** tests if the difference of the means of two samples is “big enough”

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_X^2}{N_X} + \frac{\sigma_Y^2}{N_Y}}}$$

- Large **t-value** (**effect size**):
 - Large difference between the means
 - Small variance in the samples (more accurate measurements)
 - Large sample sizes (more reliable)

Statistical tests – The Student t-test

- The Student t-test produces a **p-value**: Measures the probability of the null hypothesis that the two distributions have zero difference in mean
 - This is what we care about, the t-value is usually not looked at
- Student t-test assumptions:
 - (near) Gaussian distribution of the data,
 - (near) same variance,
 - similar sample sizes.
- There is **paired** and **unpaired** Student t-test
 - Example of paired: behavior before and after a treatment.

Statistical tests – The KS-test

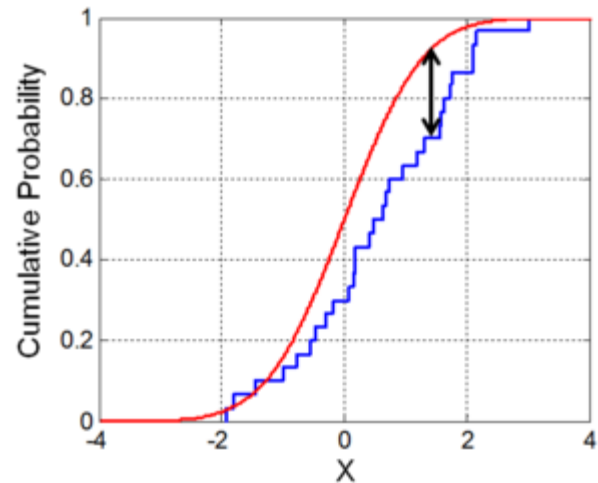
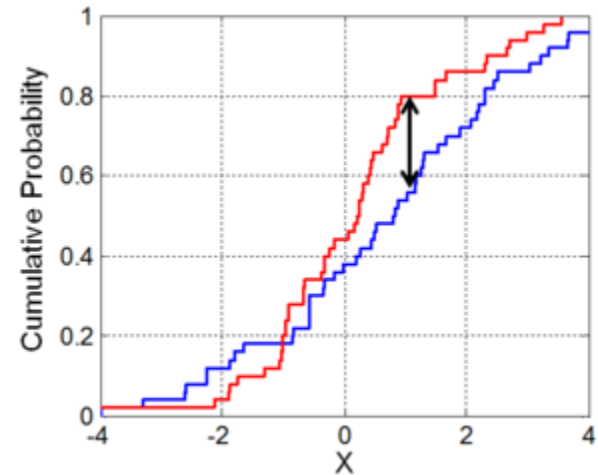
- The **Kolmogorov-Smirnov** (KS) test, tests if two samples come from the same distribution (or come from a specific distribution)
 - Take the **cumulative distribution function** (CDF) of the two distributions
 - Compute:

$$D(C_1, C_2) = \max_x |C_1(x) - C_2(x)|$$

- We can reject the null hypothesis if:

$$D(C_1, C_2) > c(\alpha) \sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$

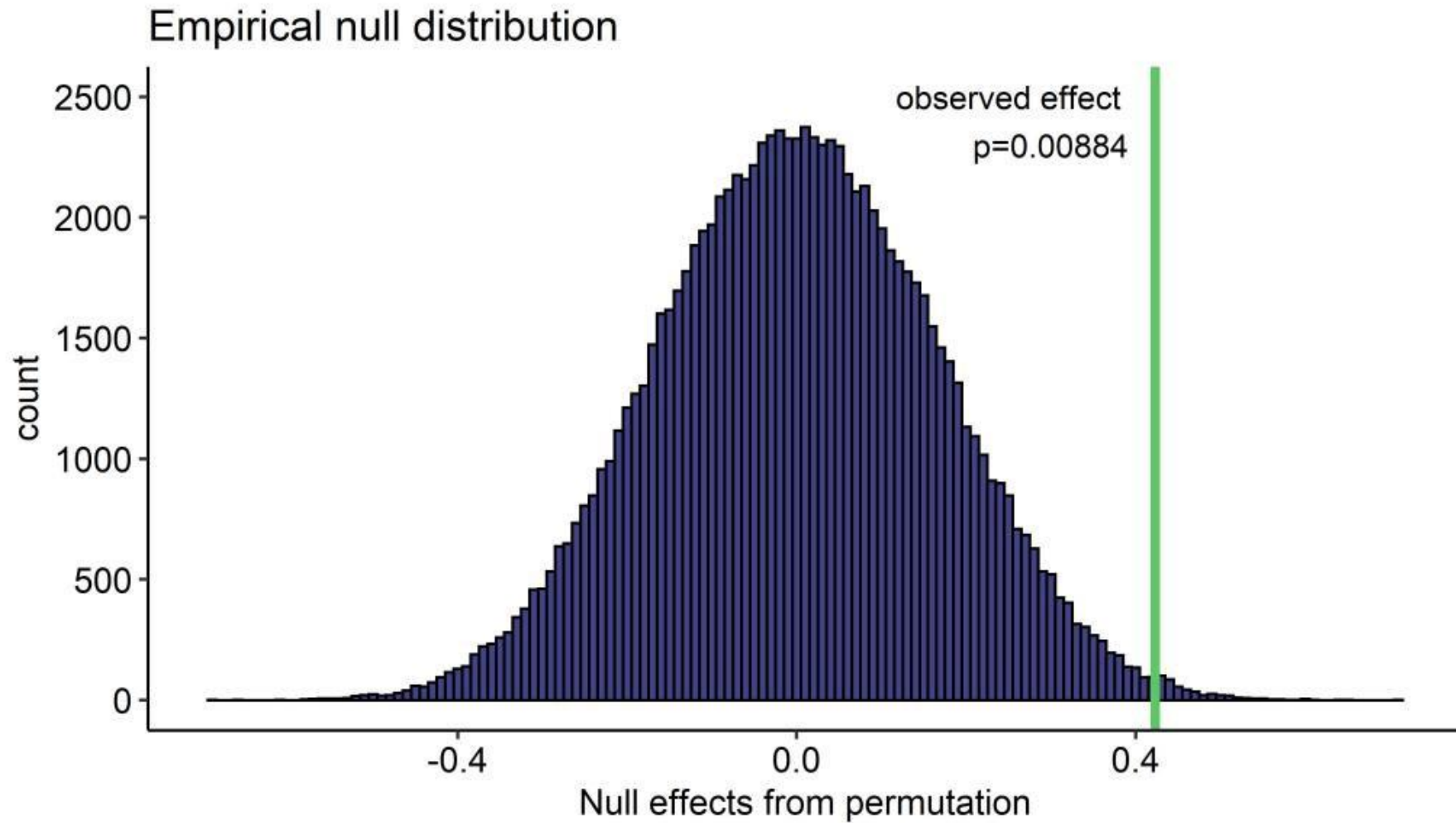
- α is the **confidence level**, $c(\alpha)$ is given by some tables



Statistical tests – Permutation testing

- Most tests make some assumption about the underlying distribution of the data.
- A **non-parametric statistical test** is the **permutation test**
- Create random instances of the data by randomly permuting values
 - E.g., permute the Cheat labels randomly
- Compute a statistic of interest for the permuted data
 - E.g., the average income of the cheaters
- Repeat this several times (at least 1000)
- Compute the **empirical p-value**: the fraction of permutations where we have a value that is equal or more extreme than the one observed.

Example

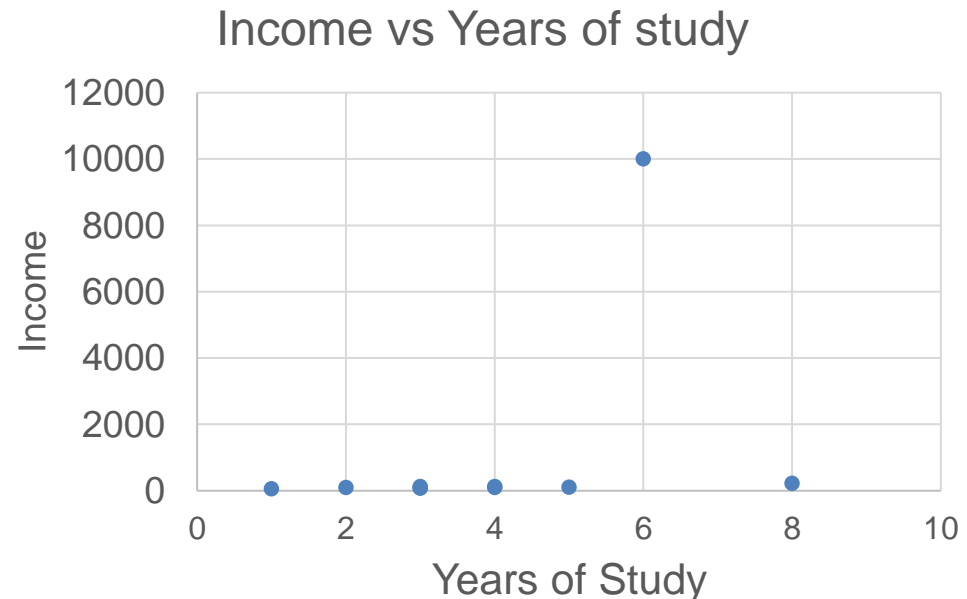


Correlating numerical attributes

<i>Tid</i>	Refund	Marital Status	Taxable Income	Years of Study
1	Yes	Single	125K	4
2	No	Married	100K	5
3	No	Single	70K	3
4	Yes	Married	120K	3
5	No	Divorced	10000K	6
6	No	NULL	60K	1
7	Yes	Divorced	220K	8
8	No	Single	85K	3
9	No	Married	90K	2
10	No	Single	90K	4

Scatter plot:

X axis is one attribute, Y axis is the other
For each entry we have two values
Plot the entries as two-dimensional points



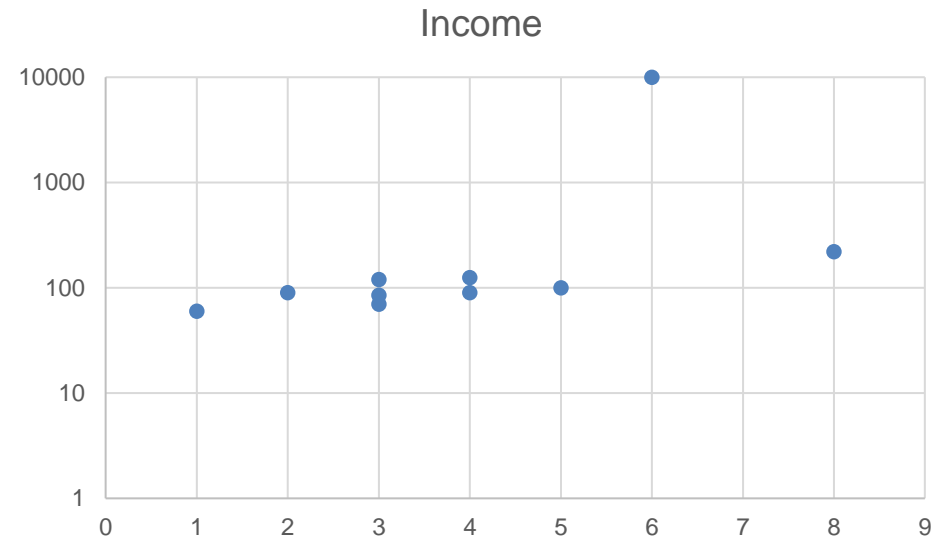
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Scatter plot:

X axis is one attribute, Y axis is the other
For each entry we have two values
Plot the entries as two-dimensional points

Log-scale in y-axis makes the plot look a little better



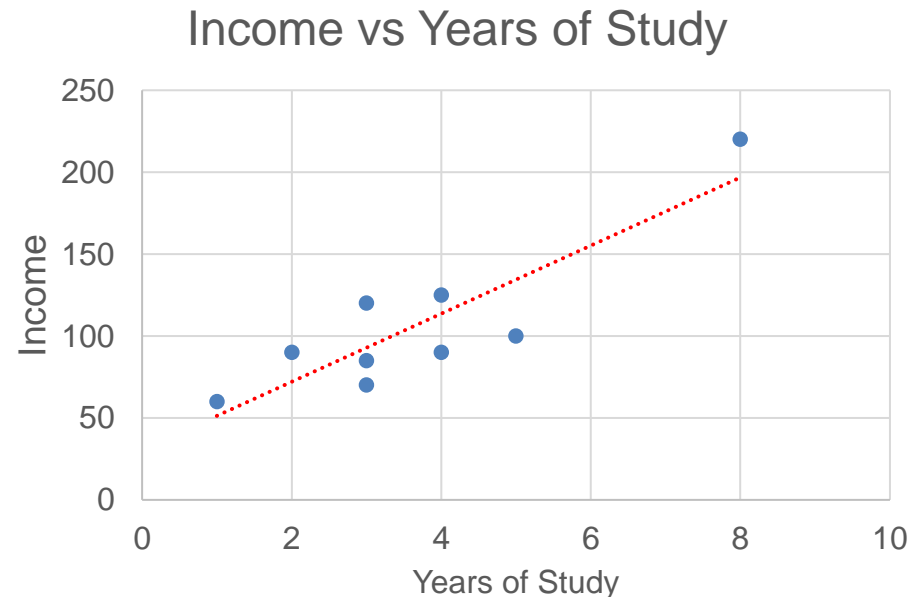
Plotting attributes against each other

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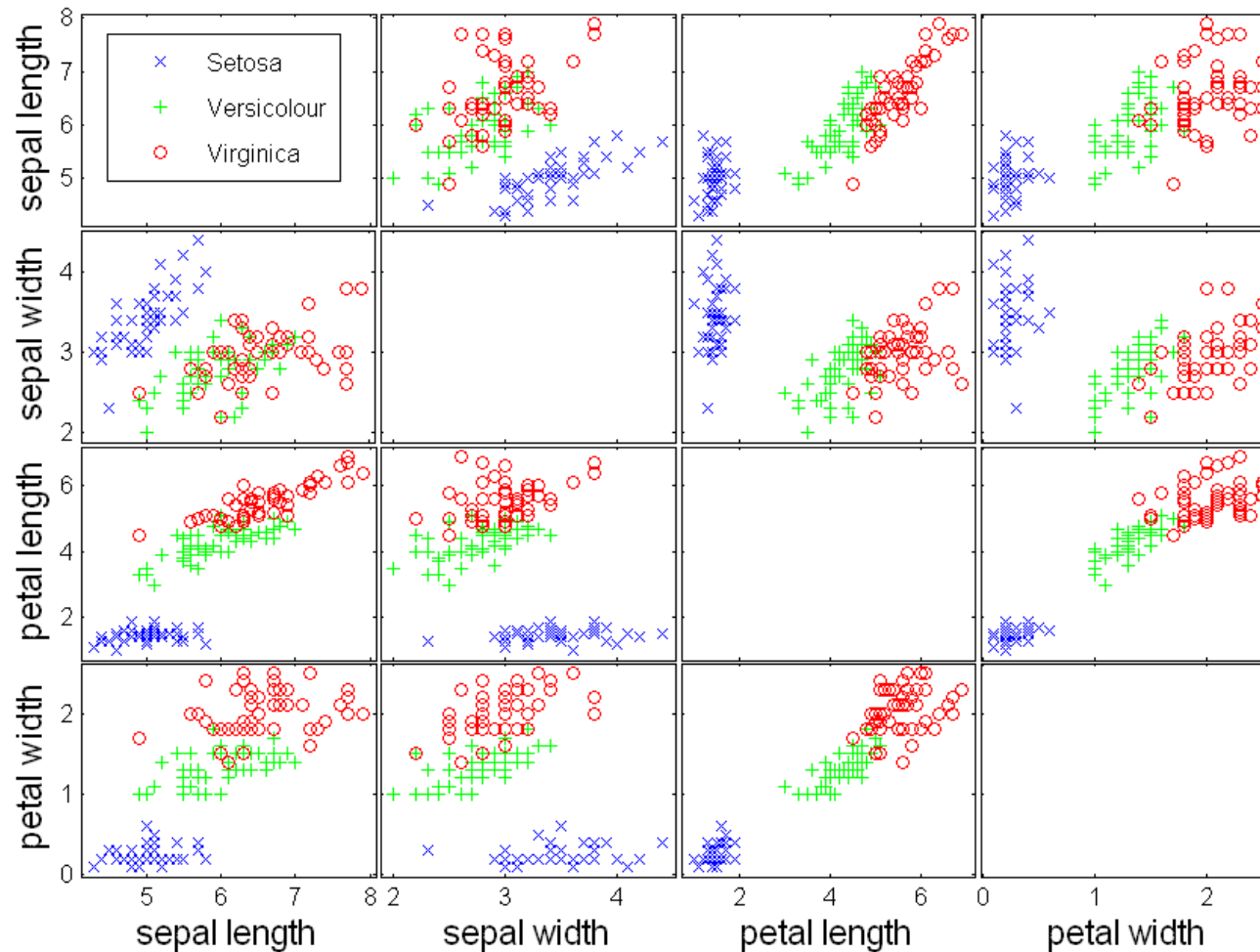
Scatter plot:

X axis is one attribute, Y axis is the other
For each entry we have two values
Plot the entries as two-dimensional points

After removing the outlier value there is a clear correlation



Scatter Plot Array of Iris Attributes



Measuring correlation

- **Pearson correlation coefficient**: measures the extent to which two variables are **linearly correlated**

- $X = \{x_1, \dots, x_n\}$

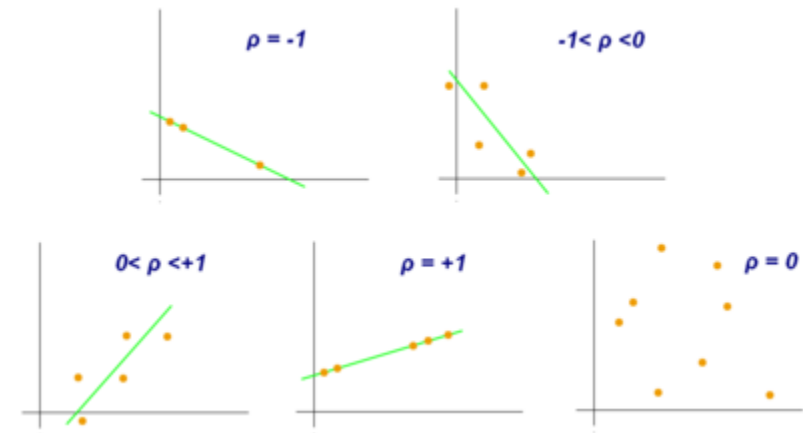
- $Y = \{y_1, \dots, y_n\}$

Must have **pairs** of observations

- $$\text{corr}(X, Y) = \frac{\sum_i (x_i - \mu_X)(y_i - \mu_Y)}{\sqrt{\sum_i (x_i - \mu_X)^2} \sqrt{\sum_i (y_i - \mu_Y)^2}}$$

- It comes with a **p-value**

- The p-value is the probability that the correlation was by chance.



Pearson correlation

- Assumptions:
 - Variables are normally distributed
 - No outliers
 - A linear relationship between the variables
- Caveats
 - For large samples p-values will always be small
 - Except for the p-value we need to also look at the **effect size**: the value of $r = \text{corr}(X, Y)$
- Interpretation
 - The value of r^2 measures the fraction of variance in one variable that is explained by the values of the other variable (**shared variance**)

Rank correlation

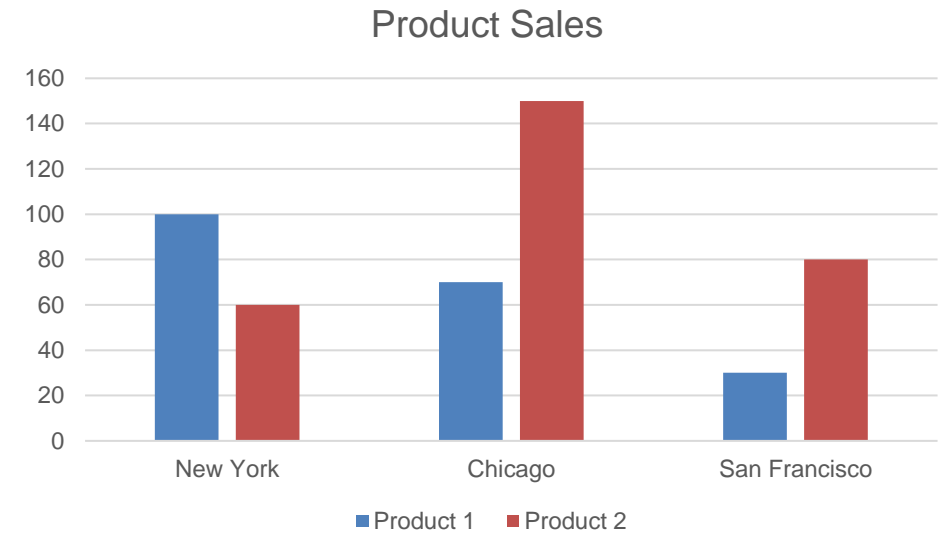
- **Spearman rank correlation coefficient**: tells us if two variable are rank-correlated
 - They place items in the same order – Pearson correlation of the rank vectors
 - For ranking without ties it looks at the differences between the ranks of the same items
- Spearman coefficient also comes with a p-value
- Spearman coefficient does not assume a linear relationship, but a **monotonic** one

Statistical significance vs Scientific significance

- Statistics place a lot of emphasis on the **p-values** and the statistical significance
- However, p-values may be small but the finding to not be of **scientific interest**
 - A difference or a correlation may be statistically significant, but too small to be of scientific interest
 - We need to evaluate the results beyond simply looking at the p-values.
 - We also need to look at the **effect size**, or the impact of the computed difference.

Plotting attributes together

City	Product 1	Product 2
New York	100	60
Chicago	70	150
San Francisco	30	80



How would you visualize the differences between the product sales per city?

Plotting attributes together

Year	Product 1	Product 2
2011	100	200
2012	200	250
2013	180	300
2014	300	350
2015	500	490
2016	600	500
2017	650	550
2018	640	540
2019	700	500
2020	200	100

How would you visualize the differences between the product sales over time?

