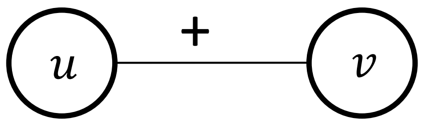
Online Social Networks and Media

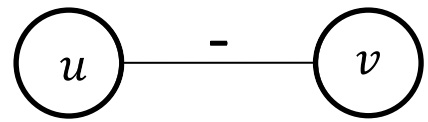
Signed Graphs

Positive and Negative Ties

Signed networks



Positive
friendship



Negative
foe

Examples

- Sign expresses agreement/disagreement between users
- Trust/distrust relationships
- Edit conflicts in Wikipedia
- Relationships between countries
- Synonyms and antonym relation between words

Structural Balance Theory

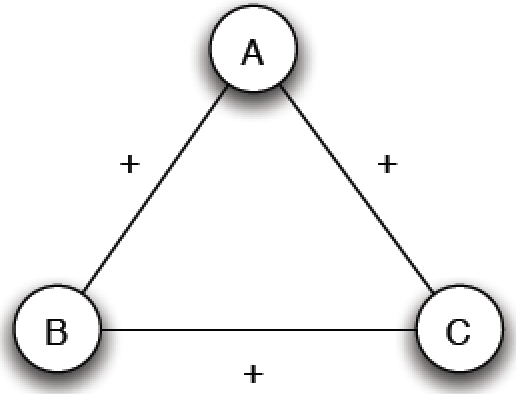
- originated in social psychology, by Heider in the 1940s
- graph-theoretic approach by Cartwright and Harary in the 1960s

Considers the possible ways in which triangles on three individuals can be signed

- All possible relationships *between 3 people* => 4 cases

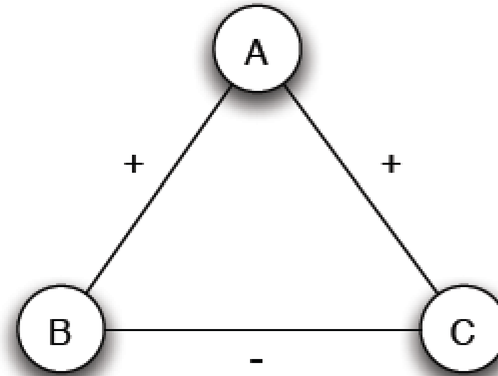
Structural Balance

3 +



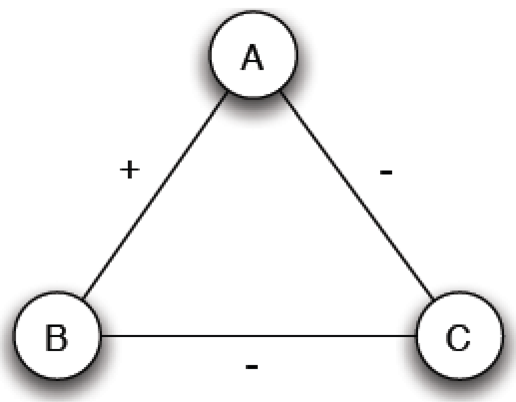
Mutual friends

2 +, 1 -



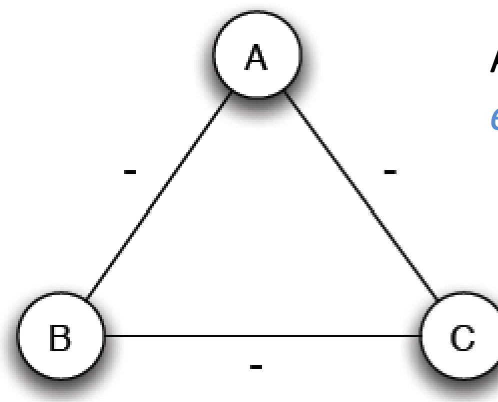
A is friend with B and C, but B and C do not get well together

1 +, 2 -



A and B are friends with a mutual enemy

3 -



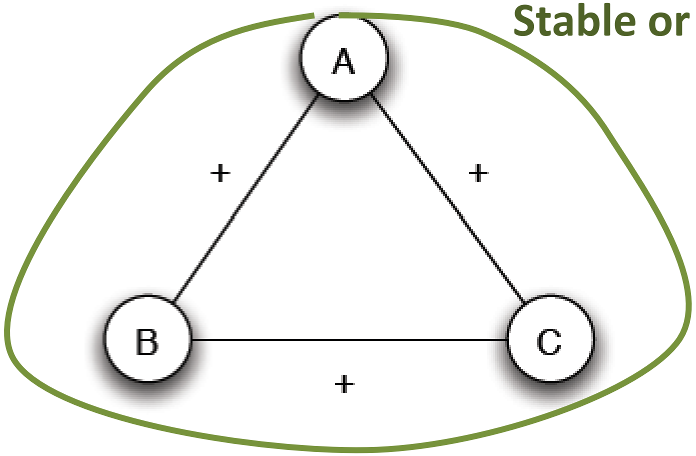
Mutual enemies

Are there all *equally possible?*

Structural Balance

3 +

Stable or balanced

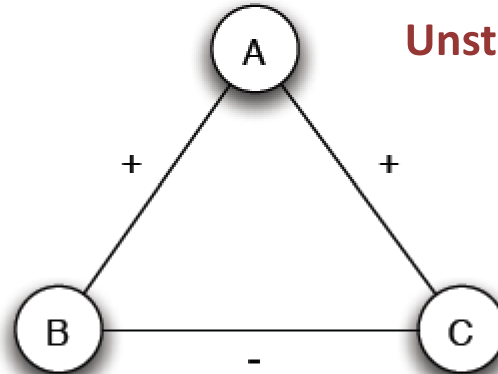


Mutual friends

"the friend of my friend is my friend"

2 +, 1 -

Unstable

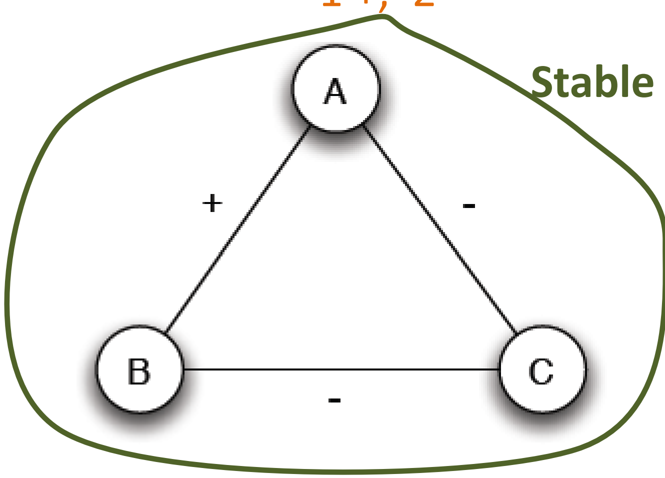


A is friend with B and C, but B and C do not get well together

Implicit force to make B and C friends (- => +) or turn one of the + to -

1 +, 2 -

Stable or balanced

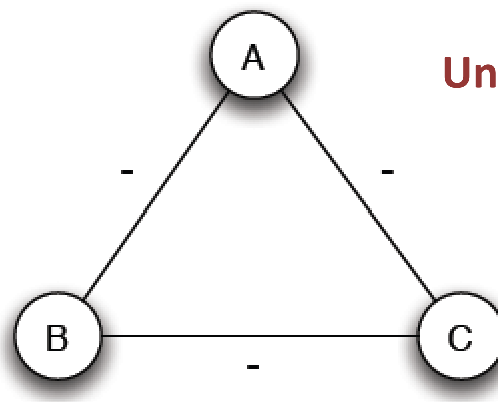


A and B are friends with a mutual enemy

"the enemy of my enemy is my friend"

3 -

Unstable



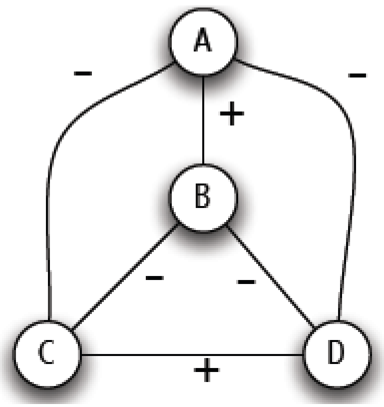
Mutual enemies

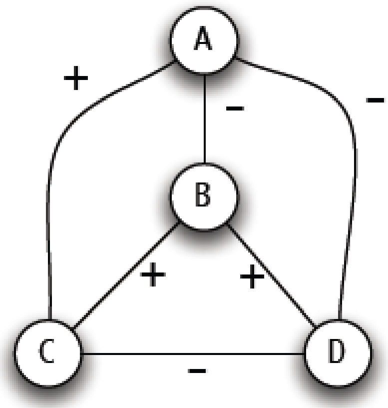
Forces to team up against the third (turn one of the – to +)

Structural Balance

A labeled complete graph is balanced if every one of its triangles is balanced

Structural Balance Property: For every set of three nodes, if we consider the three edges connecting them, either all three of these are labeled +, or else exactly one of them is labeled +, aka odd number of +





local property, individual triangles

What does a balanced network look like? (global property)

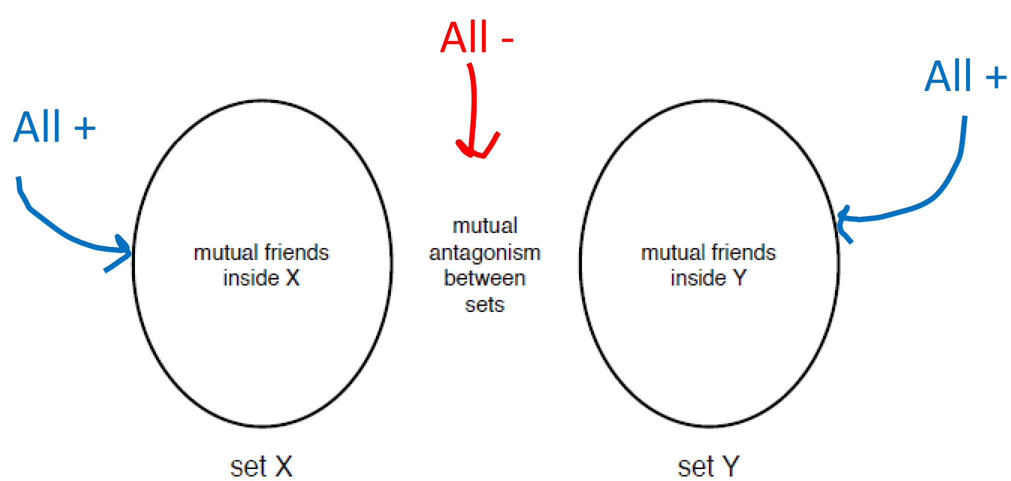
The Structure of Balanced Networks

Balance Theorem: If a labeled *complete* graph is balanced,

(a) all pairs of nodes are friends, or

(b) the nodes can be divided *into two groups* X and Y, such that every pair of nodes in X like each other, every pair of nodes in Y like each other, and everyone in X is the enemy of everyone in Y.

From a local to a global property



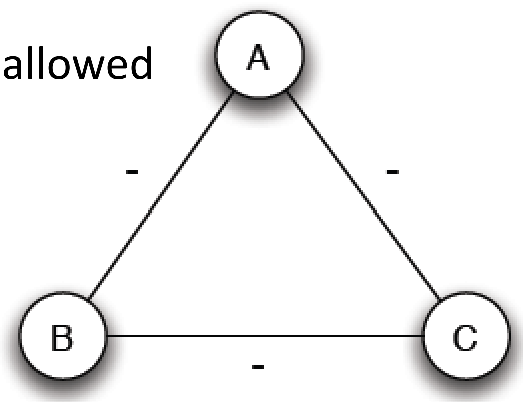
Applications of Structural Balance

- How a network evolves over time
- Political science: International relationships
    - Example: the separation of Bangladesh from Pakistan

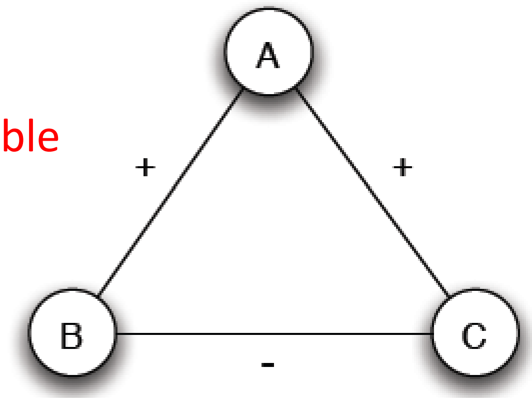
A Weaker Form of Structural Balance

Weak Structural Balance Property: There is no set of three nodes such that the edges among them consist of exactly two positive edges and one negative edge

This is allowed



Unstable

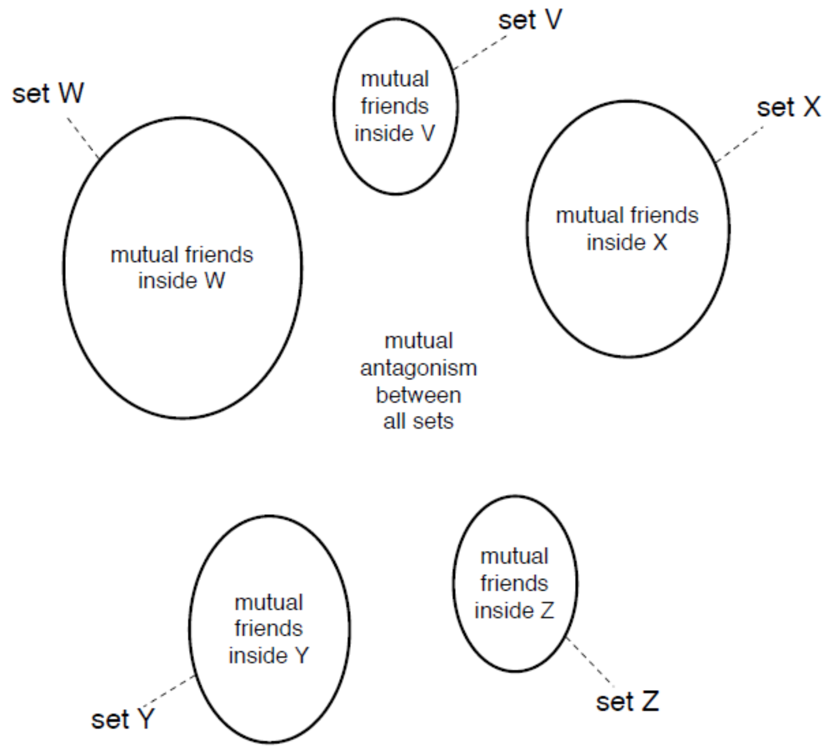


A Weaker Form of Structural Balance

Weakly Balance Theorem: If a labeled complete graph is weakly balanced, its nodes can be divided *into groups* in such a way that every two nodes belonging to the same group are friends, and every two nodes belonging to different groups are enemies.

From a local to a global property

A Weaker Form of Structural Balance



Generalizations

1. Non-complete graphs

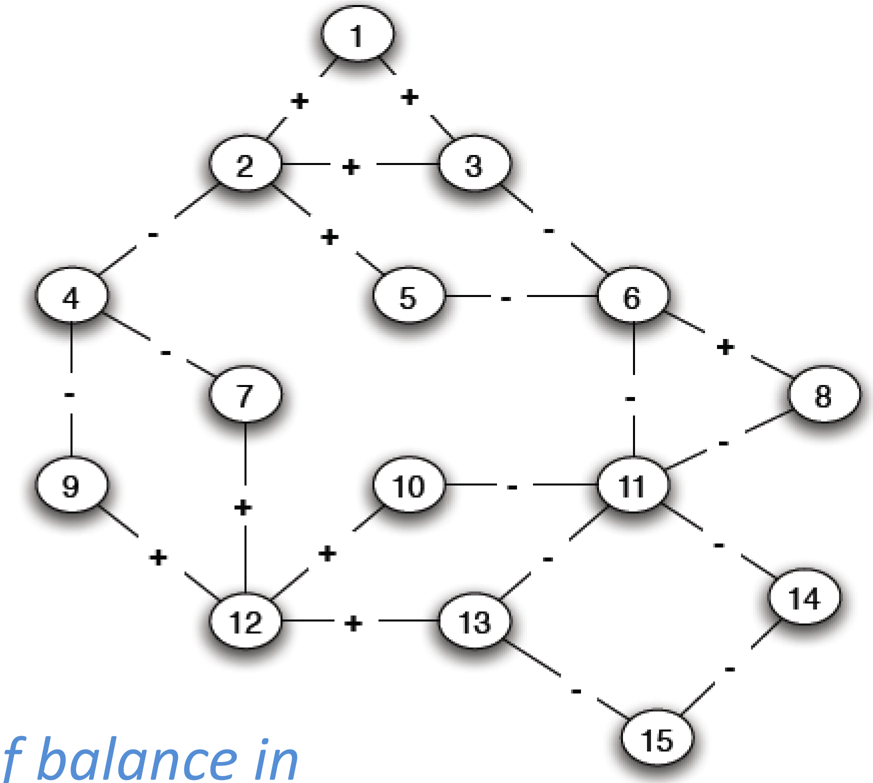
2. Instead of all triangles, “most” triangles, approximately divide the graph

We shall use the original (“non-weak”) definition of structural balance

Structural Balance in Arbitrary Graphs

Three possible relations

- Positive edge
- Negative edge
- Absence of an edge



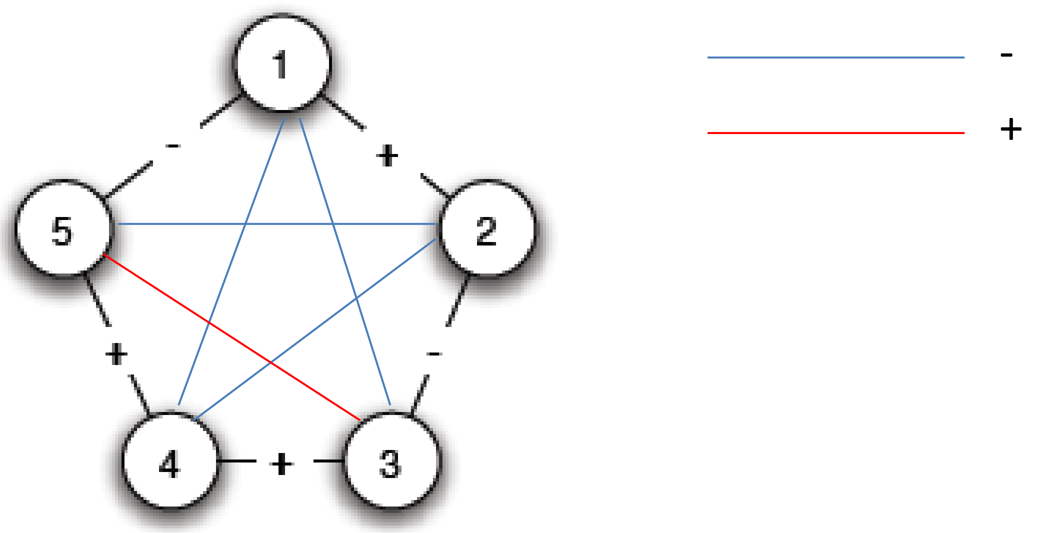
What is a good definition of balance in a non-complete graph?

Balance Definition for General Graphs

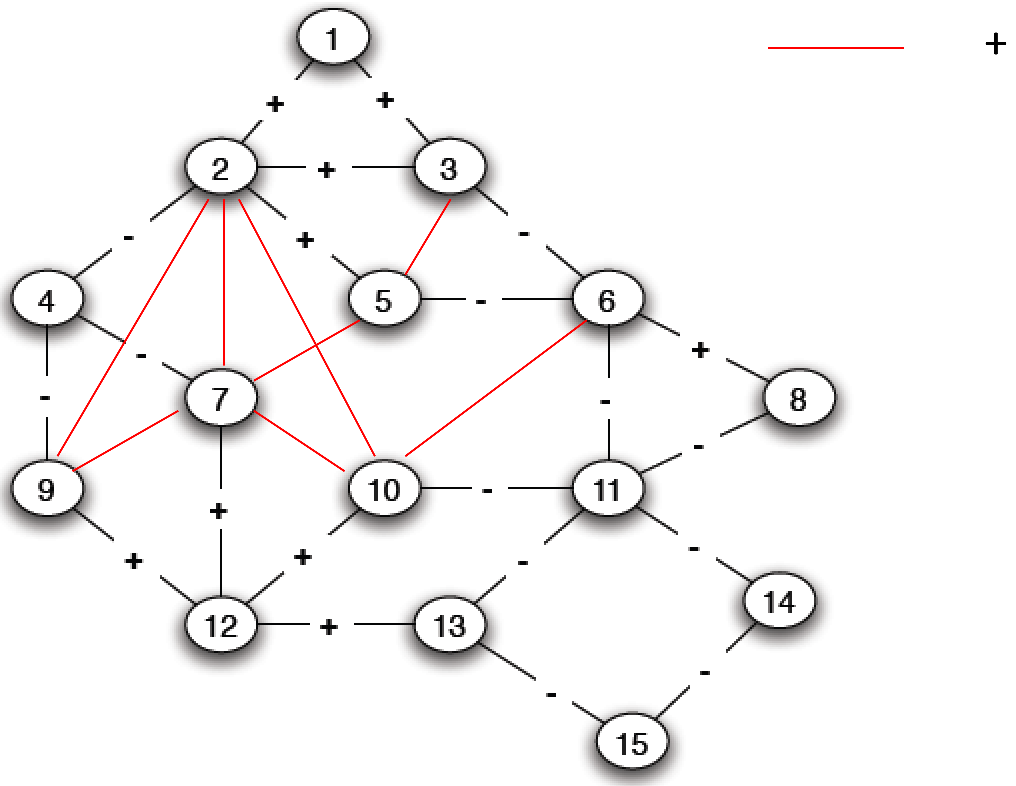
1. Based on triangles (local view)
2. Division of the network (global view)

Balance for General Graphs: local

A (non-complete) graph is balanced if it can be completed by adding edges to form a signed complete graph that is balanced

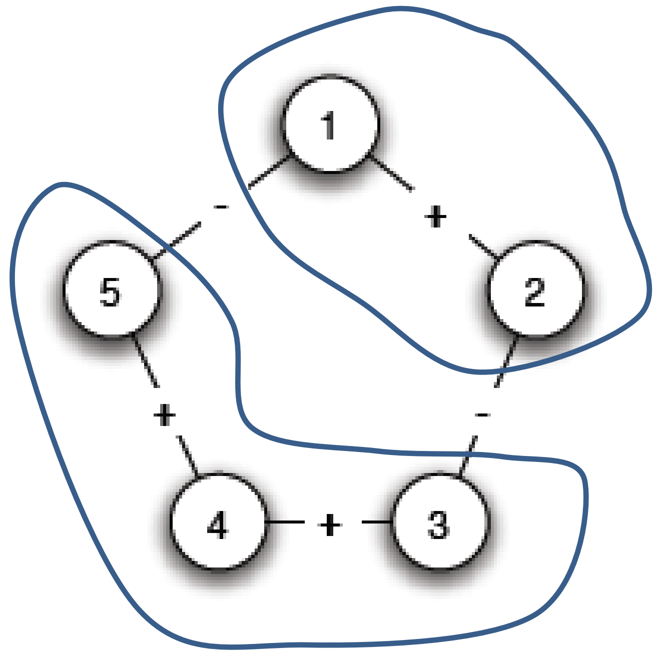


Balance for General Graphs: local



Balance for General Graphs: global

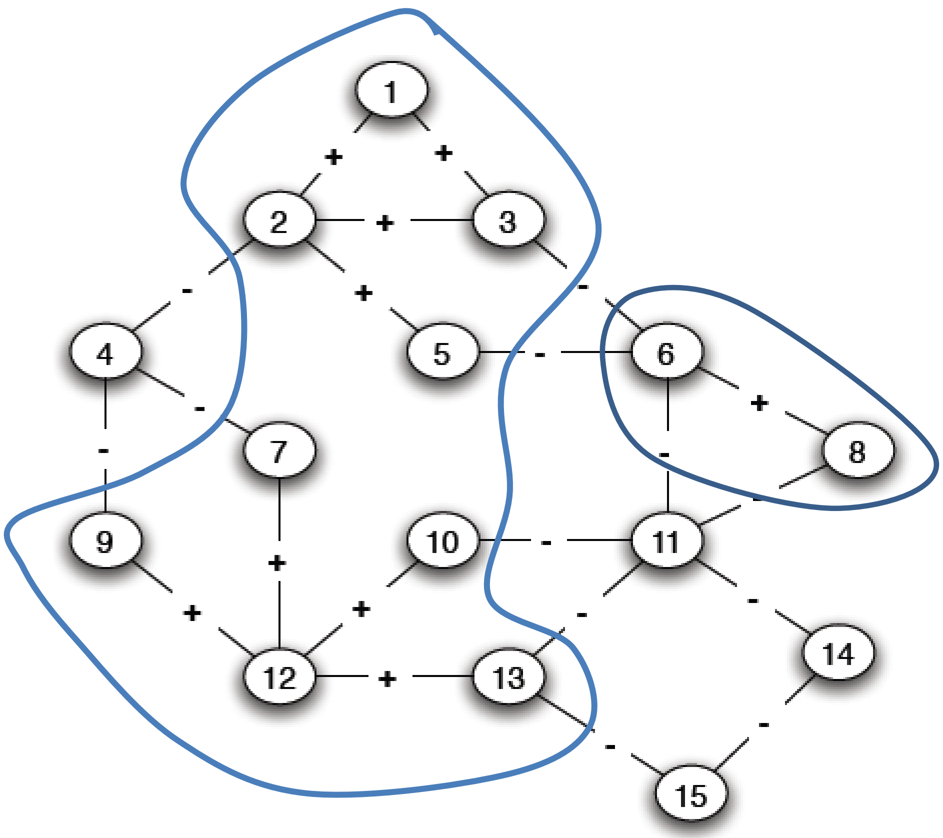
A (non-complete) graph is balanced if it possible to divide the nodes into two sets X and Y, such that any edge with both ends inside X or both ends inside Y is positive and any edge with one end in X and one end in Y is negative



The two definitions are equivalent:
An arbitrary signed graph is balanced under *the first definition*, *<u>if and only if</u>*, it is balanced under the *second definition*

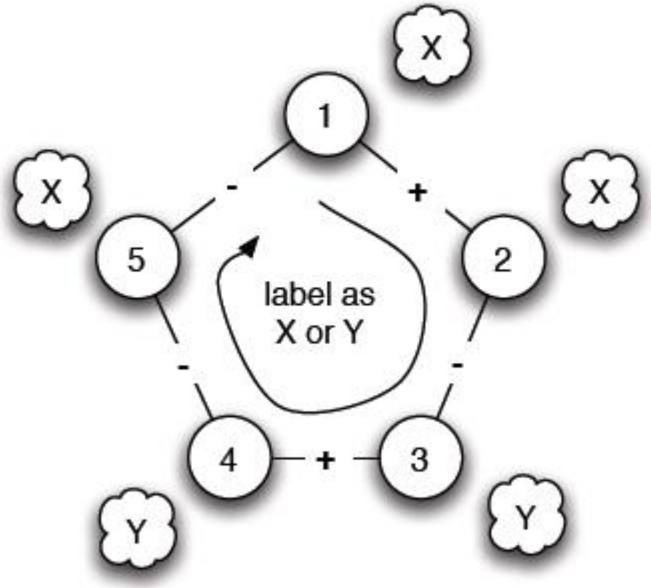
Balance Definition for General Graphs

Algorithm for dividing the nodes?



Balance Characterization

What prevents a network from being balanced?

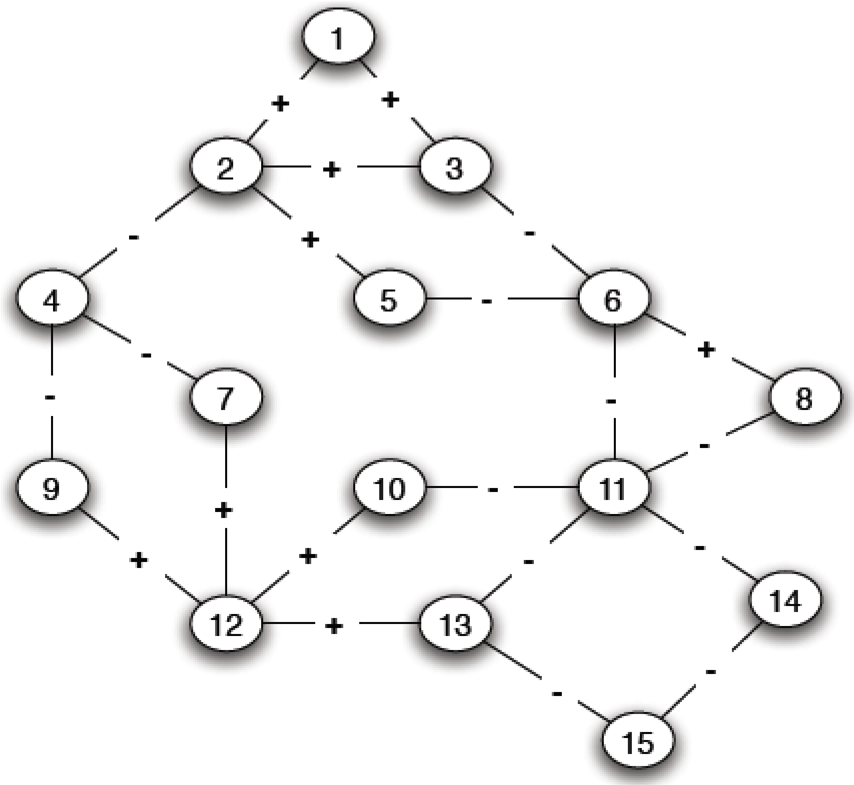


- Start from a node and place nodes in X or Y
- Every time we cross a negative edge, change the set

Cycle with odd number of negative edges

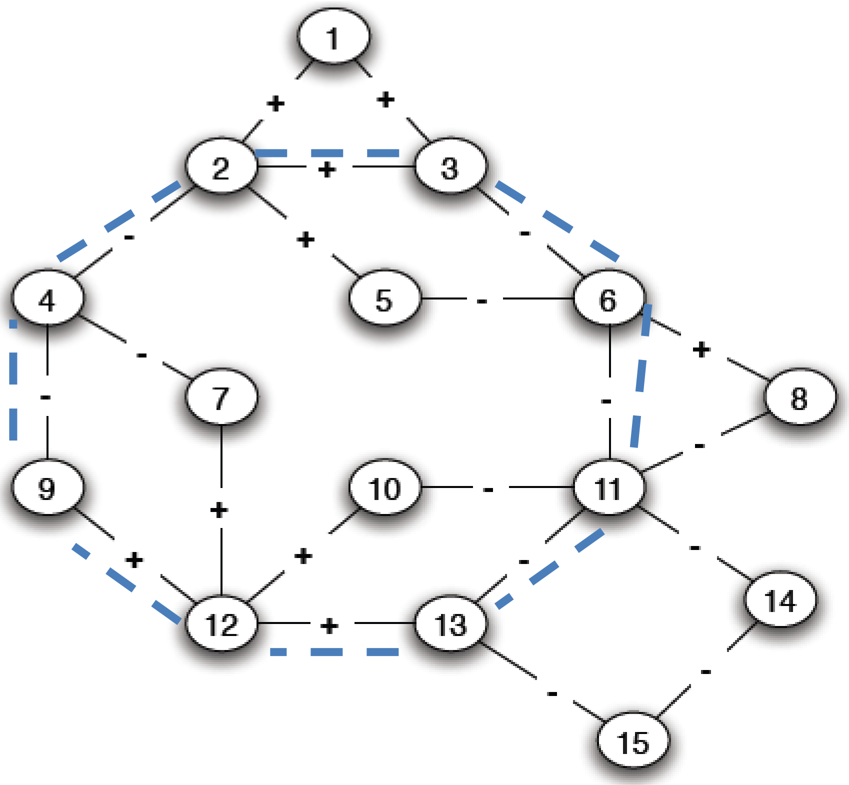
Balance Definition for General Graphs

Is there such a cycle with an odd number of -?



Balance Definition for General Graphs

Is there such a cycle with an odd number of -?



Balance Characterization

Claim: A signed graph is balanced, if and only if, it contains no cycles with an odd number of negative edges

Proof by construction

Find *a balanced division:* partition into sets X and Y, all edges inside X and Y positive, crossing edges negative

Either succeeds or Stops with a cycle containing an odd number of -

Two steps:

1. Convert the graph into a reduced one with only negative edges
2. Solve the problem in the reduced graph

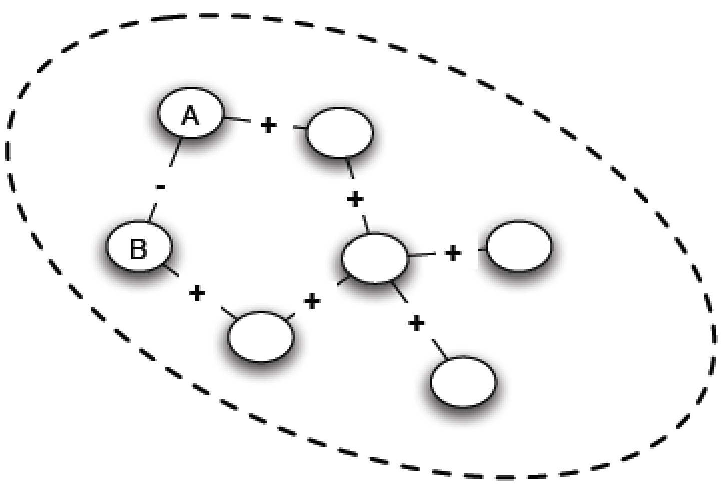
Balance Characterization: Step 1

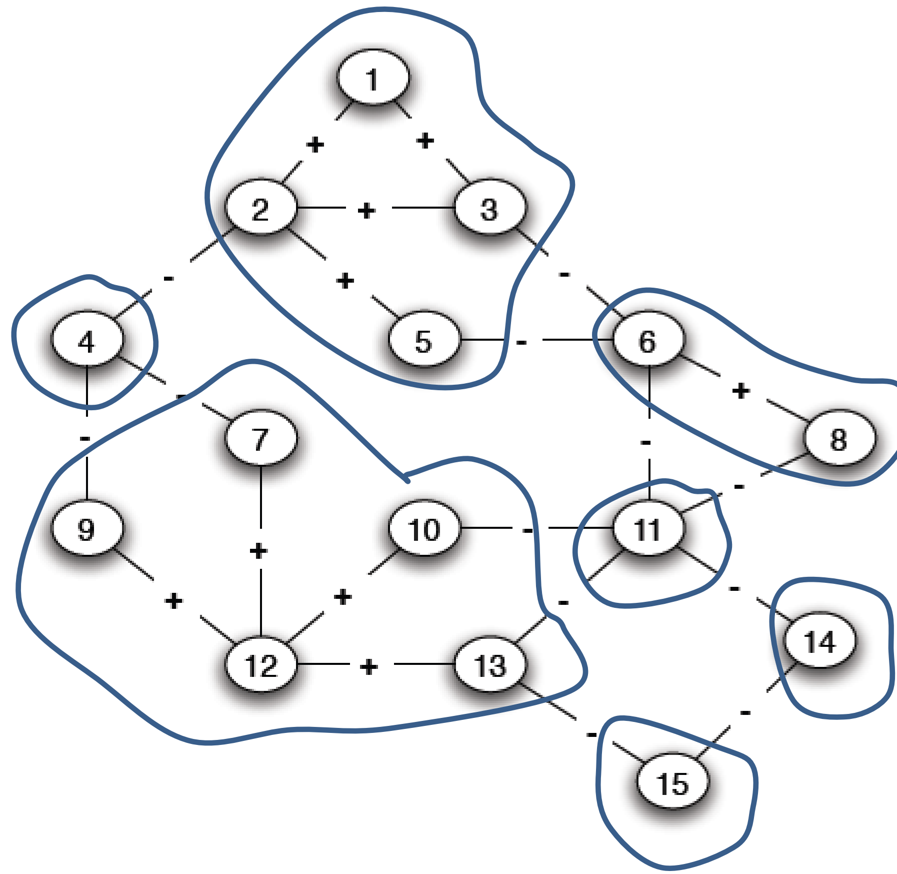
Find *connected components of G* by considering only positive edges, let us call then *supernodes*

Check: Do supernodes *contain a negative edge* between any pair of their nodes

If Yes, G is unbalanced

Proof: say - between A and B, A and B connected by an all-positive path -> odd cycle

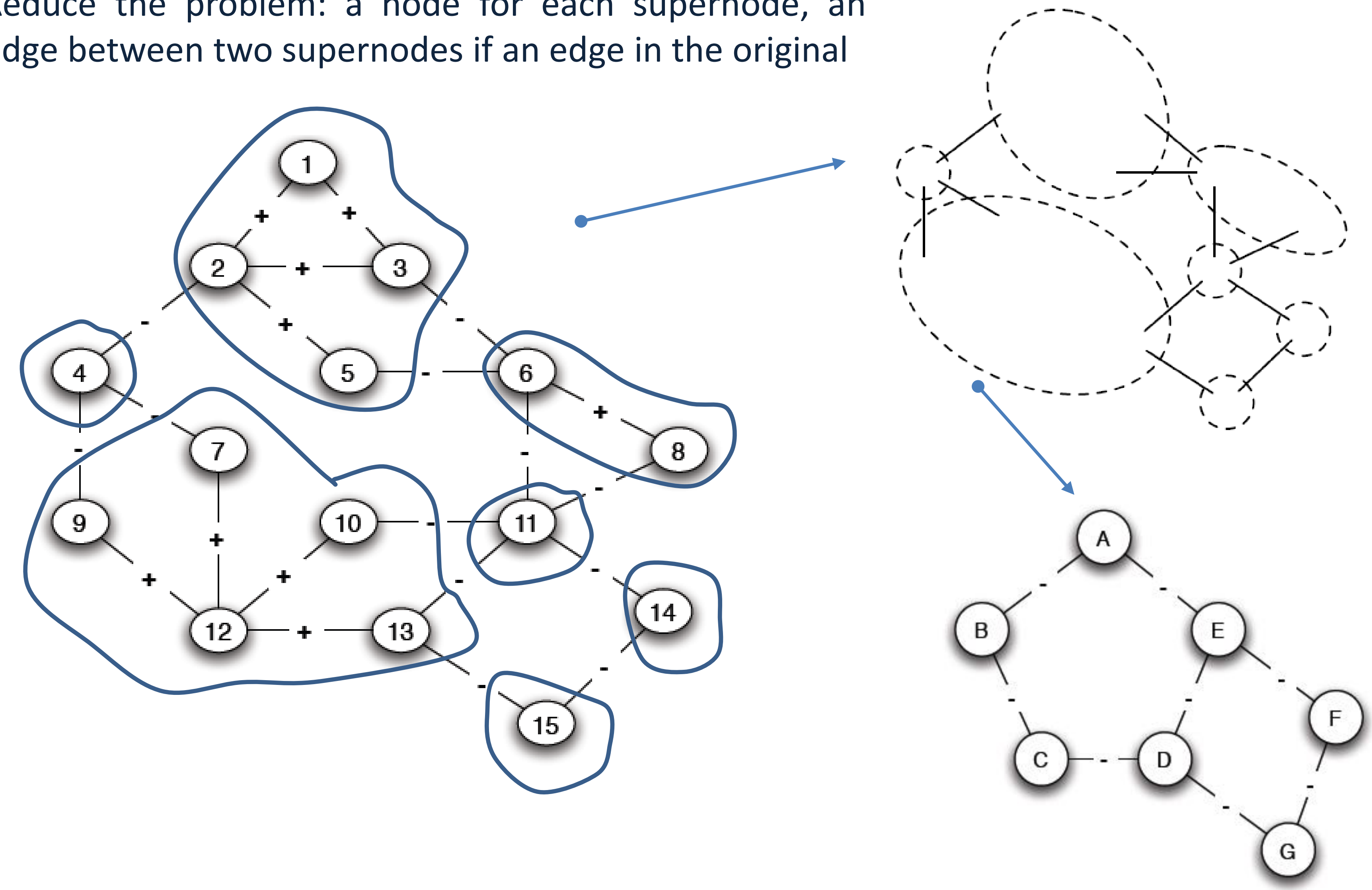




Balance Characterization: Step 1

Else:

Reduce the problem: a node for each supernode, an edge between two supernodes if an edge in the original

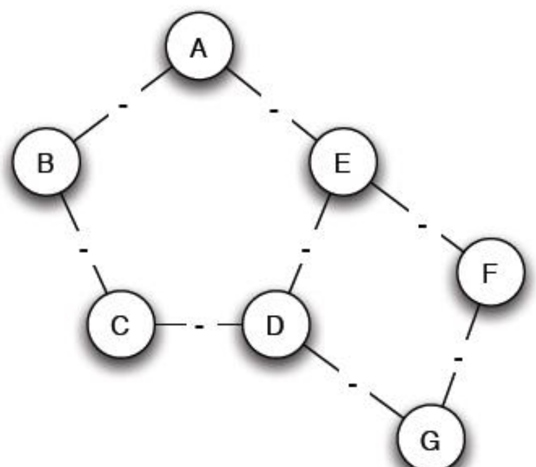


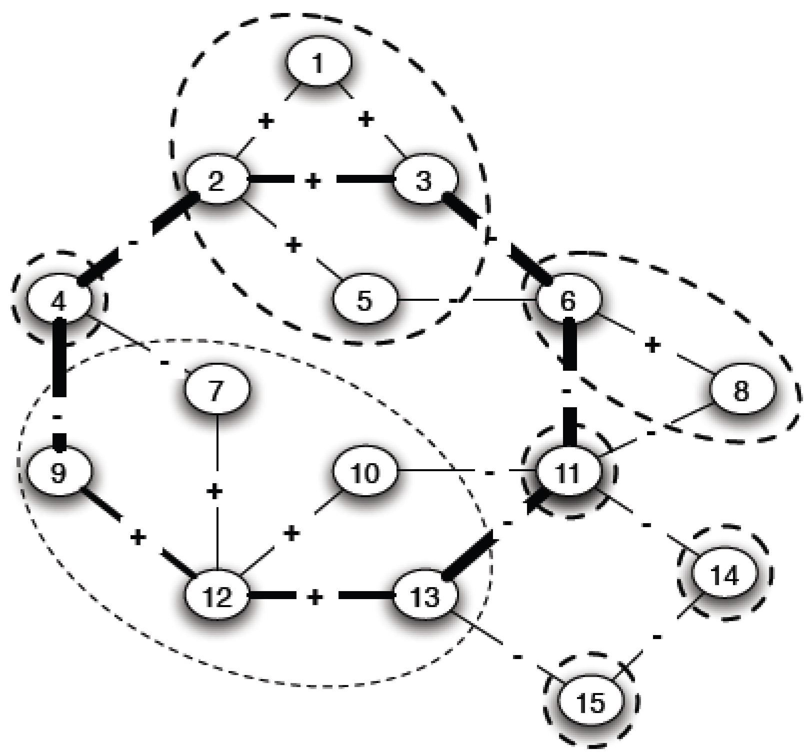
Balance Characterization: Step 2

Note: Only negative edges among supernodes

Start labeling each supernode by either X and Y
If successful, then label the nodes of the supernode correspondingly

- A cycle with an odd number, corresponds to a (possibly larger) odd cycle in the original





Balance Characterization: Step 2

Odd cycle
Determining whether the graph is bipartite
there is no edge between nodes in X or Y, *the only edges are from nodes in* X *to nodes in* Y

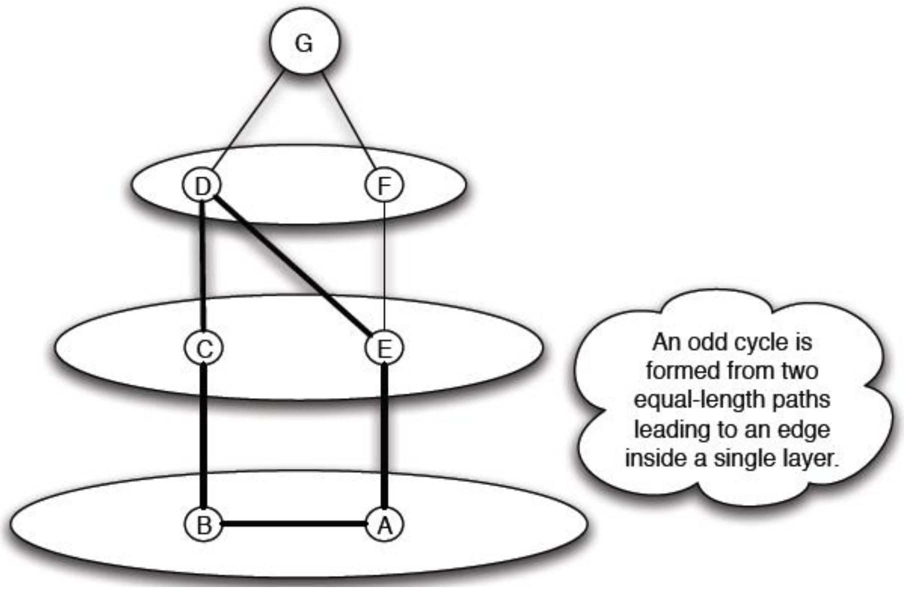
Use Breadth-First-Search (BFS)

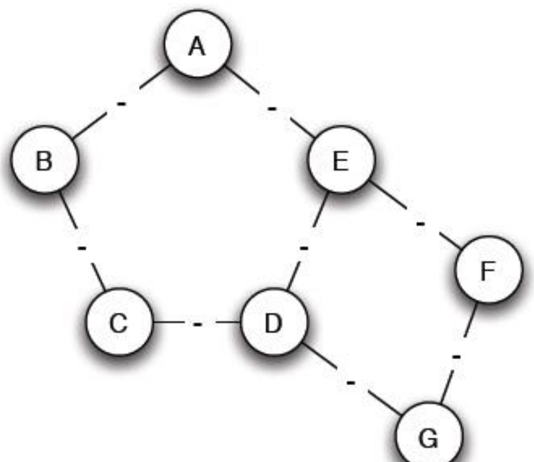
Two type of edges: (1) between nodes in adjacent levels (2) between nodes in the same level

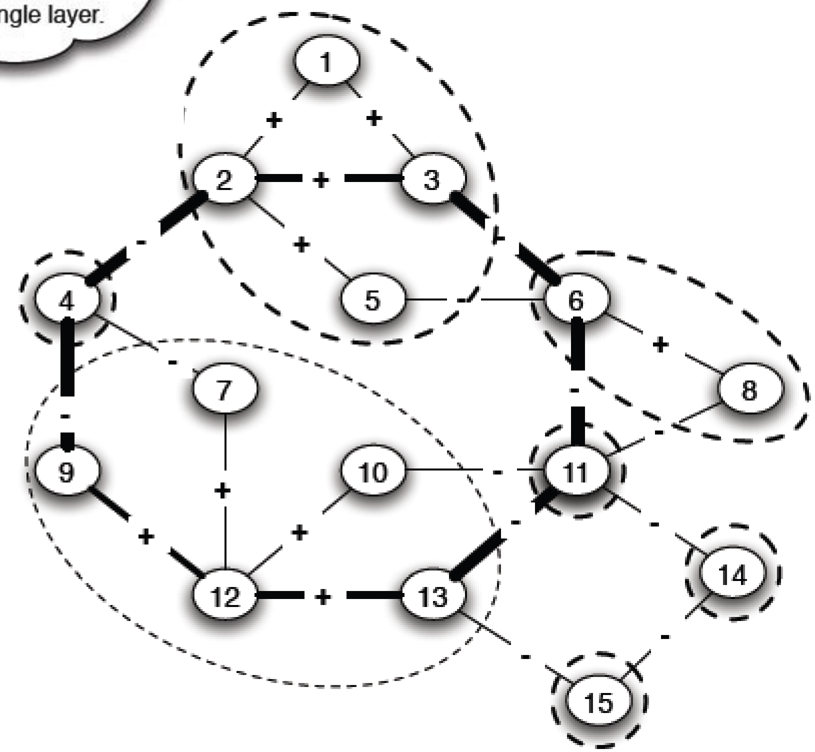
If only type (1), alternate X and Y labels at each level

If type (2), then odd cycle

Balance Characterization: Step 2





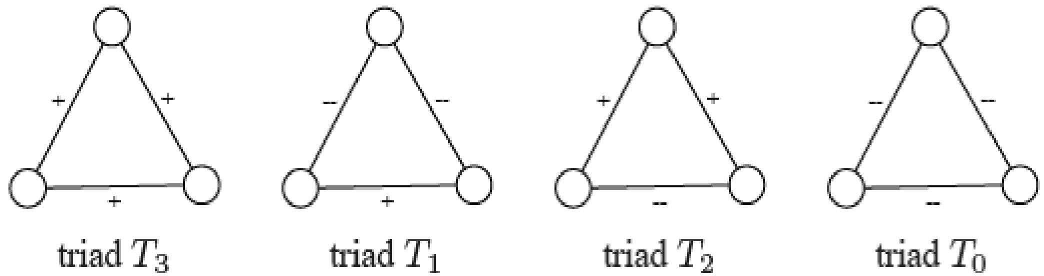


Status theory in practice

	Epinions	Slashdot	Wikipedia
Nodes	119,217	82,144	7,118
Edges	841,200	549,202	103,747
+ edges	85.0%	77.4%	78.7%
− edges	15.0%	22.6%	21.2%
Triads	13,375,407	1,508,105	790,532

- **Epinions:** product review Web site, where users can indicate their *trust* or *distrust* of the reviews
- **Slashdot:** the social network of the blog where a signed link indicates that one user *likes* or *dislikes* the *comments*
- **Wikipedia:** its voting network where a signed link indicates a positive or negative *vote* by one user *on the promotion* to admin status of another.

Structural balance theory in practice



Triad T_i		$\lvert T_i \rvert$	$p(T_i)$	$p_0(T_i)$	$s(T_i)$
Epinions					
T_3	$+++$	11,640,257	0.870	0.621	1881.1
T_1	$+--$	947,855	0.071	0.055	249.4
T_2	$++-$	698,023	0.052	0.321	-2104.8
T_0	$---$	89,272	0.007	0.003	227.5
Slashdot					
T_3	$+++$	1,266,646	0.840	0.464	926.5
T_1	$+--$	109,303	0.072	0.119	-175.2
T_2	$++-$	115,884	0.077	0.406	-823.5
T_0	$---$	16,272	0.011	0.012	-8.7
Wikipedia					
T_3	$+++$	555,300	0.702	0.489	379.6
T_1	$+--$	163,328	0.207	0.106	289.1
T_2	$++-$	63,425	0.080	0.395	-572.6
T_0	$---$	8,479	0.011	0.010	10.8

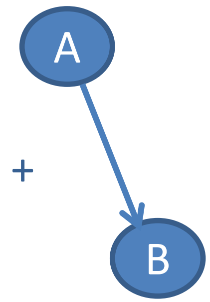
Symbol	Meaning
T_i	Signed triad, also the number of triads of type T_i
Δ	Total number of triads in the network
p	Fraction of positive edges in the network
$p(T_i)$	Fraction of triads T_i, $p(T_i) = T_i/\Delta$
$p_0(T_i)$	A priori prob. of T_i (based on sign distribution)
$E[T_i]$	Expected number of triads T_i, $E[T_i] = p_0(T_i)\Delta$
$s(T_i)$	Surprise, $s(T_i) = (T_i - E[T_i])/\sqrt{\Delta p_0(T_i)(1-p_0(T_i))}$

- All-positive triad T_3 is *heavily overrepresented* in all three datasets. T_3 tends to be overrepresented by about 40% in all three datasets
- Triad T_2 consisting of two enemies with a common friend is *heavily underrepresented*. T_2 is underrepresented by about 75%in Epinions and Slashdot and 50% in Wikipedia
- More consistent with weak structural balance

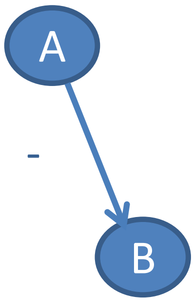
A theory of status

Directed networks

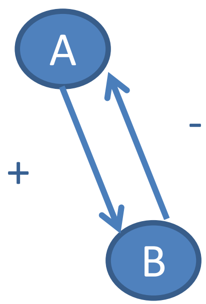
A *positive edge* (*A, B*) means that *A* regards *B* as having *higher status* than A



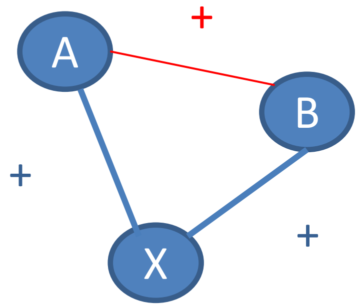
A *negative edge* (*A, B*) means that *A* regards *B* as having lower status than A



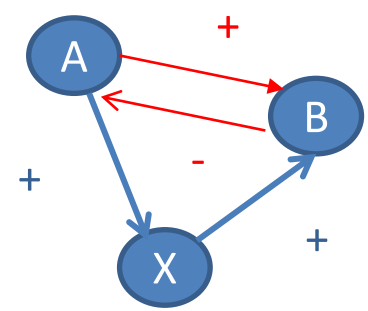
Assuming that all participants agree on status ordering, status theory predicts that when the direction of an edge is flipped, its sign should flip as well.

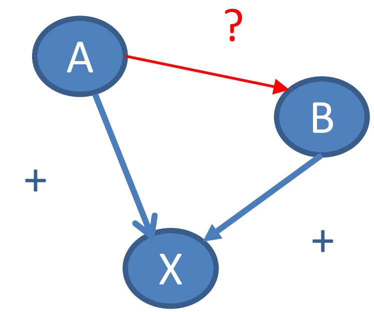


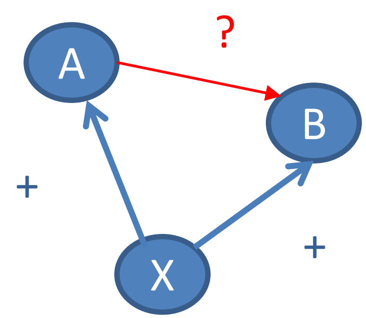
A theory of status

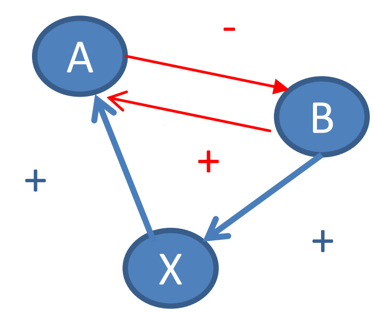


Structural balance









A theory of status: local property

For any **edge (u, v),** and any third **node w**, possible to assign distinct numerical "*status values*" to u, v, and w in such a way that the *positive edges* among them (if any) go from nodes of lower status to nodes of higher status, and the *negative edges* among them (if any) go from nodes of higher status to nodes of lower status.

Three nodes *u, v,* and *w* are *status-consistent* if this condition holds.

A theory of status: global property

Let G be a signed, directed graph, and suppose that all sets of three nodes in G are status-consistent.
Then it possible to order the nodes of G as $v_1, v_2, \ldots, v_n$ in such a way that each positive edge (vi, v_j) satisfies $i < j$, and each negative edge (vi, v_j) satisfies $i > j$.

Summary

Signed networks
Two interpretations

- Friendship/Foe (undirected)
- Status (directed)

Both at a local and global level

Questions?

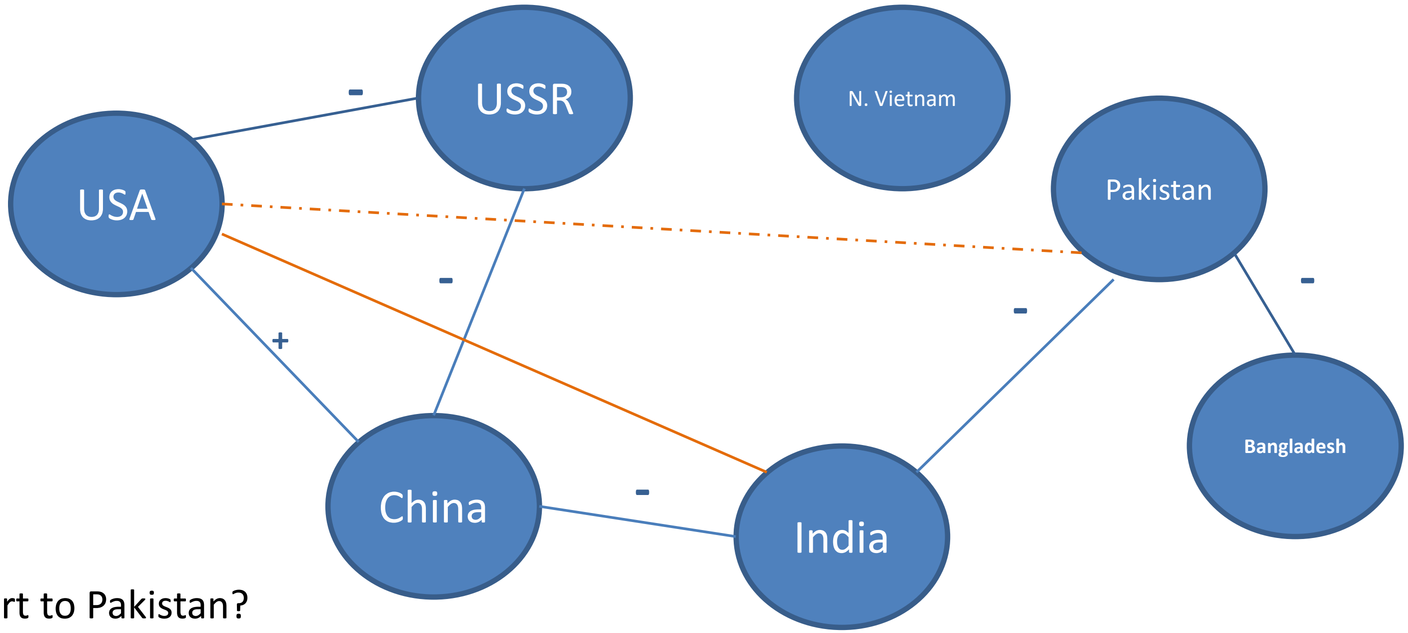
References

Networks, Crowds, and Markets (Chapter 5)

Extra material

Applications of Structural Balance

The conflict of Bangladesh's separation from Pakistan in 1972 (1)



USA support to Pakistan?

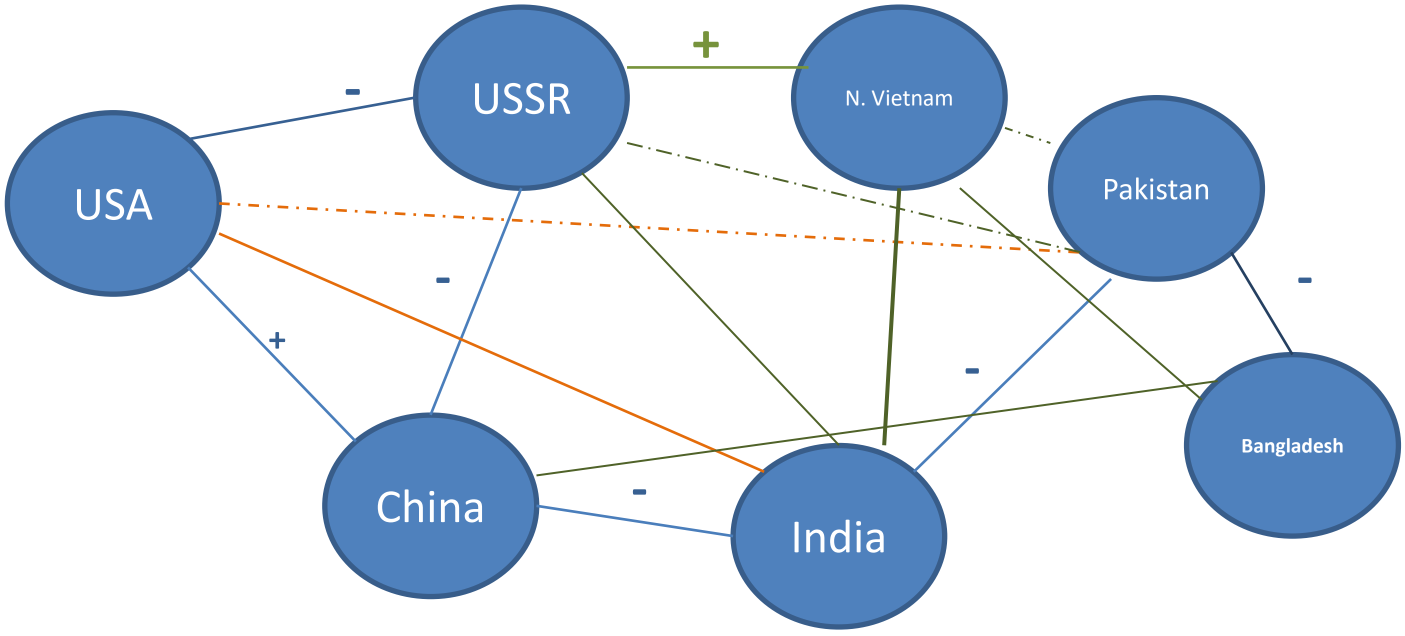
"[T]he United States's somewhat surprising support of Pakistan ... becomes less surprising when one considers that the USSR was China's enemy, China was India's foe, and India had traditionally bad relations with Pakistan. Since the U.S. was at that time improving its relations with China, it **supported the enemies of China's enemies**.

Further reverberations of this strange political constellation became inevitable: North Vietnam made friendly gestures toward India, Pakistan severed diplomatic relations with those countries of the Eastern Bloc which recognized Bangladesh, and China vetoed the acceptance of Bangladesh into the U.N."

Applications of Structural Balance

✓ International relationships (I)

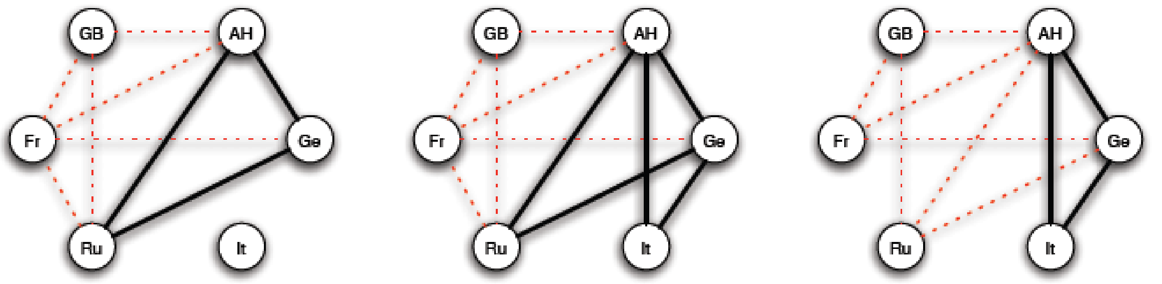
The conflict of Bangladesh's separation from Pakistan in 1972 (II)



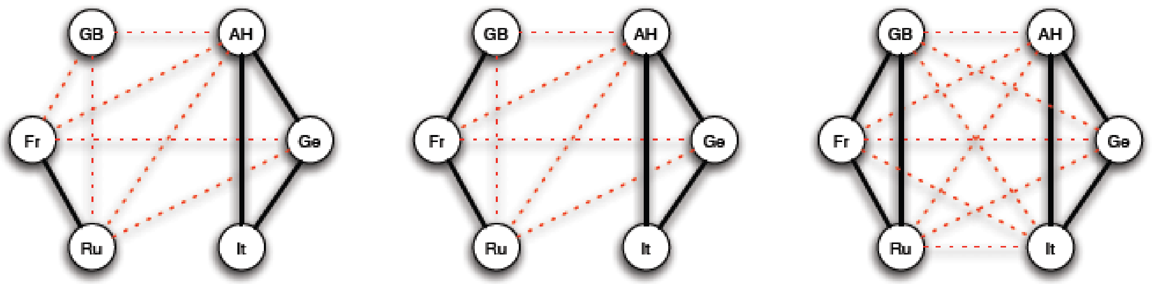
China?

Applications of Structural Balance

International relationships (II)



(a) *Three Emperors' League 1872–81*
(b) *Triple Alliance 1882*
(c) *German-Russian Lapse 1890*



(d) *French-Russian Alliance 1891–94*
(e) *Entente Cordiale 1904*
(f) *British Russian Alliance 1907*

Figure 5.5: The evolution of alliances in Europe, 1872-1907 (the nations GB, Fr, Ru, It, Ge, and AH are Great Britain, France, Russia, Italy, Germany, and Austria-Hungary respectively). Solid dark edges indicate friendship while dotted red edges indicate enmity. Note how the network slides into a balanced labeling — and into World War I. This figure and example are from Antal, Krapivsky, and Redner [20].