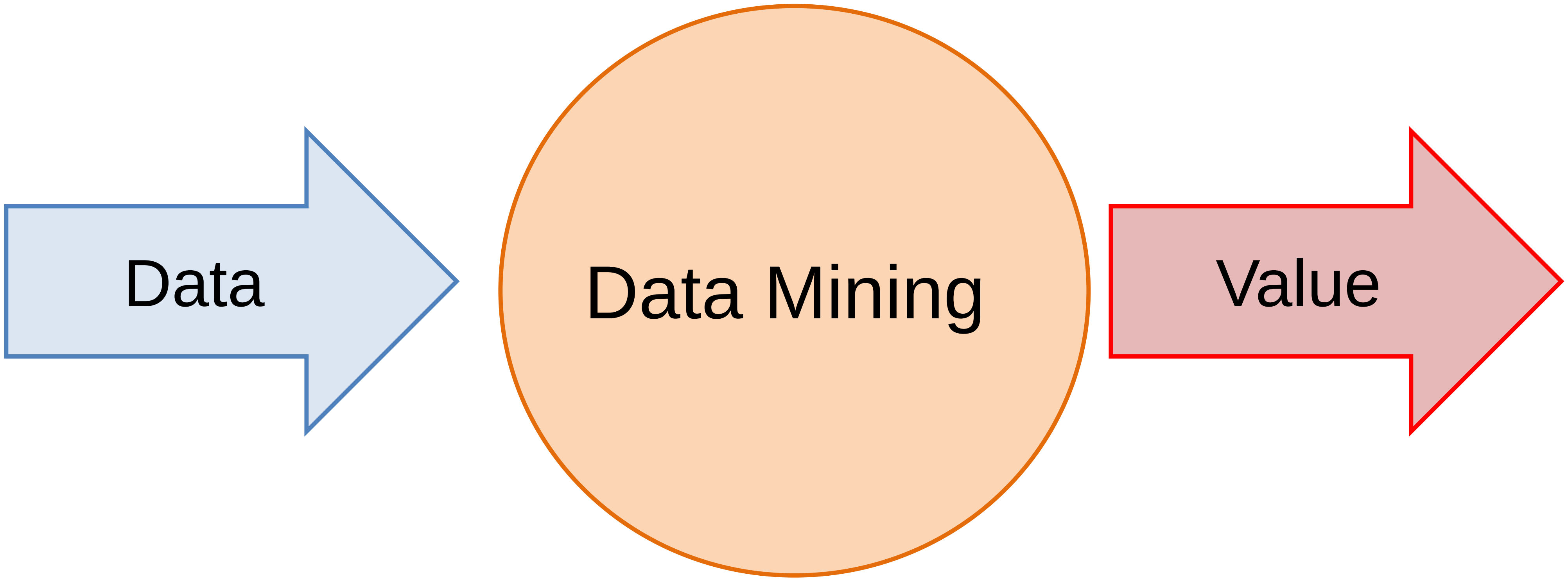
DATA MINING
THE DATA MINING PIPELINE

What is data?
The data mining pipeline: collection, preprocessing, mining, and post-processing
Sampling, feature extraction and normalization
Exploratory analysis of data – basic statistics

What is data mining again?

- "Data Mining is the study of collecting, processing, analyzing, and gaining useful insights from data" – Charu Aggarwal



- Essentially, anything that has to do with data is data mining

What is Data Mining?

- Data mining is the use of efficient techniques for the analysis of very large collections of data and the extraction of useful and possibly unexpected patterns in data.

- “Data mining is the analysis of (often large) observational data sets to find unsuspected relationships and to summarize the data in novel ways that are both understandable and useful to the data analyst” (Hand, Mannila, Smyth)

- “Data mining is the discovery of models for data” (Rajaraman, Ullman)
  - We can have the following types of models
    - Models that explain the data (e.g., a single function)
    - Models that predict the future data instances.
    - Models that summarize the data
    - Models the extract the most prominent features of the data.

Why do we need data mining?

- Really huge amounts of complex data generated from multiple sources and interconnected in different ways
  - Scientific data from different disciplines
    - Weather, astronomy, physics, biological microarrays, genomics
  - Huge text collections
    - The Web, scientific articles, news, tweets, facebook postings.
  - Transaction data
    - Retail store records, credit card records
  - Behavioral data
    - Mobile phone data, query logs, browsing behavior, ad clicks
  - Networked data
    - The Web, Social Networks, IM networks, email network, biological networks.
  - All these types of data can be combined in many ways
    - Facebook has a network, text, images, user behavior, ad transactions.
- We need to analyze this data to extract knowledge
  - Knowledge can be used for commercial or scientific purposes.
  - Our solutions should scale to the size of the data
- "*Data is the new oil*" – Clive Humby
  - Data Science: Use data to improve any process.

What is Data?

- Collection of data objects and their attributes
- An attribute is a property or characteristic of an object
  - Examples: name, date of birth, height, occupation.
  - Attribute is also known as variable, field, characteristic, or feature
- For each object the attributes take some values.
- The collection of attribute-value pairs describes a specific object
  - Object is also known as record, point, case, sample, entity, or instance

Attributes

Objects

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	95K	**Yes**
6	No	Married	60K	**No**
7	Yes	Divorced	220K	**No**
8	No	Single	85K	**Yes**
9	No	Married	75K	**No**
10	No	Single	90K	**Yes**

Size (n): Number of objects
Dimensionality (d): Number of attributes
Sparsity: Number of populated object-attribute pairs

Relational data

- The term comes from DataBases, where we assume data is stored in a relational table with a fixed schema (fixed set of attributes)
  - In Databases, it is usually assumed that the table is dense (few null values)
- There are a lot of data in this form
  - E.g., census data
- There are also a lot of data which do not fit well in this form
  - Sparse data: Many missing values
  - Not easy to define a fixed schema

Example of a relational table

Attributes = Table columns

Objects = Table rows

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	NULL
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	NULL	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Types of Attributes

- There are different types of attributes
  - Numeric
    - Examples: dates, temperature, time, length, value, count.
    - Discrete (counts) vs Continuous (temperature)
    - Special case: Binary/Boolean attributes (yes/no, exists/not exists)
  - Categorical
    - Examples: eye color, zip codes, strings, rankings (e.g, good, fair, bad), height in {tall, medium, short}
    - Nominal (no order or comparison) vs Ordinal (order but not comparable)

Numeric Relational Data

- If data objects have the same fixed set of numeric attributes, then the data objects can be thought of as points/vectors in a multi-dimensional space, where each dimension represents a distinct attribute

- Such data set can be represented by an n-by-d data matrix, where there are n rows, one for each object, and d columns, one for each attribute

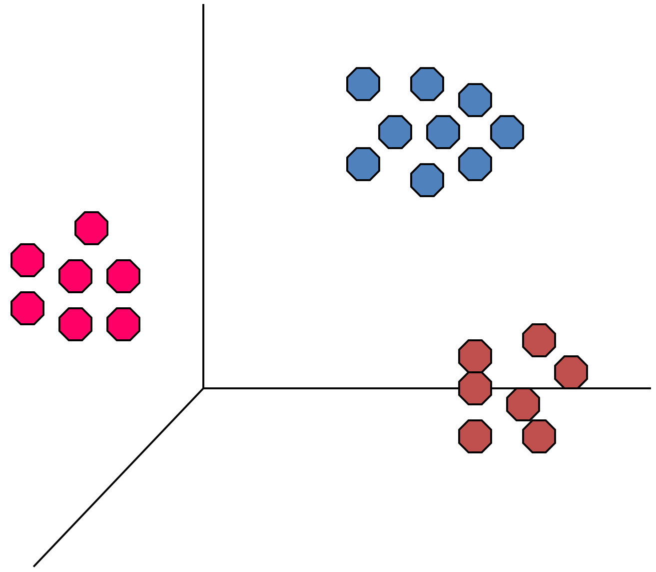
	Temperature	Humidity	Pressure
O1	30	0.8	90
O2	32	0.5	80
O3	24	0.3	95

30	0.8	90
32	0.5	80
24	0.3	95

Numeric data

- Thinking of numeric data as points or vectors is very convenient

- For small dimensions we can plot the data
- We can use geometric analogues to define concepts like distance or similarity
- We can use linear algebra to process the data matrix
- We will often talk about points or vectors



Categorical Relational Data

- Data that consists of a collection of records, each of which consists of a fixed set of categorical attributes

ID Number	Zip Code	Marital Status	Income Bracket
1129842	45221	Single	High
2342345	45223	Married	Low
1234542	45221	Divorced	High
1243535	45224	Single	Medium

Mixed Relational Data

- Data that consists of a collection of records, each of which consists of a fixed set of both numeric and categorical attributes

ID Number	Zip Code	Age	Marital Status	Income	Income Bracket
1129842	45221	55	Single	250000	High
2342345	45223	25	Married	30000	Low
1234542	45221	45	Divorced	200000	High
1243535	45224	43	Single	150000	Medium

Mixed Relational Data

- Data that consists of a collection of records, each of which consists of a fixed set of both numeric and categorical attributes

ID Number	Zip Code	Age	Marital Status	Income	Income Bracket	Refund
1129842	45221	55	Single	250000	High	No
2342345	45223	25	Married	30000	Low	Yes
1234542	45221	45	Divorced	200000	High	No
1243535	45224	43	Single	150000	Medium	No

Mixed Relational Data

- Data that consists of a collection of records, each of which consists of a fixed set of both numeric and categorical attributes

Takes numerical values but it is actually categorical

ID Number	Zip Code	Age	Marital Status	Income	Income Bracket	Refund
1129842	45221	55	Single	250000	High	0
2342345	45223	25	Married	30000	Low	1
1234542	45221	45	Divorced	200000	High	0
1243535	45224	43	Single	150000	Medium	0

Boolean attributes can be thought as both numeric and categorical

When appearing together with other attributes they make more sense as categorical

They are often represented as numeric though

Mixed Relational Data

- Some times it is convenient to represent categorical attributes as boolean.
  - Add a Boolean attribute for each possible value of the attribute

ID	Zip 45221	Zip 45223	Zip 45224	Age	Single	Married	Divorced	Income	Refund
1129842	1	0	0	55	0	0	0	250000	0
2342345	0	1	0	25	0	1	0	30000	1
1234542	1	0	0	45	0	0	1	200000	0
1243535	0	0	1	43	0	0	0	150000	0

We can now view the whole vector as numeric

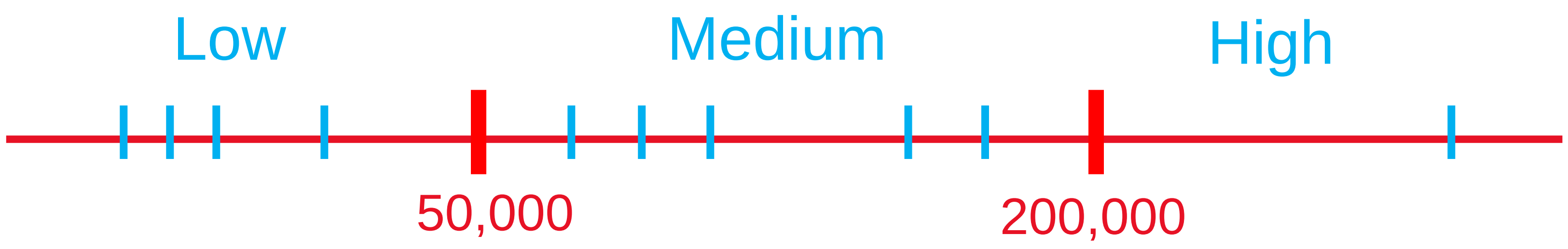
Mixed Relational Data

- Some times it is convenient to represent numerical attributes as categorical.
  - Group the values of the numerical attributes into bins

ID Number	Zip Code	Age	Marital Status	Income	Income Bracket	Refund
1129842	45221	50s	Single	High	High	0
2342345	45223	20s	Married	Low	Low	1
1234542	45221	40s	Divorced	High	High	0
1243535	45224	40s	Single	Medium	Medium	0

Binning

- Idea: split the range of the domain of the numerical attribute into bins (intervals).
- Every bucket defines a categorical value

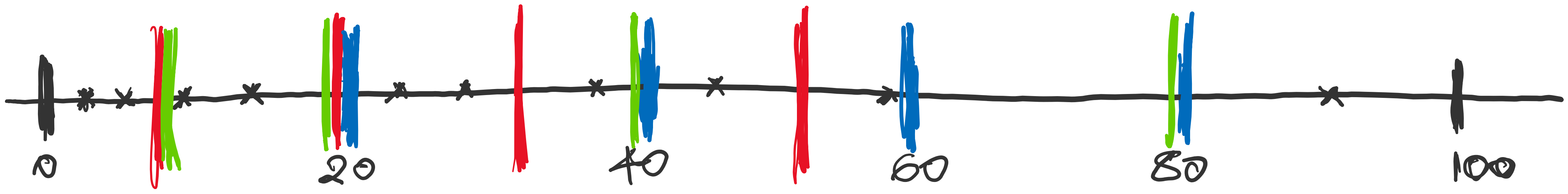


- How do we decide the number of bins?
  - Depends on the granularity of the data that we want

Bucketization

- How do we decide the size of the bucket?
  - Depends on the data and our application
- Equi-width bins: All bins have the same size
  - Example: split time into decades
  - Problem: some bins may be very sparse or empty
- Equi-size (depth) bins: Select the bins so that they all contain the same number of elements
  - This splits data into quantiles: top-10%, second 10% etc
  - Some bins may be very small
- Equi-log bins: $\log end - \log start$ is constant
  - The size of the previous bin is a fraction of the current one
  - Better for skewed distributions
- Optimized bins: Use a 1-dimensional clustering algorithm to create the bins

Example



Blue: Equi-width [20,40,60,80]

Red: Equi-depth (2 points per bin)

Green: Equi-log ($\frac{end}{start} = 2$)

Physical data storage

- Stored in a Relational Database
  - Assumes a strict schema and relatively dense data (few missing/Null values)
- Tab or Comma separated files (TSV/CSV), Excel sheets, relational tables
  - Assumes a strict schema and relatively dense data (few missing/Null values)
- Flat file with triplets (record id, attribute, attribute value)
  - A very flexible data format, allows multiple values for the same attribute (e.g., phone number)
- JSON, XML format
  - Standards for data description that are more flexible than relational tables
  - There exist parsers for reading such data.

Examples

Comma Separated File

```
id,Name,Surname,Age,Zip
1,John,Smith,25,10021
2,Mary,Jones,50,96107
3,Joe ,Doe,80,80235
```

- Can be processed with simple parsers, or loaded to excel or a database

Triple-store

```
1, Name, John
1, Surname, Smith
1, Age, 25
1, Zip, 10021
2, Name, Mary
2, Surname, Jones
2, Age, 50
2, Zip, 96107
3, Name, Joe
3, Surname, Doe
3, Age, 80
3, Zip, 80235
```

- Easy to deal with missing values

Examples

JSON EXAMPLE – Record of a person

```json
{
  "firstName": "John",
  "lastName": "Smith",
  "isAlive": true,
  "age": 25,
  "address": {
    "streetAddress": "21 2nd Street",
    "city": "New York",
    "state": "NY",
    "postalCode": "10021-3100"
  },
  "phoneNumbers": [
    {
      "type": "home",
      "number": "212 555-1234"
    },
    {
      "type": "office",
      "number": "646 555-4567"
    }
  ],
  "children": [],
  "spouse": null
}
```

XML EXAMPLE – Record of a person

```xml
<person>
  <firstName>John</firstName>
  <lastName>Smith</lastName>
  <age>25</age>
  <address>
    <streetAddress>21 2nd
Street</streetAddress>
    <city>New York</city>
    <state>NY</state>
    <postalCode>10021</postalCode>
  </address>
  <phoneNumbers>
    <phoneNumber>
      <type>home</type>
      <number>212 555-1234</number>
    </phoneNumber>
    <phoneNumber>
      <type>fax</type>
      <number>646 555-4567</number>
    </phoneNumber>
  </phoneNumbers>
  <gender>
    <type>male</type>
  </gender>
</person>
```

Beyond relational data: Set data

- Each record is a set of items from a space of possible items
- Example: Transaction data
  - Also called market-basket data

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Set data

- Each record is a set of items from a space of possible items
- Example: Document data
  - Also called bag-of-words representation

Doc Id	Words
1	the, dog, followed, the, cat
2	the, cat, chased, the, cat
3	the, man, walked, the, dog

Vector representation of market-basket data

- Market-basket data can be represented, or thought of, as numeric vector data
  - The vector is defined over the set of all possible items
  - The values are binary (the item appears or not in the set)

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

TID	Bread	Coke	Milk	Beer	Diaper
1	1	1	1	0	0
2	1	0	0	1	0
3	0	1	1	1	1
4	1	0	1	1	1
5	0	1	1	0	1

Sparsity: Most entries are zero. Most baskets contain few items

Vector representation of document data

- Document data can be represented, or thought of, as numeric vector data
  - The vector is defined over the set of all possible words
  - The values are the counts (number of times a word appears in the document)

Doc Id	Words
1	the, dog, follows, the, cat
2	the, cat, chases, the, cat
3	the, man, walks, the, dog

Doc Id	the	dog	follows	cat	chases	man	walks
1	2	1	1	1	0	0	0
2	2	0	0	2	1	0	0
3	1	1	0	0	0	1	1

Sparsity: Most entries are zero. Most documents contain few of the words

Physical data storage

- Usually set data is stored in flat files
  - One line per set

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29
30 31 32
33 34 35
36 37 38 39 40 41 42 43 44 45 46
38 39 47 48
38 39 48 49 50 51 52 53 54 55 56 57 58
32 41 59 60 61 62
3 39 48
```

- I heard so many good things about this place so I was pretty juiced to try it. I'm from Cali and I heard Shake Shack is comparable to IN-N-OUT and I gotta say, Shake Shake wins hands down. Surprisingly, the line was short and we waited about 10 MIN. to order. I ordered a regular cheeseburger, fries and a black/white shake. So yummerz. I love the location too! It's in the middle of the city and the view is breathtaking. Definitely one of my favorite places to eat in NYC.
- I'm from California and I must say, Shake Shack is better than IN-N-OUT, all day, err'day.

Dependent data

- In tables we usually consider each object independent of each other.
- In some cases, there are explicit dependencies between the data
  - Ordered/Temporal data: We know the time order of the data
  - Spatial data: Data that is placed on specific locations
  - Spatiotemporal data: data with location and time
  - Networked/Graph data: data with pairwise relationships between entities

Ordered Data

- Genomic sequence data

```
GGTTCCGCCTTCAGCCCCGCGCC
CGCAGGGCCCGCCCCGCGCCGTC
GAGAAGGGCCCGCCTGGCGGGCG
GGGGGAGGCGGGGCCGCCCGAGC
CCAACCGAGTCCGACCAGGTGCC
CCCTCTGCTCGGCCTAGACCTGA
GCTCATTAGGCGGCAGCGGACAG
GCCAAGTAGAACACGCGAAGCGC
TGGGCTGCCTGCTGCGACCAGGG
```

- Data is a long ordered string

Ordered Data

- Time series
  - Sequence of ordered (over "time") numeric values.

Ordered Data

- Sequence data: Similar to the time series but in this case we have categorical values rather than numerical ones.
- Example: Event logs

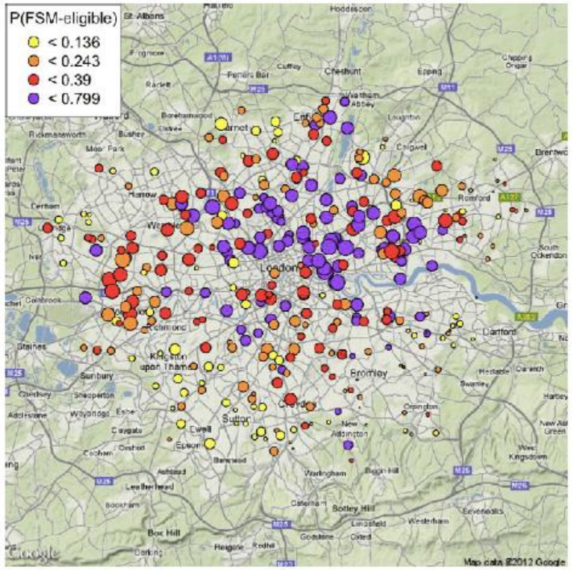
```
fcrawler.looksmart.com - - [26/Apr/2000:00:00:12 -0400] "GET /contacts.html HTTP/1.0" 200 4595 "-" "FAST-WebCrawl
fcrawler.looksmart.com - - [26/Apr/2000:00:17:19 -0400] "GET /news/news.html HTTP/1.0" 200 16716 "-" "FAST-WebCra

ppp931.on.bellglobal.com - - [26/Apr/2000:00:16:12 -0400] "GET /download/windows/asctab31.zip HTTP/1.0" 200 1540

123.123.123.123 - - [26/Apr/2000:00:23:48 -0400] "GET /pics/wpaper.gif HTTP/1.0" 200 6248 "http://www.jafsoft.co
123.123.123.123 - - [26/Apr/2000:00:23:47 -0400] "GET /asctortf/ HTTP/1.0" 200 8130 "http://search.netscape.com/
123.123.123.123 - - [26/Apr/2000:00:23:48 -0400] "GET /pics/5star2000.gif HTTP/1.0" 200 4005 "http://www.jafsoft
123.123.123.123 - - [26/Apr/2000:00:23:50 -0400] "GET /pics/5star.gif HTTP/1.0" 200 1031 "http://www.jafsoft.com
123.123.123.123 - - [26/Apr/2000:00:23:51 -0400] "GET /pics/a2hlogo.jpg HTTP/1.0" 200 4282 "http://www.jafsoft.c
123.123.123.123 - - [26/Apr/2000:00:23:51 -0400] "GET /cgi-bin/newcount?jafsof3&width=4&font=digital&noshow HTTP
```

Spatial data

- Attribute values that can be arranged with geographic co-ordinates
  - Measurements of temperature/pressure in different locations.
  - Sales numbers in different stores
  - The majority party in the country states (categorical)
- Such data can be nicely visualized.



Spatiotemporal data

- Data that have both spatial and temporal aspects
  - Measurements in different locations over time
    - Pressure, Temperature, Humidity
  - Measurements that move in space over time
    - Traffic, Trajectories of moving objects

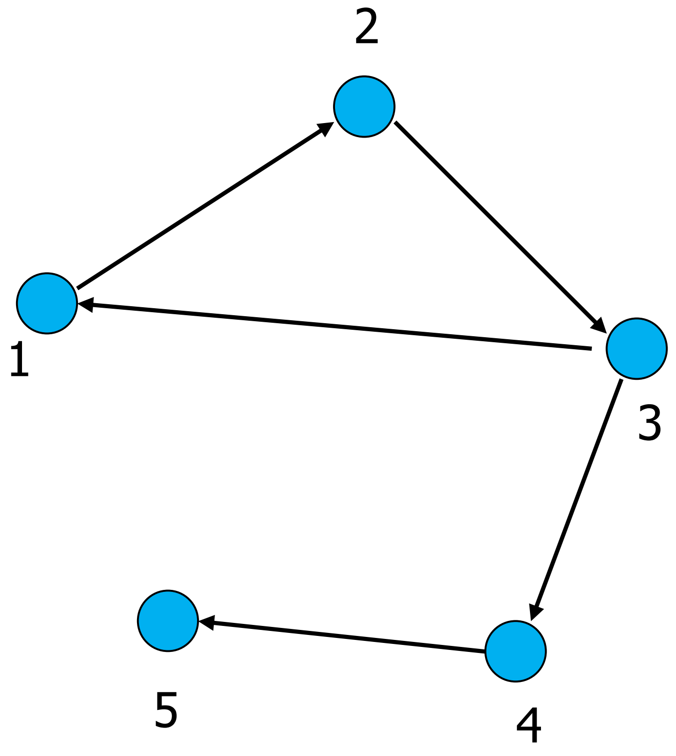
Graph Data

- Graph data: a collection of entities and their pairwise relationships.
- Examples:
  - Web pages and hyperlinks
  - Facebook users and friendships
  - The connections between brain neurons
  - Genes that regulate each oterh

In this case the data consists of pairs:

Who links to whom

We may have directed links



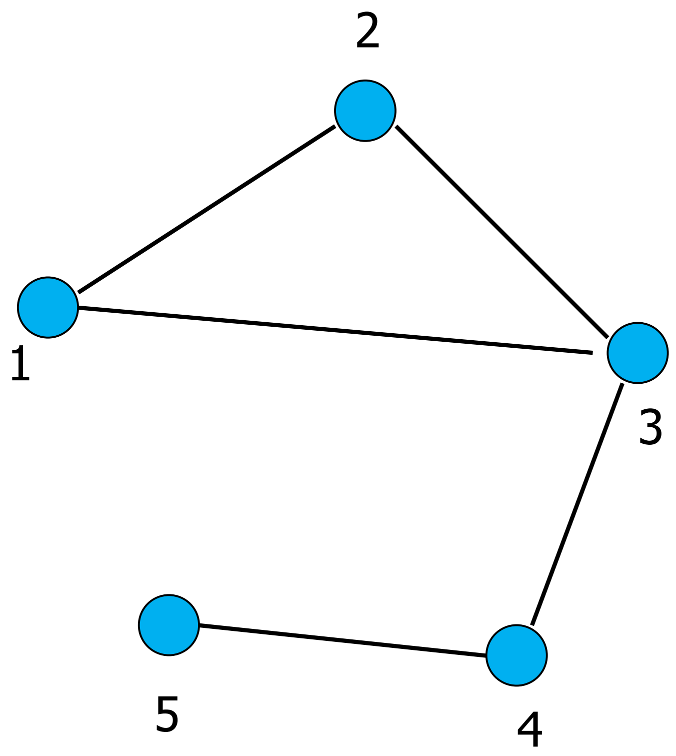
Graph Data

- Graph data: a collection of entities and their pairwise relationships.
- Examples:
  - Web pages and hyperlinks
  - Facebook users and friendships
  - The connections between brain neurons
  - Genes that regulate each oterh

In this case the data consists of pairs:

Who links to whom

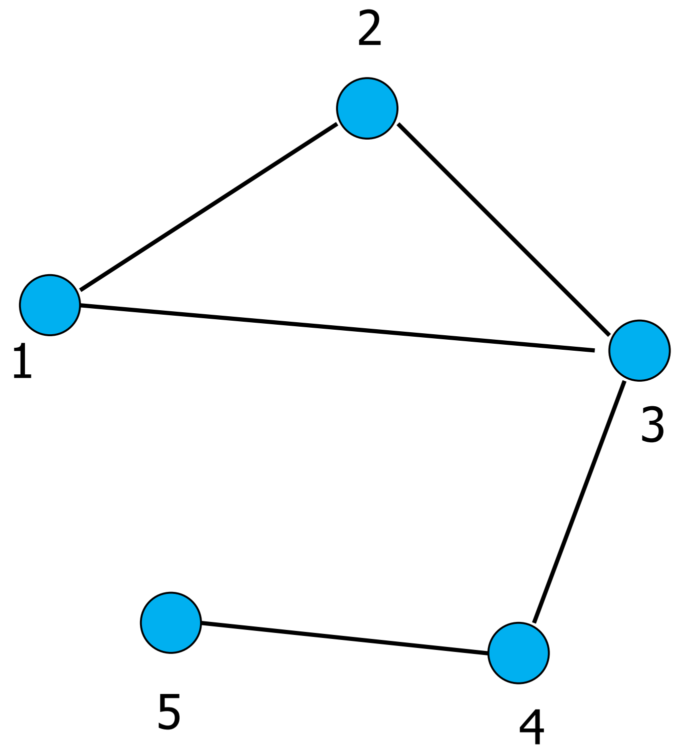
Or undirected links



Representation

- Adjacency matrix
  - Very sparse, very wasteful, but useful conceptually

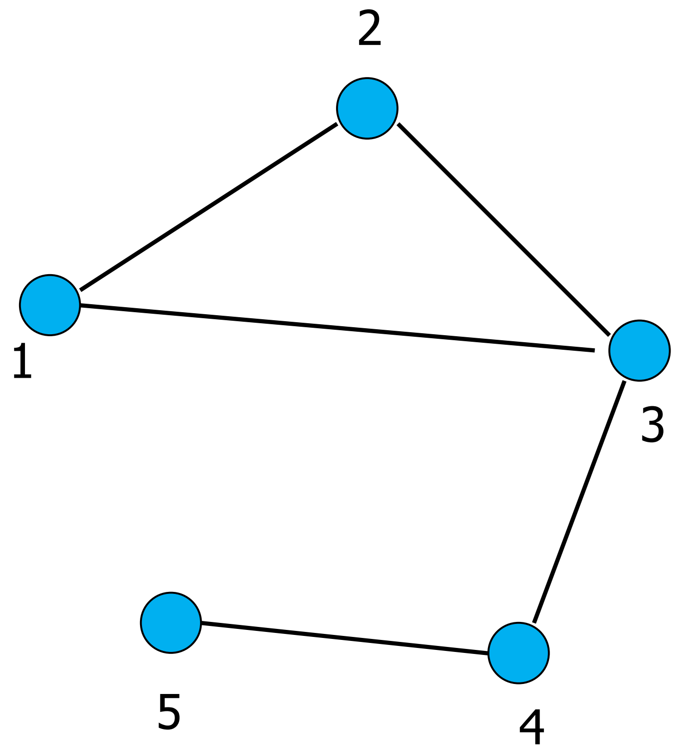
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



Representation

- Adjacency list
  - Not so easy to maintain

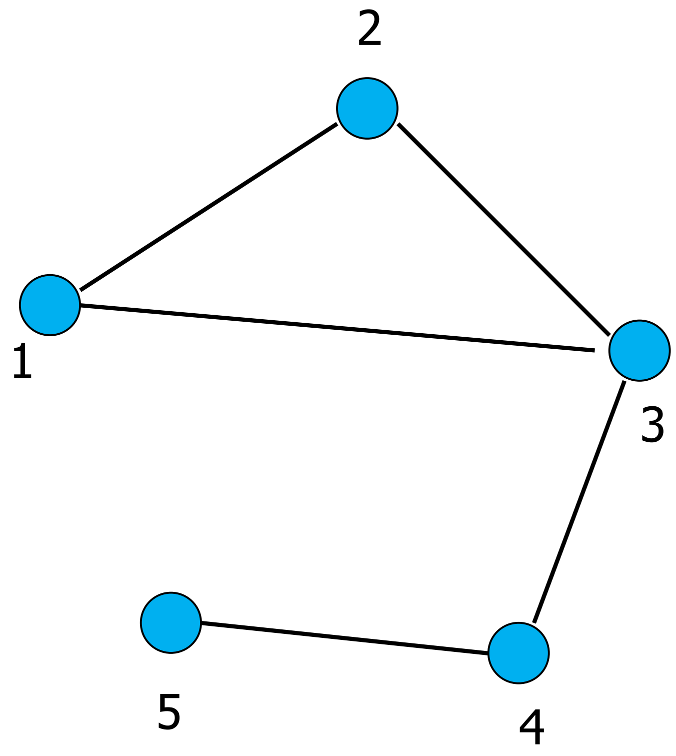
1: [2, 3]
2: [1, 3]
3: [1, 2, 4]
4: [3, 5]
5: [4]



Representation

- List of pairs
  - The simplest and most efficient representation

(1,2)
(2,3)
(1,3)
(3,4)
(4,5)

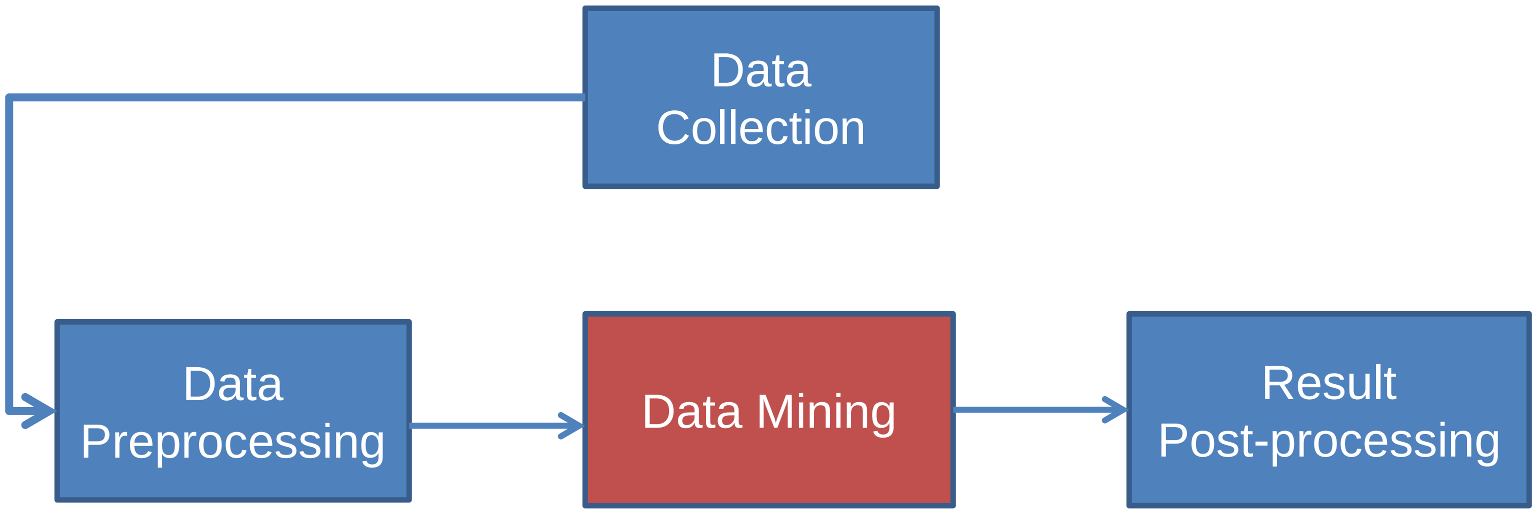


Types of data: summary

- Numeric data: Each object is a point in a multidimensional space
- Categorical data: Each object is a vector of categorical values
- Set data: Each object is a set of values (with or without counts)
  - Sets can also be represented as binary vectors, or vectors of counts
- Dependent data:
  - Ordered sequences: Each object is an ordered sequence of values.
  - Spatial data: objects are fixed on specific geographic locations
  - Graph data: A collection of pairwise relationships

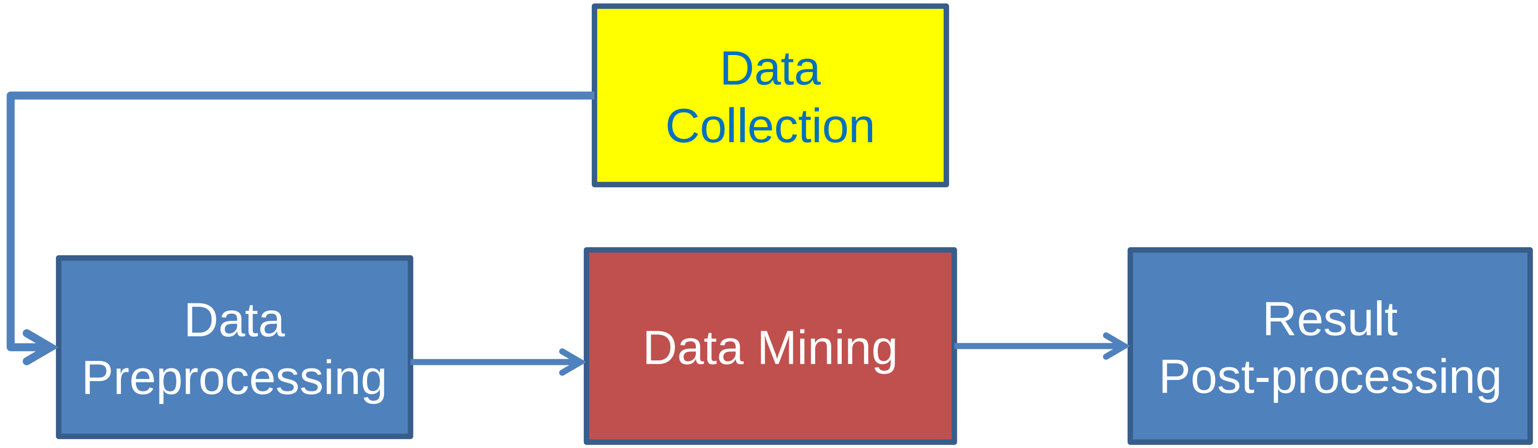
The data analysis pipeline

Mining is not the only step in the analysis process



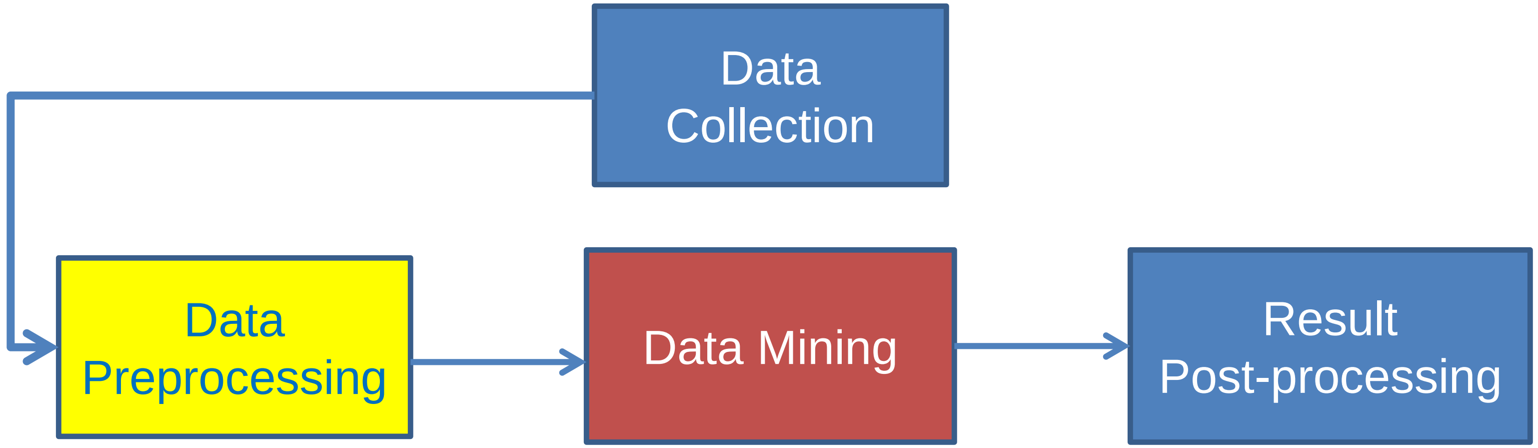
The data mining part is about the analytical methods and algorithms for extracting useful knowledge from the data.

The data analysis pipeline



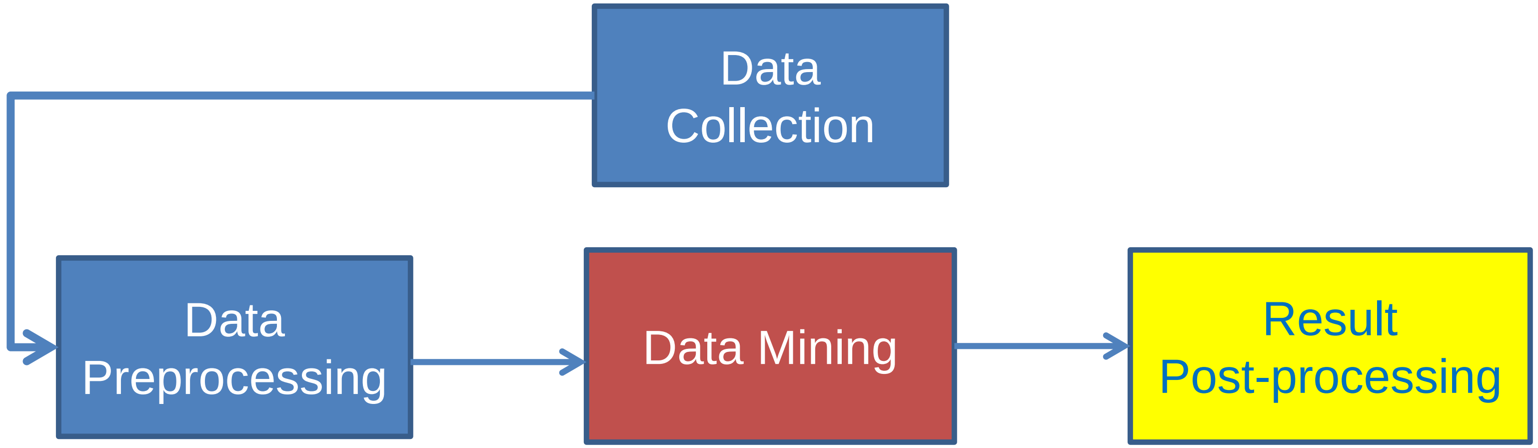
- Today there is an abundance of data online (Twitter, Wikipedia, Web, Open data initiatives, etc)
- Collecting the data is a separate task
  - Customized crawlers, use of public APIs. Respect of crawling etiquette
- Which data should we collect?
  - We cannot necessarily collect everything so we need to make some choices before starting.
- How should we store them?
- In many cases when collecting data we also need to label them
  - E.g., how do we identify fraudulent transactions?
  - E.g., how do we elicit user preferences?

The data analysis pipeline



- Preprocessing: Real data is large, noisy, incomplete and inconsistent.
  - Reducing the data: Sampling, Dimensionality Reduction
  - Data cleaning: deal with missing or inconsistent information
  - Feature extraction and selection: create a useful representation of the data by extracting useful features
- The preprocessing step determines the input to the data mining algorithm
  - A dirty work, but someone has to do it.
  - It is often the most important step for the analysis

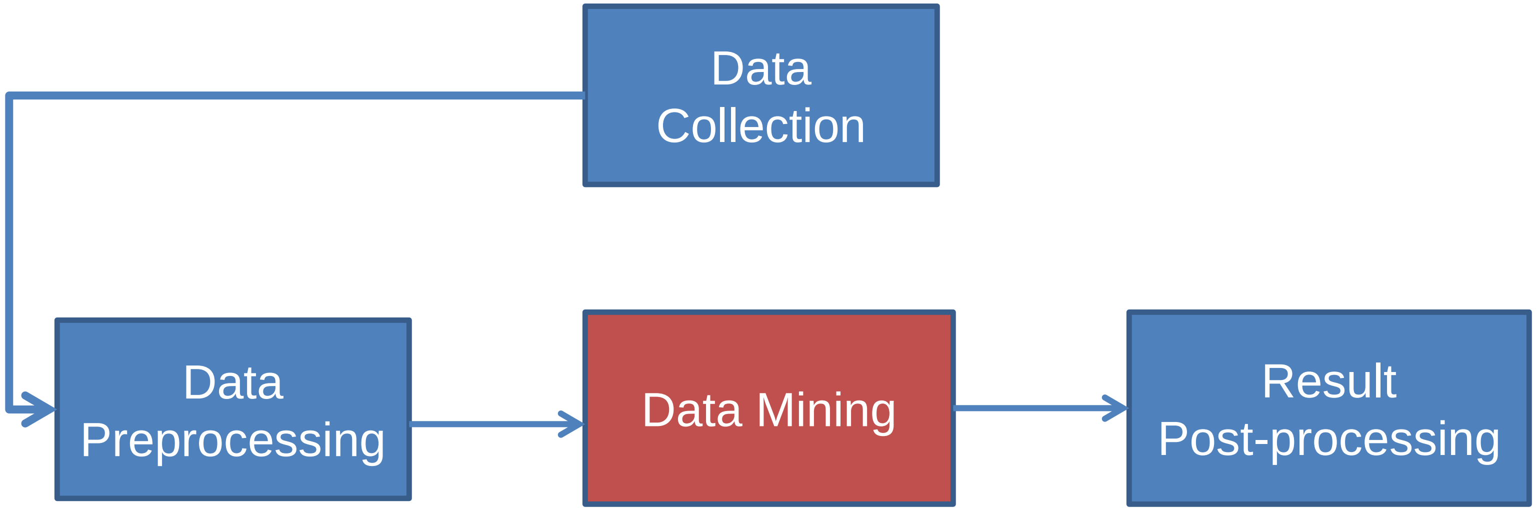
The data analysis pipeline



- Post-Processing: Make the data actionable and useful to the user
  - Statistical analysis of importance of results
  - Visualization

The data analysis pipeline

Mining is not the only step in the analysis process



- Pre- and Post-processing are often data mining tasks as well

Data collection

- Suppose that you want to collect data from Twitter about the elections in USA
  - How do you go about it?
- Twitter Streaming/Search API:
  - Get a sample of all tweets that are posted on Twitter
  - Example of JSON object
- REST API:
  - Get information about specific users.
- There are several decisions that we need to make before we start collecting the data.
  - Time and Storage resources

Data Quality

- Examples of data quality problems:
  - Noise and outliers
  - Missing values
  - Duplicate data

A mistake or a millionaire?

Missing values

Inconsistent duplicate entries

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9	No	Single	90K	**No**

Sampling

- Sampling is the main technique employed for data selection.
  - It is often used for both the preliminary investigation of the data and the final data analysis.

- Statisticians sample because obtaining the entire set of data of interest is too expensive or time consuming.
  - Example: What is the average height of a person in Greece?
    - We cannot measure the height of everybody

- Sampling is used in data mining because processing the entire set of data of interest is too expensive or time consuming.
  - Example: We have 1M documents. What fraction of pairs has at least 100 words in common?
    - Computing number of common words for all pairs requires 10^{12} comparisons
  - Example: What fraction of tweets in a year contain the word “Greece”?
    - 500M tweets per day, if 100 characters on average, 86.5TB to store all tweets

Sampling …

- The key principle for effective sampling is the following:
  - using a sample will work almost as well as using the entire data sets, if the sample is representative
  - A sample is representative if it has approximately the same property (of interest) as the original set of data
  - Otherwise we say that the sample introduces some bias
  - What happens if we take a sample from the university campus to compute the average height of a person at Ioannina?

Types of Sampling

- Simple Random Sampling
  - There is an equal probability of selecting any particular item
- Sampling without replacement
  - As each item is selected, it is removed from the population
- Sampling with replacement
  - Objects are not removed from the population as they are selected for the sample.
    - In sampling with replacement, the same object can be picked up more than once. This makes analytical computation of probabilities easier
    - E.g., we have 100 people, 51 are women $P(W) = 0.51$, 49 men $P(M) = 0.49$. If I pick two persons what is the probability $P(W,W)$ that both are women?
      - Sampling with replacement: $P(W,W) = 0.51^2$
      - Sampling without replacement: $P(W,W) = 51/100 * 50/99$

Types of Sampling

- Stratified sampling
  - Split the data into several groups; then draw random samples from each group.
    - Ensures that all groups are represented.
  - Example 1. I want to understand the differences between legitimate and fraudulent credit card transactions. 0.1% of transactions are fraudulent. What happens if I select 1000 transactions at random?
    - I get 1 fraudulent transaction (in expectation). Not enough to draw any conclusions. Solution: sample 1000 legitimate and 1000 fraudulent transactions

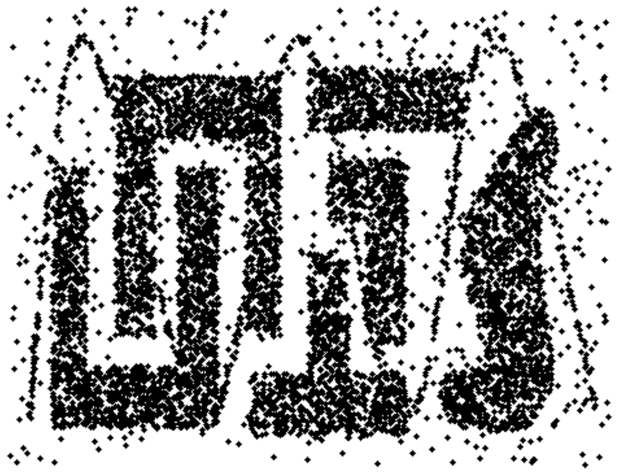
Probability Reminder: If an event has probability p of happening and I do N trials, the expected number of times the event occurs is pN

  - Example 2. I want to answer the question: Do web pages that are linked have on average more words in common than those that are not? I have 1M pages, and 1M links, what happens if I select 10K pairs of pages at random?
    - Most likely I will not get any links.
    - Solution: sample 10K random pairs, and 10K links

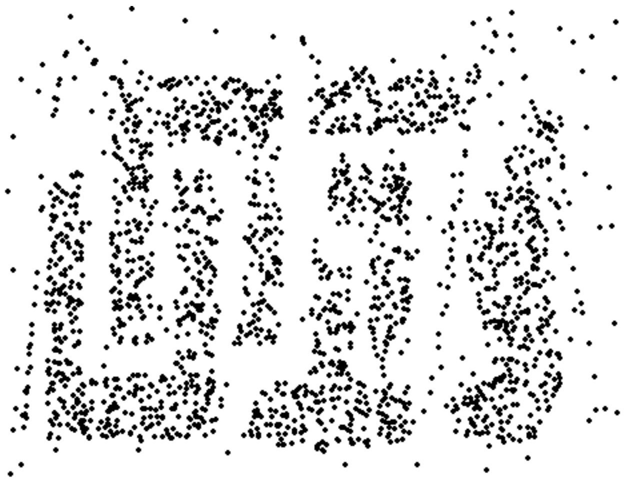
Biased sampling

- Some times we want to bias our sample towards some subset of the data
  - Stratified sampling is one example
- Example: When sampling temporal data, we want to increase the probability of sampling recent data
  - Introduce recency bias
- Make the sampling probability to be a function of time, or the age of an item
  - Typical: Probability decreases exponentially with time
  - For item x_t after time t select with probability $p(x_t) \propto e^{-t}$

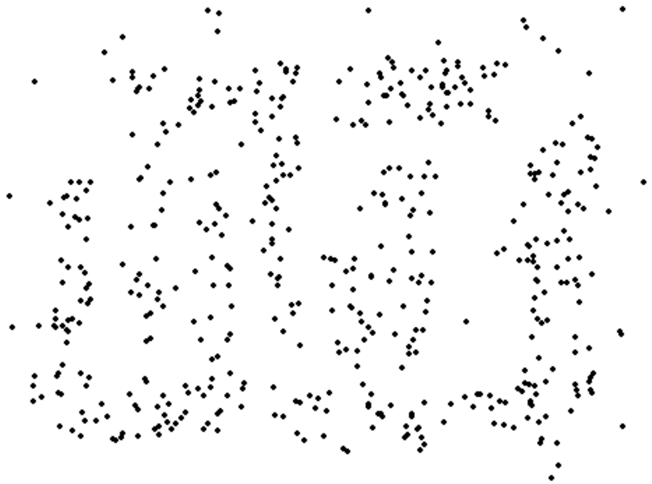
Sample Size



8000 points



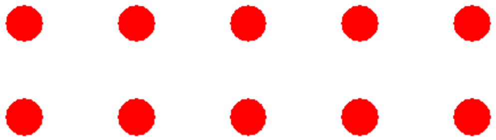
2000 Points

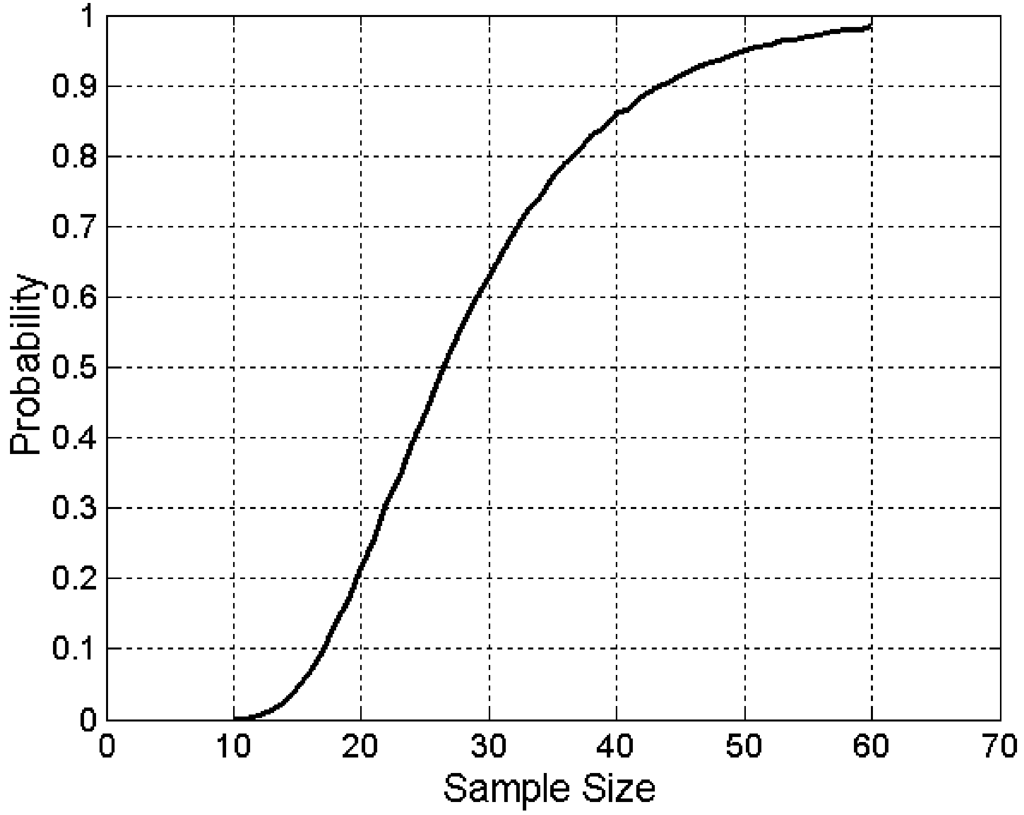


500 Points

Sample Size

- What sample size is necessary to get at least one object from each of 10 groups.





A data mining challenge

- You have N items and you want to sample one item uniformly at random. How do you do that?

- The items are coming in a stream: you do not know the size of the stream in advance, and there is not enough memory to store the stream in memory. You can only keep a constant amount of items in memory
- How do you sample?
  - Hint: if the stream ends after reading k items the last item in the stream should have probability 1/k to be selected.

- Reservoir Sampling:
  - Standard interview question for many companies

Reservoir sampling

- Algorithm: With probability 1/k select the k-th item of the stream and replace the previous choice.
- Claim: Every item has probability 1/N to be selected after N items have been read.
- Proof
  - What is the probability of the k-th item to be selected?
    - $\frac{1}{k}$
  - What is the probability of the k-th item to survive for $N-k$ rounds?
    - $\frac{1}{k}\left(1-\frac{1}{k+1}\right)\left(1-\frac{1}{k+2}\right)\cdots\left(1-\frac{1}{N}\right)=\frac{1}{\mathrm{N}}$

Proof by Induction

- We want to show that the probability the k-th item is selected after $n \geq k$ items have been seen is $\frac{1}{n}$
- Induction on the number of steps
  - Base of the induction: For $n = k$, the probability that the k-th item is selected is $\frac{1}{k}$
  - Inductive Hypothesis: Assume that it is true for N
  - Inductive Step: The probability that the item is still selected after $N + 1$ items is

$$\frac{1}{N}\left(1 - \frac{1}{N+1}\right) = \frac{1}{N+1}$$

Data preprocessing: feature extraction

- The data we obtain are not necessarily as a relational table
- Data may be in a very raw format
  - Examples: text, speech, mouse movements, etc
- We need to extract the features from the data

- Feature extraction:
  - Selecting the characteristics by which we want to represent our data
  - It requires some domain knowledge about the data
  - It depends on the application
- Deep learning: eliminates this step.

A data preprocessing example

- Suppose we want to mine the comments/reviews of people on Yelp or Foursquare.

Mining Task

- Collect all reviews for the top-10 most reviewed restaurants in NY in Yelp


```
{"votes": {"funny": 0, "useful": 2, "cool": 1},
 "user_id": "Xqd0DzHaiyRqVH3WRG7hzg",
 "review_id": "15SdjuK7DmYqUAj6rjGowg",
 "stars": 5, "date": "2007-05-17",
 "text": "I heard so many good things about this place so I was pretty juiced to try
it.  I'm from Cali and I heard Shake Shack is comparable to IN-N-OUT and I gotta
say,  Shake Shake wins hands down.    Surprisingly, the line was short and we waited
about 10 MIN. to order.  I ordered a regular cheeseburger, fries and a black/white
shake.  So yummerz.   I love the location too!  It's in the middle of the city and
the view is breathtaking.   Definitely one of my favorite places to eat in NYC.",
 "type": "review",
 "business_id": "vcNAWiLM4dR7D2nwwJ7nCA"}
```


- Feature extraction: Find few terms that best describe the restaurants.

Example data

I heard so many good things about this place so I was pretty juiced to try it. I'm from Cali and I heard Shake Shack is comparable to IN-N-OUT and I gotta say, Shake Shake wins hands down.   Surprisingly, the line was short and we waited about 10 MIN. to order. I ordered a regular cheeseburger, fries and a black/white shake. So yummerz.   I love the location too! It's in the middle of the city and the view is breathtaking.   Definitely one of my favorite places to eat in NYC.

I'm from California and I must say, Shake Shack is better than IN-N-OUT, all day, err'day.

Would I pay $15+ for a burger here? No. But for the price point they are asking for, this is a definite bang for your buck (though for some, the opportunity cost of waiting in line might outweigh the cost savings) Thankfully, I came in before the lunch swarm descended and I ordered a shake shack (the special burger with the patty + fried cheese & portabella topping) and a coffee milk shake. The beef patty was very juicy and snugly packed within a soft potato roll. On the downside, I could do without the fried portabella-thingy, as the crispy taste conflicted with the juicy, tender burger. How does shake shack compare with in-and-out or 5-guys? I say a very close tie, and I think it comes down to personal affliations. On the shake side, true to its name, the shake was well churned and very thick and luscious. The coffee flavor added a tangy taste and complemented the vanilla shake well. Situated in an open space in NYC, the open air sitting allows you to munch on your burger while watching people zoom by around the city. It's an oddly calming experience, or perhaps it was the food

First cut

- Do simple processing to "normalize" the data (remove punctuation, make into lower case, clear white spaces, other?)
- Break into words, keep the most popular words

the 27514	the 16710	the 16010	the 14241
and 14508	and 9139	and 9504	and 8237
i 13088	a 8583	i 7966	a 8182
a 12152	i 8415	to 6524	i 7001
to 10672	to 7003	a 6370	to 6727
of 8702	in 5363	it 5169	of 4874
ramen 8518	it 4606	of 5159	you 4515
was 8274	of 4365	is 4519	it 4308
is 6835	is 4340	sauce 4020	is 4016
it 6802	burger 432	in 3951	was 3791
in 6402	was 4070	this 3519	pastrami 3748
for 6145	for 3441	was 3453	in 3508
but 5254	but 3284	for 3327	for 3424
that 4540	shack 3278	you 3220	sandwich 2928
you 4366	shake 3172	that 2769	that 2728
with 4181	that 3005	but 2590	but 2715
pork 4115	you 2985	food 2497	on 2247
my 3841	my 2514	on 2350	this 2099
this 3487	line 2389	my 2311	my 2064
wait 3184	this 2242	cart 2236	with 2040
not 3016	fries 2240	chicken 2220	not 1655
we 2984	on 2204	with 2195	your 1622
at 2980	are 2142	rice 2049	so 1610
on 2922	with 2095	so 1825	have 1585

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it 6802	**burger 432**	in 3951	was 3791
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pork 4115	you 2985	food 2497	on 2247
my 3841	my 2514		
this 3487	line 2389		
wait 3184	this 2242	**cart 2236**	
not 3016	**fries 2240**	**chicken 2220**	not 1655
we 2984	on 2204	with 2195	your 1622
at 2980	are 2142	rice 2049	so 1610
on 2922	with 2095	so 1825	have 1585

Most frequent words are **stop words**

Second cut

- Remove stop words
  - Stop-word lists can be found online.

```
a,about,above,after,again,against,all,am,an,and,any,are,aren't,as,at,be,because
,been,before,being,below,between,both,but,by,can't,cannot,could,couldn't,did,di
dn't,do,does,doesn't,doing,don't,down,during,each,few,for,from,further,had,hadn
't,has,hasn't,have,haven't,having,he,he'd,he'll,he's,her,here,here's,hers,herse
lf,him,himself,his,how,how's,i,i'd,i'll,i'm,i've,if,in,into,is,isn't,it,it's,it
s,itself,let's,me,more,most,mustn't,my,myself,no,nor,not,of,off,on,once,only,or
,other,ought,our,ours,ourselves,out,over,own,same,shan't,she,she'd,she'll,she's
,should,shouldn't,so,some,such,than,that,that's,the,their,theirs,them,themselve
s,then,there,there's,these,they,they'd,they'll,they're,they've,this,those,throu
gh,to,too,under,until,up,very,was,wasn't,we,we'd,we'll,we're,we've,were,weren't
,what,what's,when,when's,where,where's,which,while,who,who's,whom,why,why's,wit
h,won't,would,wouldn't,you,you'd,you'll,you're,you've,your,yours,yourself,yours
elves,
```

Second cut

- Remove stop words
  - Stop-word lists can be found online.

ramen 8572	burger 4340	sauce 4023	pastrami 3782
pork 4152	shack 3291	food 2507	sandwich 2934
wait 3195	shake 3221	cart 2239	place 1480
good 2867	line 2397	chicken 2238	good 1341
place 2361	fries 2260	rice 2052	get 1251
noodles 2279	good 1920	hot 1835	katz's 1223
ippudo 2261	burgers 1643	white 1782	just 1214
buns 2251	wait 1508	line 1755	like 1207
broth 2041	just 1412	good 1629	meat 1168
like 1902	cheese 1307	lamb 1422	one 1071
just 1896	like 1204	halal 1343	deli 984
get 1641	food 1175	just 1338	best 965
time 1613	get 1162	get 1332	go 961
one 1460	place 1159	one 1222	ticket 955
really 1437	one 1118	like 1096	food 896
go 1366	long 1013	place 1052	sandwiches 813
food 1296	go 995	go 965	can 812
bowl 1272	time 951	can 878	beef 768
can 1256	park 887	night 832	order 720
great 1172	can 860	time 794	pickles 699
best 1167	best 849	long 792	time 662
		people 790	

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pork 4152	shack 3291	food 2507	sandwich 2934
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place 2361	fries 2260	rice 2052	**get** 1251
noodles 2279	good 1920	hot 1835	katz's 1223
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get 1641	food 1175	just 1338	best 965
time 1613	**get** 1162	**get** 1332	go 961
one 1460	place 1159	one 1222	ticket 955
really 1437	one 1118	**like** 1096	food 896
go 1366	long 1013	place 1052	
food 1296			
bowl 1272			
can 1256	park 887	night 832	order 720
great 1172	can 860	time 794	pickles 699
best 1167	best 849	long 792	time 662
		people 790	

Commonly used words in reviews, not so interesting

IDF

- Important words are the ones that are unique to the document (differentiating) compared to the rest of the collection
  - All reviews use the word “like”. This is not interesting
  - We want the words that characterize the specific restaurant

- Document Frequency $DF(w)$: fraction of documents that contain word w.

$$DF(w) = \frac{D(w)}{D}$$

$D(w)$: num of docs that contain word w
D: total number of documents

- Inverse Document Frequency $IDF(w)$:

$$IDF(w) = \log\left(\frac{1}{DF(w)}\right)$$

- Maximum when unique to one document : $IDF(w) = \log(D)$
- Minimum when the word is common to all documents: $IDF(w) = 0$

TF-IDF

- The words that are best for describing a document are the ones that are important for the document, but also unique to the document.

- $TF(w, d)$: term frequency of word w in document d
  - Number of times that the word appears in the document
  - Natural measure of importance of the word for the document

- $IDF(w)$: inverse document frequency
  - Natural measure of the uniqueness of the word w

- $TF\text{-}IDF(w, d) = TF(w, d) \times IDF(w)$

Third cut

- Ordered by TF-IDF

ramen 3057.4176194	fries 806.08537330	lamb 985.655290756243	pastrami 1931.94250908298 6
akamaru 2353.24196	custard 729.607519	halal 686.038812717726	katz's 1120.62356508209 4
noodles 1579.68242	shakes 628.4738038	53rd 375.685771863491	rye 1004.28925735888 2
broth 1414.7133955	shroom 515.7790608	gyro 305.809092298788	corned 906.113544700399 2
miso 1252.60629058	burger 457.2646379	pita 304.984759446376	pickles 640.487221580035 4
hirata 709.1962086	crinkle 398.347221	cart 235.902194557873	reuben 515.779060830666 1
hakata 591.7643688	burgers 366.624854	platter 139.45990308004	matzo 430.583412389887 1
shiromaru 587.1591	madison 350.939350	chicken/lamb 135.852520	sally 428.110484707471 2
noodle 581.8446147	shackburger 292.42	carts 120.274374158359	harry 226.323810772916 4
tonkotsu 529.59457	'shroom 287.823136	hilton 84.2987473324223	mustard 216.079238853014 6
ippudo 504.5275695	portobello 239.806	lamb/chicken 82.8930633	cutter 209.535243462458 1
buns 502.296134008	custards 211.83782	yogurt 70.0078652365545	carnegie 198.655512713779 3
ippudo's 453.60926	concrete 195.16992	52nd 67.5963923222322	katz 194.387844446609 7
modern 394.8391629	bun 186.9621782983	6th 60.7930175345658 9	knish 184.206807439524 1
egg 367.3680056967	milkshakes 174.996	4am 55.4517744447956 5	sandwiches 181.415707218 8
shoyu 352.29551922	concretes 165.7861	yellow 54.4470265206673	brisket 131.945865389878 4
chashu 347.6903490	portabello 163.483	tzatziki 52.95945713886	fries 131.613054313392 7
karaka 336.1774235	shack's 159.334353	lettuce 51.32301680226	salami 127.621117258549 3
kakuni 276.3102111	patty 152.22603588	sammy's 50.656872045869	knishes 124.339595021678 1
ramens 262.4947006	ss 149.66803104461	sw 50.5668577816893 3	delicatessen 117.488967607 2
bun 236.5122638036	patties 148.068287	platters 49.90659700031	deli's 117.431839742696 1
wasabi 232.3667512	cam 105.9496067806	falafel 49.479699521204	carver 115.129254649702 1
dama 221.048168927	milkshake 103.9720	sober 49.2211422635451	brown's 109.441778045519 2
brulee 201.1797390	lamps 99.011158998	moma 48.1589121730374	matzoh 108.22149937072 1

Third cut

- TF-IDF takes care of stop words as well
- We do not need to remove the stopwords since they will get $IDF(w) = 0$
- Important: IDF is collection-dependent!
  - For some other corpus the words *get, like, eat*, may be important

Decisions, decisions…

- When mining real data you often need to make some decisions
  - What data should we collect? How much? For how long?
  - Should we throw out some data that does not seem to be useful?

An actual review

```
AAAAAAAAAAAAA
AAAAAAAAAAAAAAAAAAAAAAAAA AAAAAAAAAAAAAAAAAAAAAAAAA AAA
```

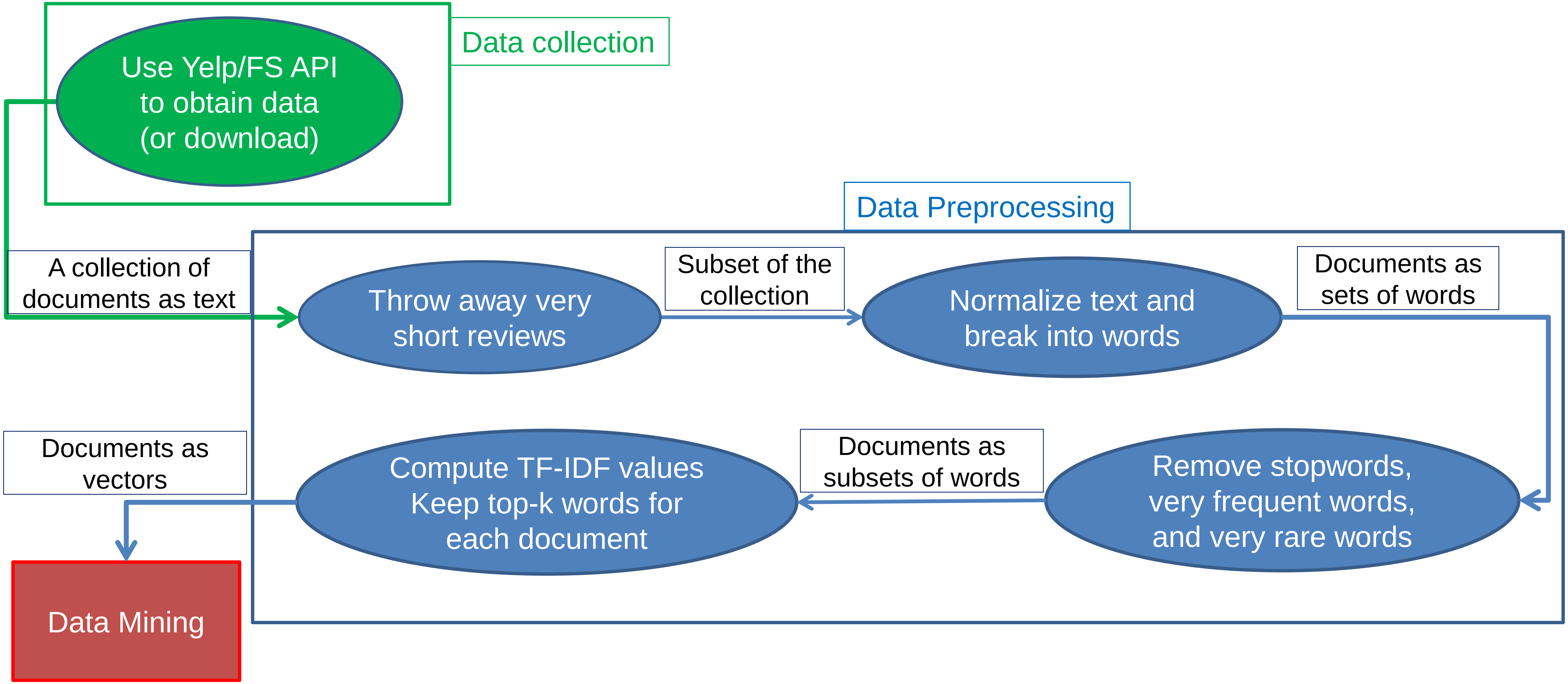
    - Too frequent data (stop words), too infrequent (errors?), erroneous data, missing data, outliers
  - How should we weight the different pieces of data?

- Most decisions are application dependent. Some information may be lost but we can usually live with it (most of the times)

- We should make our decisions clear since they affect our findings.

- Dealing with real data is hard…

The preprocessing pipeline for our text mining task



Word and document representations

- Using TF-IDF values has a very long history in text mining
  - Assigns a numerical value to each word, and a vector to a document
- Recent trend: Use word embeddings
  - Map every word into a multidimensional vector
- Use the notion of context: the words that surround a word in a phrase
  - Similar words appear in similar contexts
  - Similar words should be mapped to close-by vectors
- Example: words “movie” and “film”

The actor for the movie / film Joker is candidate for an Oscar

- Both words are likely to appear with similar words
  - director, actor, actress, scenario, script, Oscar, cinemas etc

word2vec

- Two approaches

CBOW: Learn an embedding for words so that given the context you can predict the missing word

Skip-Gram: Learn an embedding for words such that given a word you can predict the context

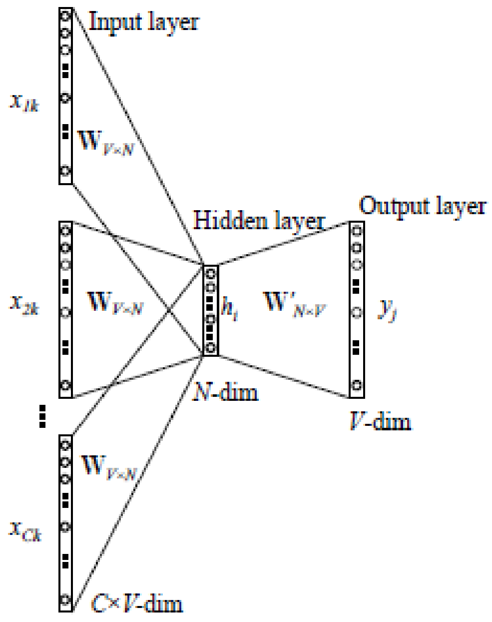


Figure 2: Continuous bag-of-word model

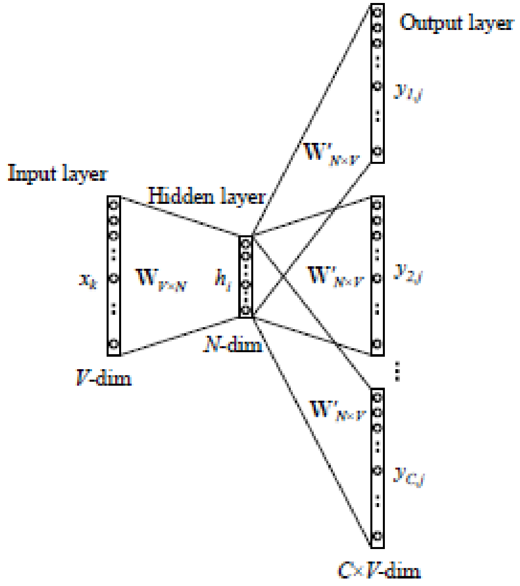


Figure 3: The skip-gram model.

Normalization of numeric data

- In many cases it is important to normalize the data rather than use the raw values
- The kind of normalization that we use depends on what we want to achieve

Column normalization

- In this data, different attributes take very different range of values. For distance/similarity the small values will disappear
- We need to make them comparable

Temperature	Humidity	Pressure
30	0.8	90
32	0.5	80
24	0.3	95

Column Normalization

- Divide (the values of a column) by the maximum value for each attribute
  - Brings everything in the [0,1] range, maximum is 1

Temperature	Humidity	Pressure
0.9375	1	0.9473
1	0.625	0.8421
0.75	0.375	1

new value = old value / max value in the column

Temperature	Humidity	Pressure
30	0.8	90
32	0.5	80
24	0.3	95

Column Normalization

- Subtract the minimum value and divide by the difference of the maximum value and minimum value for each attribute
  - Brings everything in the [0,1] range, maximum is one, minimum is zero

Temperature	Humidity	Pressure
0.75	1	0.33
1	0.6	0
0	0	1

new value = (old value – min column value) / (max col. value –min col. value)

Temperature	Humidity	Pressure
30	0.8	90
32	0.5	80
24	0.3	95

Row Normalization

- Are these documents similar?

	Word 1	Word 2	Word 3
Doc 1	28	50	22
Doc 2	12	25	13

Row Normalization

- Are these documents similar?
- Divide by the sum of values for each document (row in the matrix)
  - Transform a vector into a distribution*

	Word 1	Word 2	Word 3
Doc 1	0.28	0.5	0.22
Doc 2	0.24	0.5	0.26

new value = old value / Σ old values in the row

*For example, the value of cell (Doc1, Word2) is the probability that a randomly chosen word of Doc1 is Word2

	Word 1	Word 2	Word 3
Doc 1	28	50	22
Doc 2	12	25	13

Row Normalization

- Do these two users rate movies in a similar way?

	Movie 1	Movie 2	Movie 3
User 1	1	2	3
User 2	2	3	4

Row Normalization

- Do these two users rate movies in a similar way?
- Subtract the mean value for each user (row) – centering of data
  - Captures the deviation from the average behavior

	Movie 1	Movie 2	Movie 3
User 1	-1	0	+1
User 2	-1	0	+1

new value = (old value – mean row value) [/ (max row value –min row value)]

	Movie 1	Movie 2	Movie 3
User 1	1	2	3
User 2	2	3	4

Row Normalization

$$\text{mean}(x) = \frac{1}{N}\sum_{j=1}^{N} x_j$$

- Z-score:

$$z_i = \frac{x_i - \text{mean}(x)}{\text{std}(x)}$$

$$\text{std}(x) = \sqrt{\frac{\sum_{j=1}^{N}\left(x_j - \text{mean}(x)\right)^2}{N}}$$

Average “distance” from the mean
N may be N-1: population vs sample

- Measures the number of standard deviations away from the mean

	Movie 1	Movie 2	Movie 3
User 1	1.01	-0.87	-0.22
User 2	-1.01	0.55	0.93

	Movie 1	Movie 2	Movie 3	Mean	STD
User 1	5	2	3	3.33	1.53
User 2	1	3	4	2.66	1.53

Row Normalization

- What if we want to transform the scores into probabilities?
  - E.g., probability that the user will visit the restaurant again
  - Different from "probability that the user will select one among the three"
- One idea: Normalize by the max score:

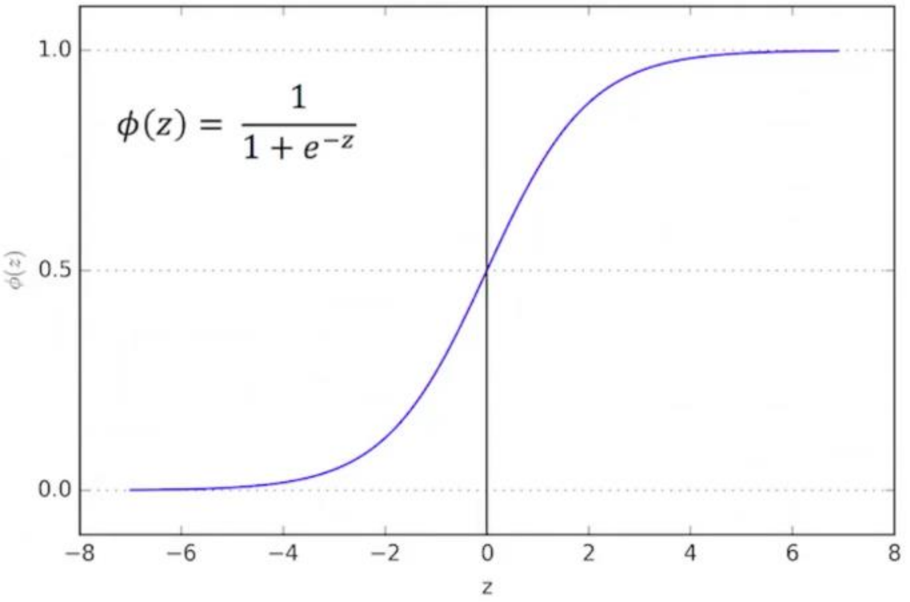
	Restaurant 1	Restaurant 2	Restaurant 3
User 1	1	0.4	0.6
User 2	0.25	0.75	1

- Problem with that?
  - We have probability 1, too strong

	Restaurant 1	Restaurant 2	Restaurant 3
User 1	5	2	3
User 2	1	3	4

Row Normalization

- Another idea: Use the logistic function:
  - Maps reals to the [0,1] range
  - Mimics the step function
  - In the class of sigmoid functions



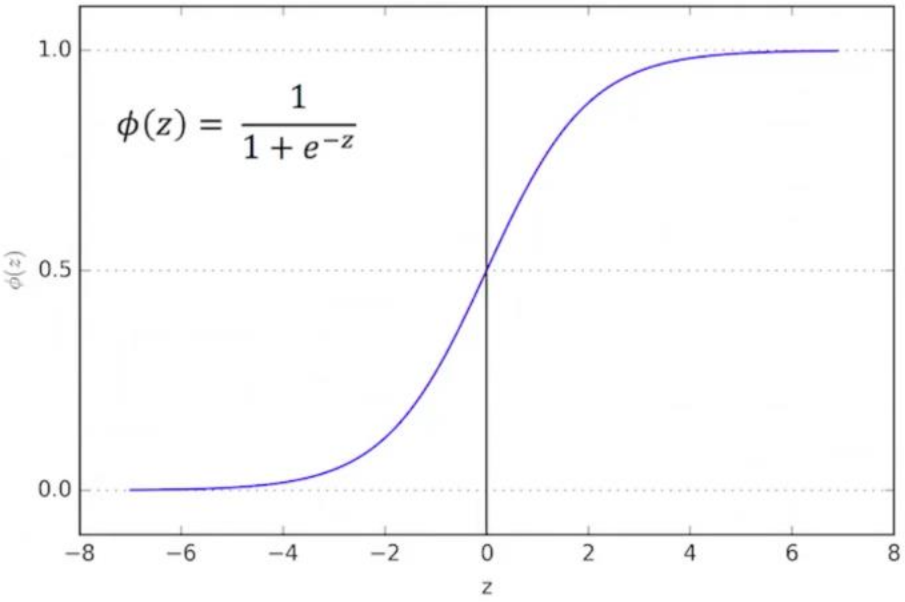
	Restaurant 1	Restaurant 2	Restaurant 3
User 1	0.99	0.88	0.95
User 2	0.73	0.95	0.98

Too big values for all restaurants

	Restaurant 1	Restaurant 2	Restaurant 3
User 1	5	2	3
User 2	1	3	4

Row Normalization

- Another idea: Use the logistic function:
  - Maps reals to the [0,1] range
  - Mimics the step function
  - In the class of sigmoid functions



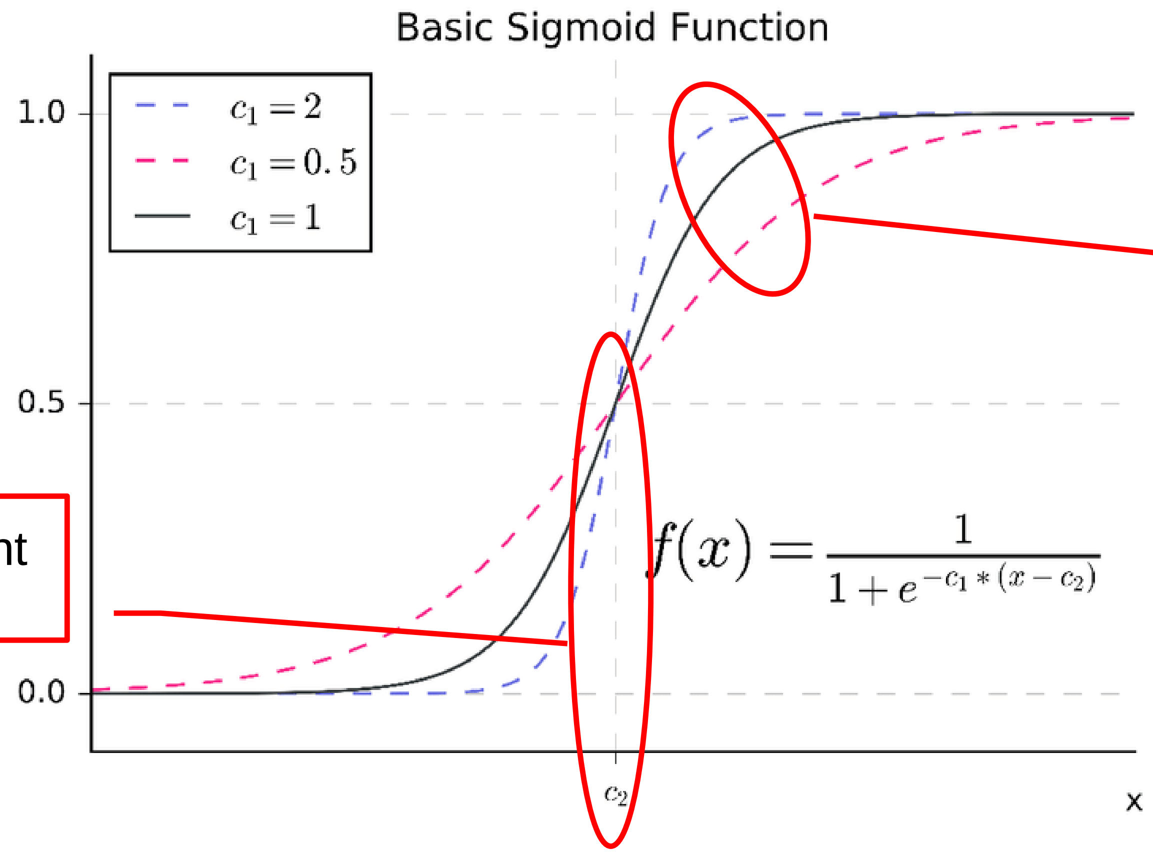
	Restaurant 1	Restaurant 2	Restaurant 3
User 1	0.99	0.88	0.95
User 2	0.73	0.95	0.98

Subtract the mean
Mean value gets 50-50 probability

	Restaurant 1	Restaurant 2	Restaurant 3
User 1	5	2	3
User 2	1	3	4

Row Normalization

- General sigmoid function:
  - We can control the zero point and the slope



Higher c_1 closer to a step function

c_2 controls the 0.5 point – change of slope

Row Normalization

- What if we want to transform the scores into probabilities that sum to one, but we capture the single selection of the user?
- Use the softmax function

$$\frac{e^{x_i}}{\sum_i e^{x_i}}$$

	Restaurant 1	Restaurant 2	Restaurant 3
User 1	0.72	0.10	0.18
User 2	0.07	0.31	0.62

	Restaurant 1	Restaurant 2	Restaurant 3
User 1	5	2	3
User 2	1	3	4

Exploratory analysis of data

- Summary statistics: numbers that summarize properties of the data
- Summarized properties include frequency, location and spread
  - Examples: location - mean
    spread - standard deviation
- Most summary statistics can be calculated in a single pass through the data
- Computing data statistics is one of the first steps in understanding our data

Frequency and Mode

- The frequency of an attribute value is the percentage of time the value occurs in the data set
  - For example, given the attribute ‘gender’ and a representative population of people, the gender ‘female’ occurs about 50% of the time.
- The mode of an attribute is the most frequent attribute value
- The notions of frequency and mode are typically used with categorical data

- We can visualize the data frequencies using a value histogram

Example

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	10000K	**Yes**
6	No	NULL	60K	**No**
7	Yes	Divorced	220K	**NULL**
8	No	Single	85K	**Yes**
9	No	Married	90K	**No**
10	No	Single	90K	**No**

Marital Status

Single	**Married**	**Divorced**	**NULL**
4	3	2	1

Mode: Single

Example

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7	Yes	Divorced	220K	**NULL**
8	No	Single	85K	**Yes**
9	No	Married	90K	**No**
10	No	Single	90K	**No**

Marital Status

Single	**Married**	**Divorced**	**NULL**
40%	30%	20%	10%

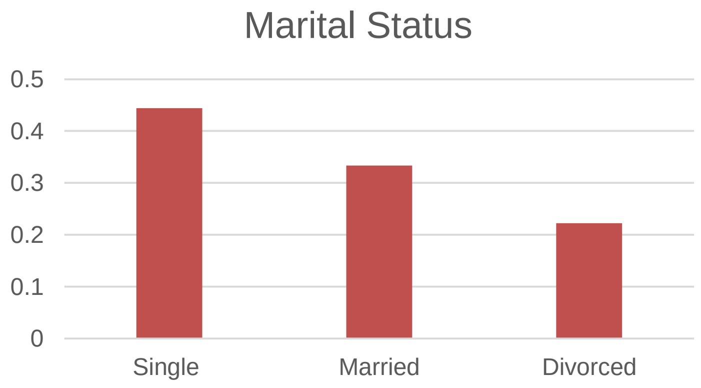
Example

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	10000K	Yes
6	No	NULL	60K	No
7	Yes	Divorced	220K	NULL
8	No	Single	85K	Yes
9	No	Married	90K	No
10	No	Single	90K	No

We can choose to ignore NULL values

Marital Status

Single	Married	Divorced
44%	33%	22%



Data histograms

Tid	Refund	Marital Status	Taxable Income	Cheat
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6	No	NULL	60K	No
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8	No	Single	85K	Yes
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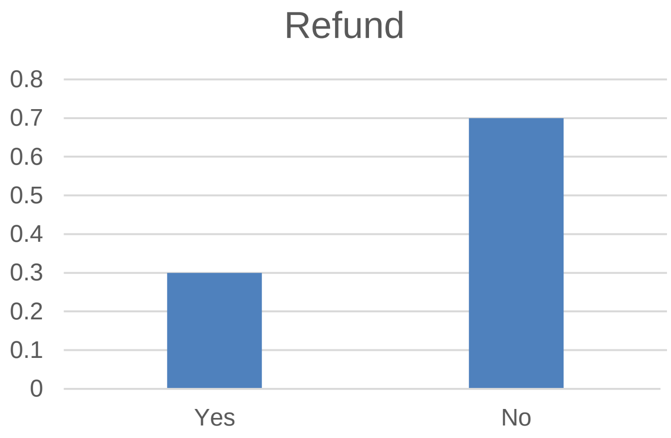
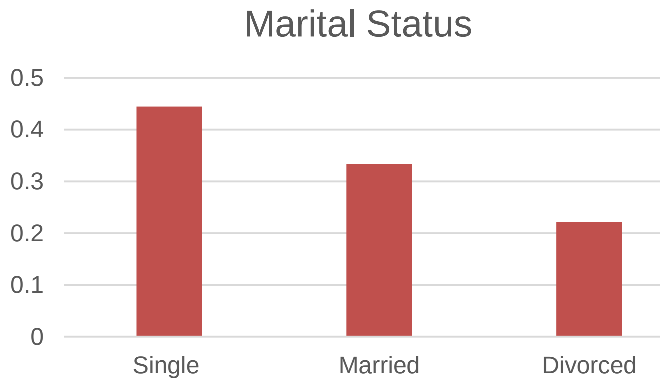
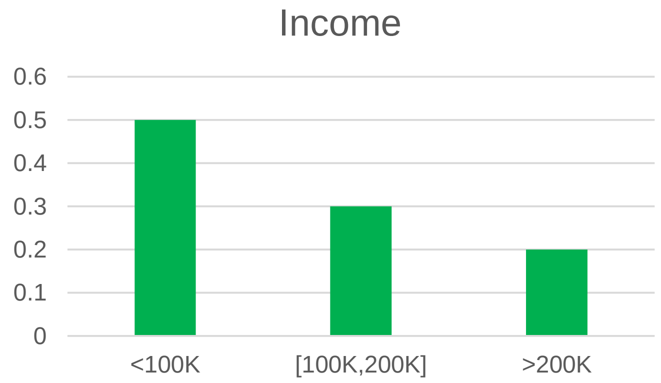
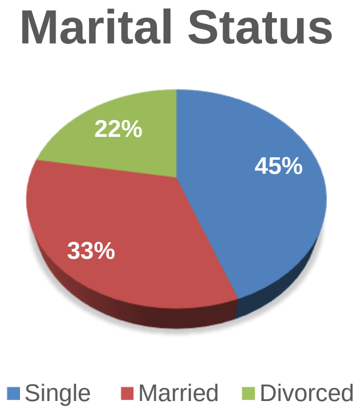
10	No	Single	90K	No

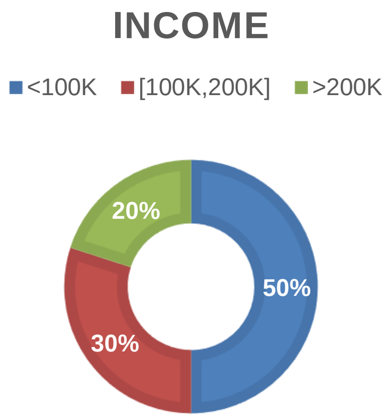
Use binning for numerical values











Percentiles

- For continuous data, the notion of a percentile is more useful.

Given an ordinal or continuous attribute x and a number p between 0 and 100, the p^{th} percentile is a value x_p of x such that $p\%$ of the observed values of x are less or equal than x_p.

- For instance, the 80th percentile is the value $x_{80\%}$ that is greater or equal than 80% of all the values of x we have in our data.

Example

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	**No**
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3	No	Single	70K	**No**
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7	Yes	Divorced	220K	**NULL**
8	No	Single	85K	**Yes**
9	No	Married	90K	**No**
10	No	Single	90K	**No**

Taxable Income
10000K
220K
125K
120K
100K
90K
90K
85K
70K
60K

$x_{80\%}$ = 125K

Measures of Location: Mean and Median

- The mean is the most common measure of the location of a set of points.

$$\text{mean}(x) = \overline{x} = \frac{1}{m}\sum_{i=1}^{m} x_i$$

- However, the mean is very sensitive to outliers.

$$\text{median}(x) = \begin{cases} x_{(r+1)} & \text{if } m \text{ is odd, i.e., } m = 2r+1 \\ \frac{1}{2}(x_{(r)} + x_{(r+1)}) & \text{if } m \text{ is even, i.e., } m = 2r \end{cases}$$

- Thus, the median or a trimmed mean is also commonly used.

Example

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	**No**
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8	No	Single	85K	**Yes**
9	No	Married	90K	**No**
10	No	Single	90K	**No**

Mean: 1090K

Trimmed mean (remove min, max): 105K

Median: (90+100)/2 = 95K

Measures of Spread: Range and Variance

- Range is the difference between the max and min

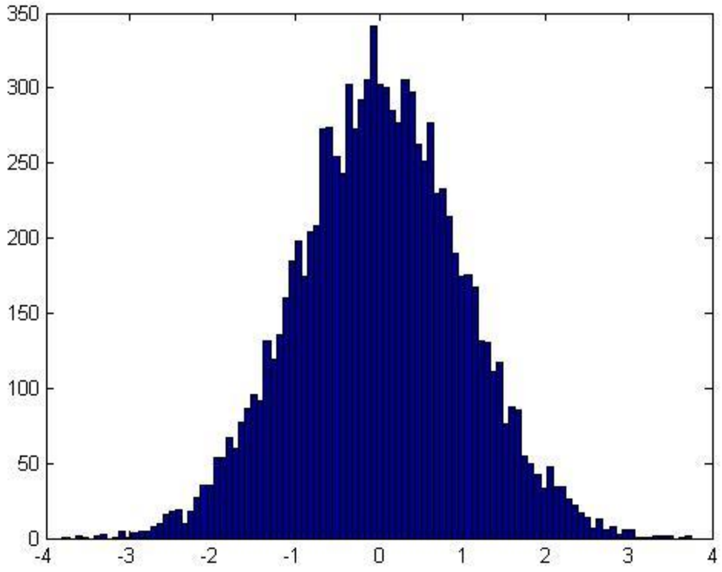
- The variance or standard deviation is the most common measure of the spread of a set of points.

$$var(x) = \frac{1}{m}\sum_{i=1}^{m}(x - \bar{x})^2$$

$$\sigma(x) = \sqrt{var(x)}$$

Normal Distribution

- $\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$



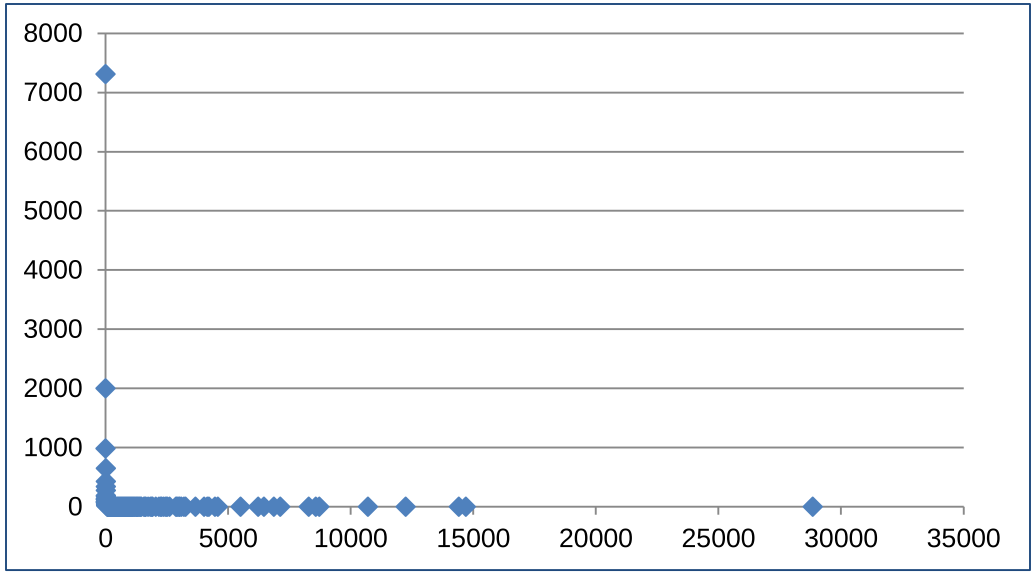
This is a value histogram

- An important distribution that characterizes many quantities and has a central role in probabilities and statistics.
- Appears also in the central limit theorem: the distribution of the sum of IID random variables.
- Fully characterized by the mean μ and standard deviation σ

Not everything is normally distributed

- Plot of number of words with x number of occurrences

y: number of words with x number of occurrences



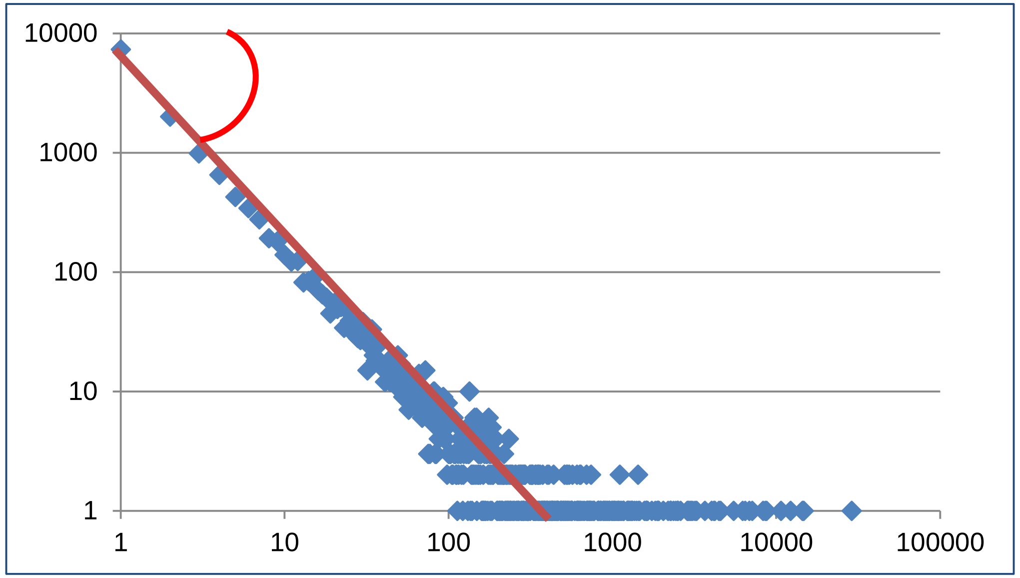
x: number of occurrences

- If this was a normal distribution we would not have number of occurrences as large as 28K

Power-law distribution

- We can understand the distribution of words if we take the log-log plot

y: logarithm of number of words with x number of occurrences



x: logarithm of number of occurrences

Power-law distribution:

$$p(k) = k^{-a}$$

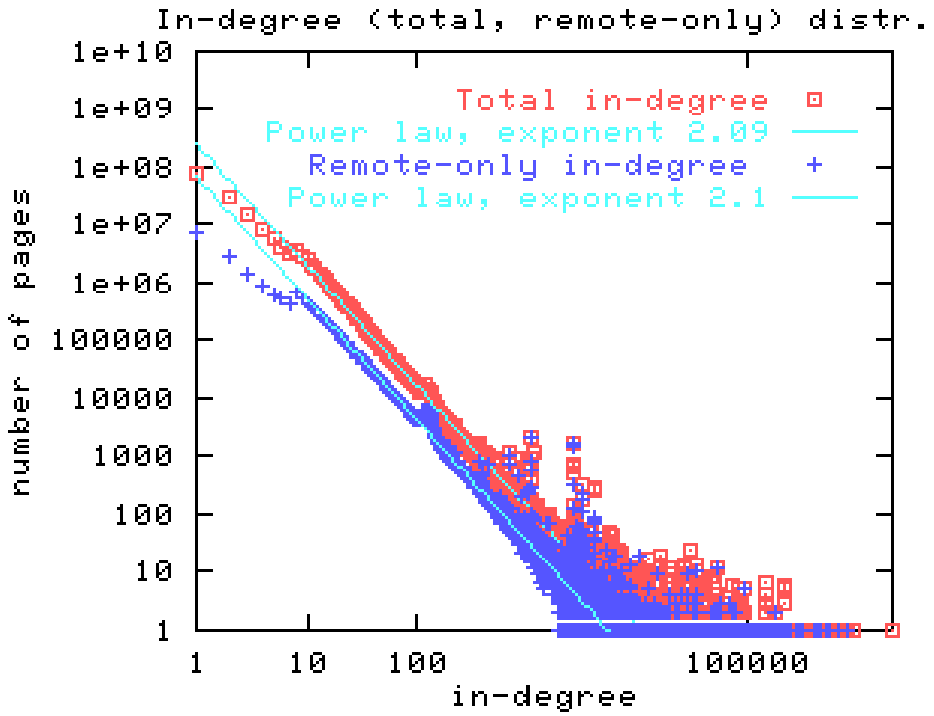
The slope of the line gives us the exponent α

Linear relationship in the log-log space

$$\log p(x = k) = -a \log k$$

Power-laws are everywhere

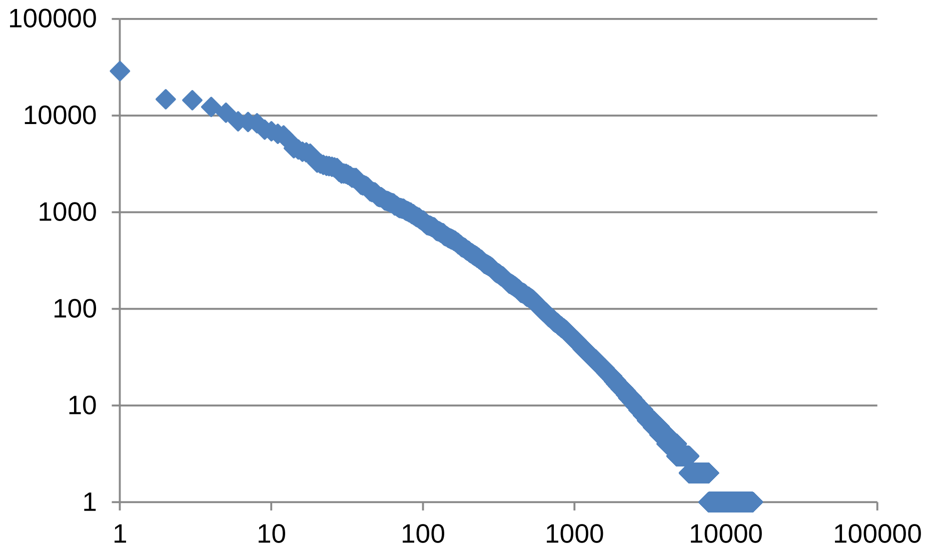
- Incoming and outgoing links of web pages, number of friends in social networks, number of occurrences of words, file sizes, city sizes, income distribution, popularity of products and movies
  - Signature of human activity?
  - A mechanism that explains everything?
  - Rich get richer process



Zipf's law

- Power laws can be detected also by a linear relationship in the log-log space for the rank-frequency plot

y: number of occurrences of the r-th most frequent word



r: rank of word according to frequency (1st, 2nd …)

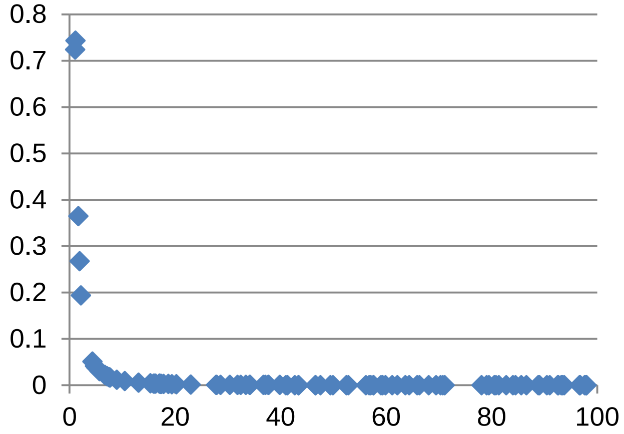
Zipf distribution:
$f(r) = r^{-\beta}$

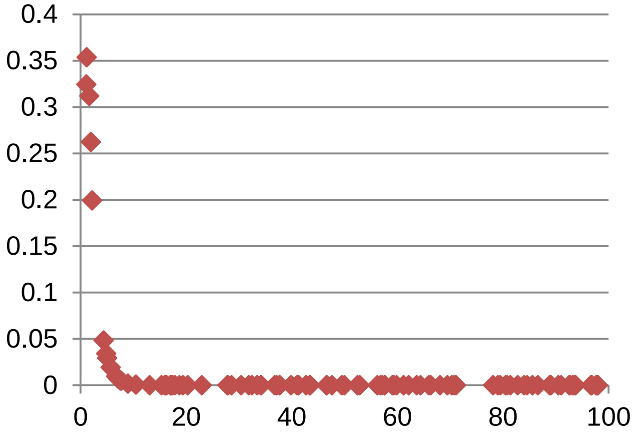
- $f(r)$: Frequency of the r-th most frequent word

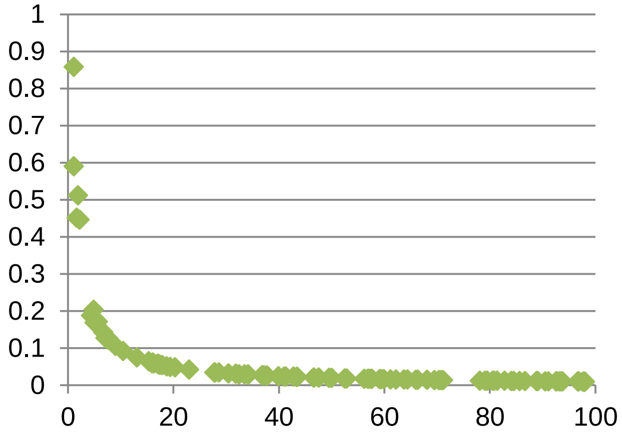
$$\log f(r) = -\beta \log r$$

The importance of correct representation

- Consider the following three plots which are histograms of values. What do you observe? What can you tell of the underlying function?

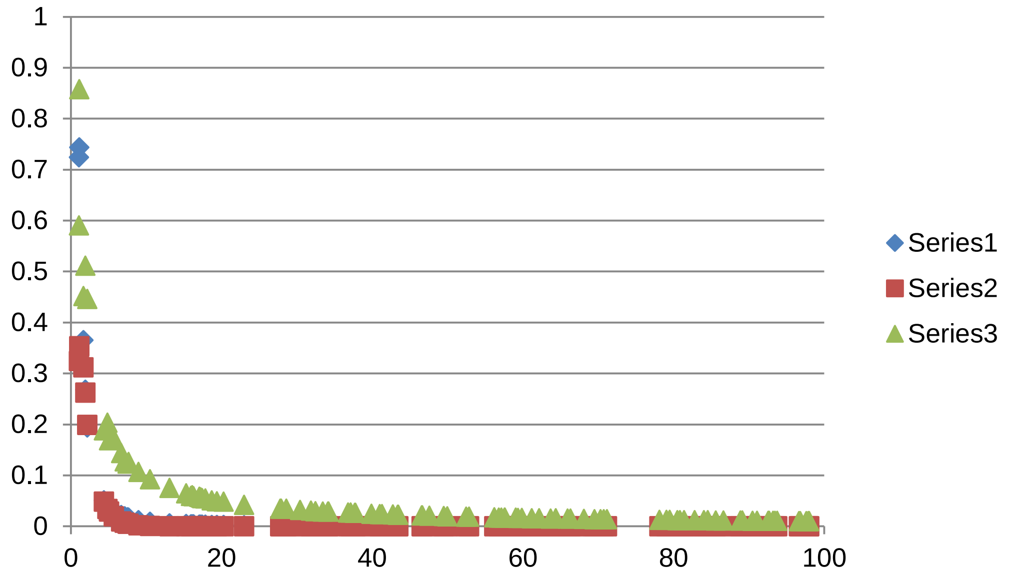






The importance of correct representation

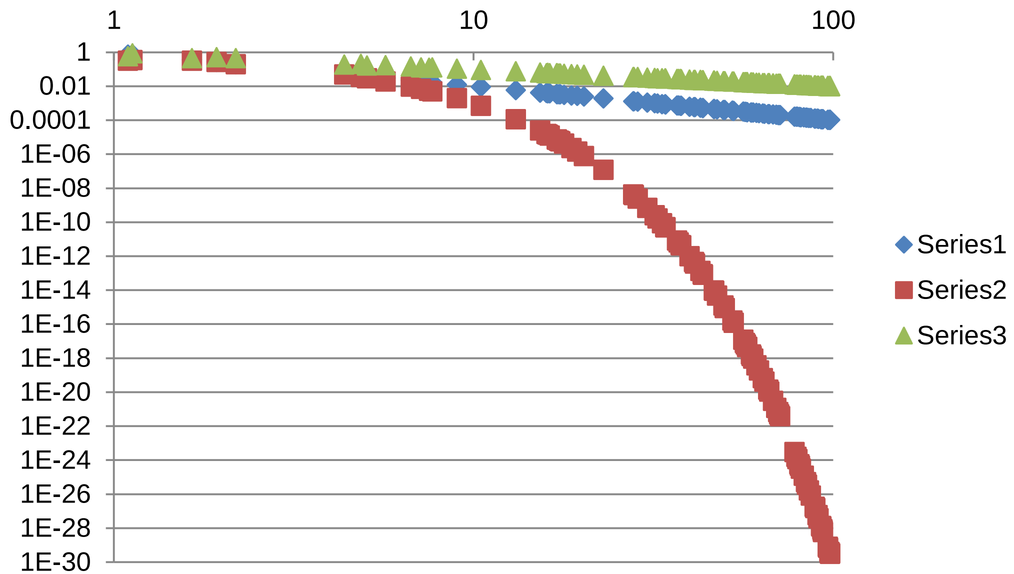
- Putting all three plots together makes it clearer to see the differences



- Green falls more slowly. Blue and Red seem more or less the same

The importance of correct representation

- Making the plot in log-log space makes the differences more clear



- Green and Blue form straight lines. Red drops exponentially.
  - $y = \frac{1}{2x+\epsilon}$ $\log y \approx -\log x + c$
  - $y = \frac{1}{x^2+\epsilon}$ $\log y \approx -2\log x + c$
  - $y = 2^{-x} + \epsilon$ $\log y \approx -x + c = -10^{\log x} + c$

Linear relationship in log-log means polynomial in linear-linear
The slope in the log-log is the exponent of the polynomial

Attribute relationships

- In many cases it is interesting to look at two attributes together to understand if they are correlated
  - E.g., how does your marital status relate with tax cheating?
  - E.g., Does refund correlate with average income?
  - Is there a relationship between years of study and income?
- How do we visualize these relationships?

Plotting attributes against each other

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	10000K	**Yes**
6	No	Married	60K	**No**
7	Yes	Divorced	220K	**No**
8	No	Single	85K	**Yes**
9	No	Married	90K	**No**
10	No	Single	90K	**No**

Confusion Matrix

	No	**Yes**
Single	2	1
Married	4	0
Divorced	1	1

Plotting attributes against each other

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Confusion Matrix

	No	**Yes**
Single	2	1
Married	4	0
Divorced	1	1

Joint Distribution Matrix

	No	**Yes**
Single	0.2	0.1
Married	0.4	0.0
Divorced	0.1	0.1

	No	**Yes**
Single	0.2	0.1
Married	0.4	0.0
Divorced	0.1	0.1

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9	No	Married	90K	**No**
10	No	Single	90K	**No**

Joint Distribution Matrix

	No	**Yes**	
Single	0.2	0.1	0.3
Married	0.4	0.0	0.4
Divorced	0.1	0.1	0.2
	0.8	0.2	1

Marginal distribution for Marital Status

Marginal distribution for Cheat

Plotting attributes against each other

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	**No**
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6	No	Married	60K	**No**
7	Yes	Divorced	220K	**No**
8	No	Single	85K	**Yes**
9	No	Married	90K	**No**
10	No	Single	90K	**No**

How do we know if there are interesting correlations?

Joint Distribution Matrix P

	No	**Yes**	
Single	0.2	0.1	0.3
Married	0.4	0.0	0.4
Divorced	0.1	0.1	0.2
	0.8	0.2	1

Independence Matrix E

	No	**Yes**	
Single	0.24	0.06	0.3
Married	0.32	0.08	0.4
Divorced	0.16	0.04	0.2
	0.8	0.2	1

Compare the values P_{xy} with E_{xy}

The product of the two marginal values 0.2*0.8

Plotting attributes against each other

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Joint Distribution Matrix P

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Single	0.24	0.06	0.3
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Divorced	0.16	0.04	0.2
	0.8	0.2	1

We can compare specific pairs of values:

- If $P(x, y) > E(x, y)$ there is positive correlation (e.g, Married, No)
- If $P(x, y) < E(x, y)$ there is negative correlation (e.g., Single, No)
- Otherwise there is no correlation

The quantity $\frac{P(x,y)}{E(x,y)} = \frac{P(x,y)}{P(x)P(y)}$ is called Lift, or Pointwise Mutual Information

Plotting attributes against each other

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Married	0.32	0.08	0.4
Divorced	0.16	0.04	0.2
	0.8	0.2	1

Or compare the two attributes:

Pearson x^2 Independence Test Statistic:

$$U = N \sum_{x} \sum_{y} \frac{\left(P_{xy} - E_{xy}\right)^2}{E_{xy}}$$

Hypothesis testing

- How important is the statistic value we computed?
- Formulate a null hypothesis H_0:
  - H_0 = the two attributes are independent
- Compute the distribution of the statistic in the case that H_0 is true
  - In this case we can show that the statistic U follows a χ^2 distribution
- For the statistic value θ we computed, compute the probability $P(U > \theta)$ under the null hypothesis
  - For most distributions there are tables that give these numbers for our data
- This is the p-value of our experiment:

> The p-value is the probability (under H_0) of observing a value of the test statistic the same as, or more extreme than what was actually observed

- We want it to be small
  - This means that the observed value is interesting

Categorical and numerical attributes

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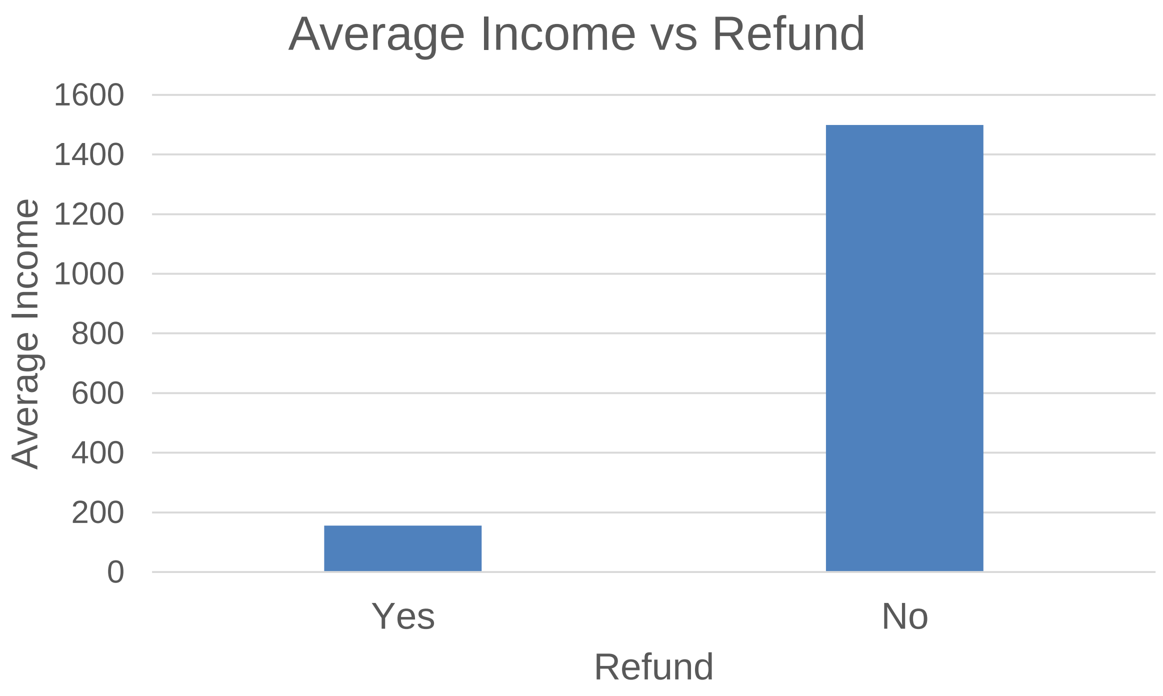

Categorical and numerical attributes

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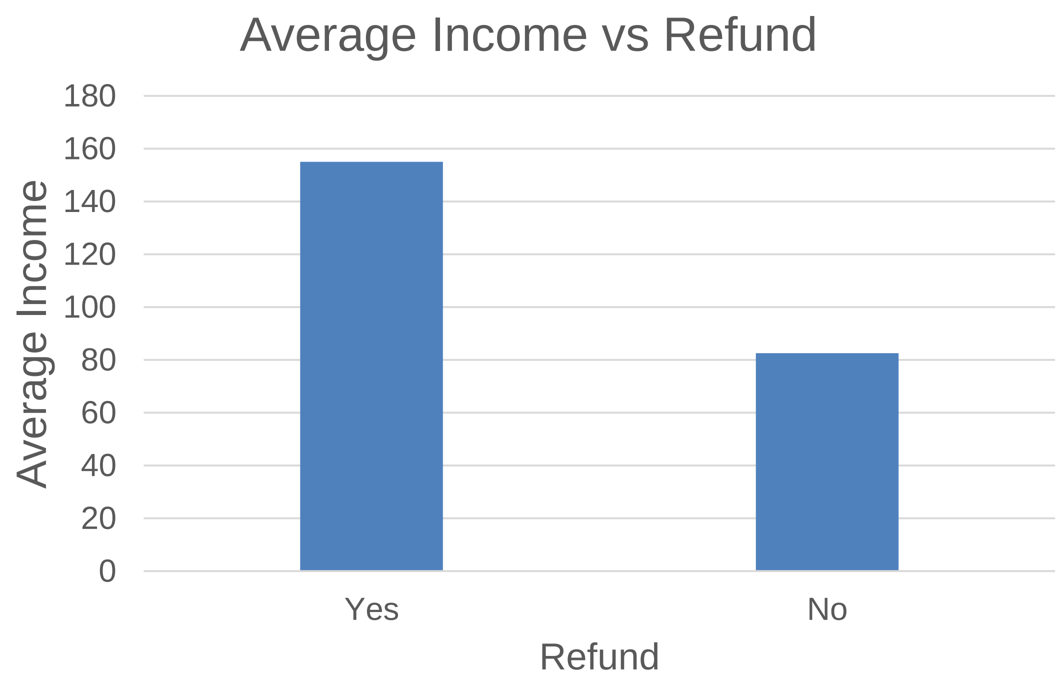
After removing the outlier value



Is this difference significant?

Categorical and numerical attributes

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
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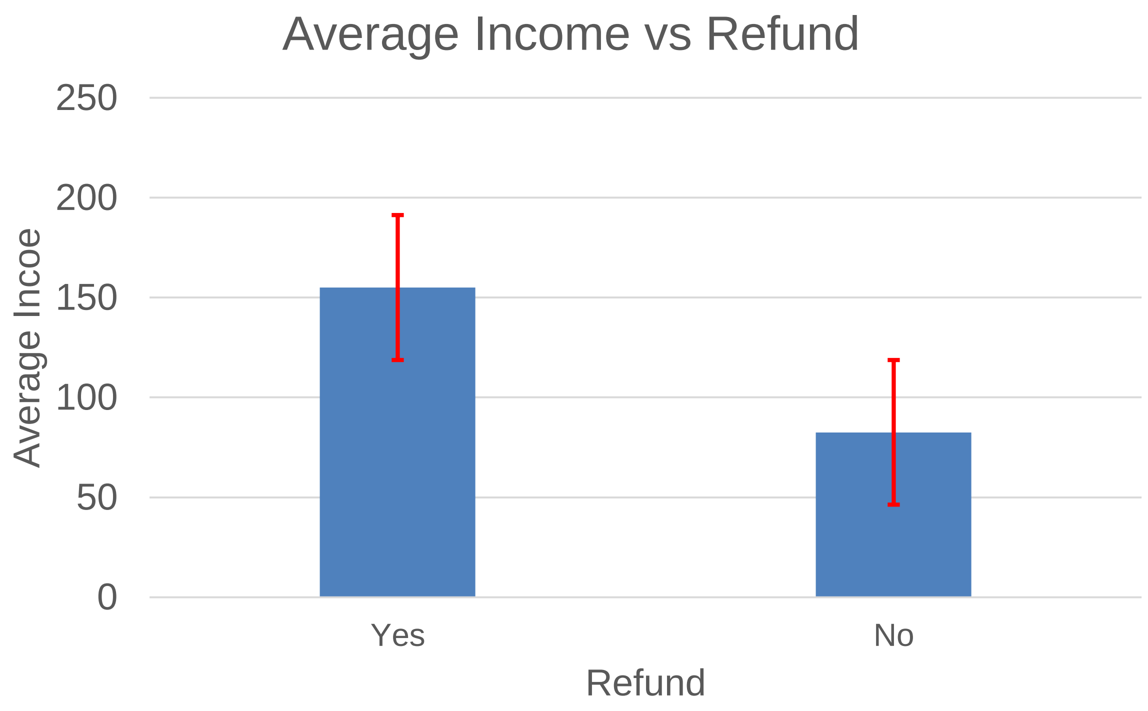
Compute error bars



Confidence interval

- We want to estimate the average income μ which is a fixed value.
- We have a set of measurements X_i of incomes and we estimate the average income as:

$$\hat{\mu} = \frac{1}{n}\sum_i X_i$$

- How good is this estimate?
- The p-confidence interval of the value μ is an interval of values C_n such that

$$P(\mu \in C_n) \geq p$$

Standard error

- If we have a measurement $\hat{\theta}$ that we estimate from the data, the standard error is defined as

$$se = \sqrt{Var(\hat{\theta})}$$

- In our case our measurement is the average income which we estimate as:

$$\hat{\mu} = \frac{1}{n}\sum_i X_i$$

  - We assume that X_i are independent samples of the income random variable X that come from the same distribution. We can show that:

$$se = \frac{\sqrt{Var(X)}}{\sqrt{n}}$$

We use the fact that:

$$Var\left(\sum_i \alpha_i X_i\right) = \sum_i \alpha_i^2 Var(X_i)$$

  - We can estimate $Var(X)$ from the data
- The value $\hat{\mu}$ follows a normal distribution for large n. For normal distributions the 95% confidence interval for the real average income μ is:

$$(\hat{\mu} - 2se, \hat{\mu} + 2se)$$

Statistical tests

- There are statistical tests for testing if two samples come from distributions with the same mean (or median)
- These tests can also provide us with a p-value
- Wald test:
  - Tests the null hypothesis that our variable takes a specific value
    - E.g., the difference of the means or medians is zero
- Student t-test:
  - Test of the means of two normal distributions
- Permutation test:
  - Sample permutations of the merged data points and compute an empirical p-value

Correlating numerical attributes

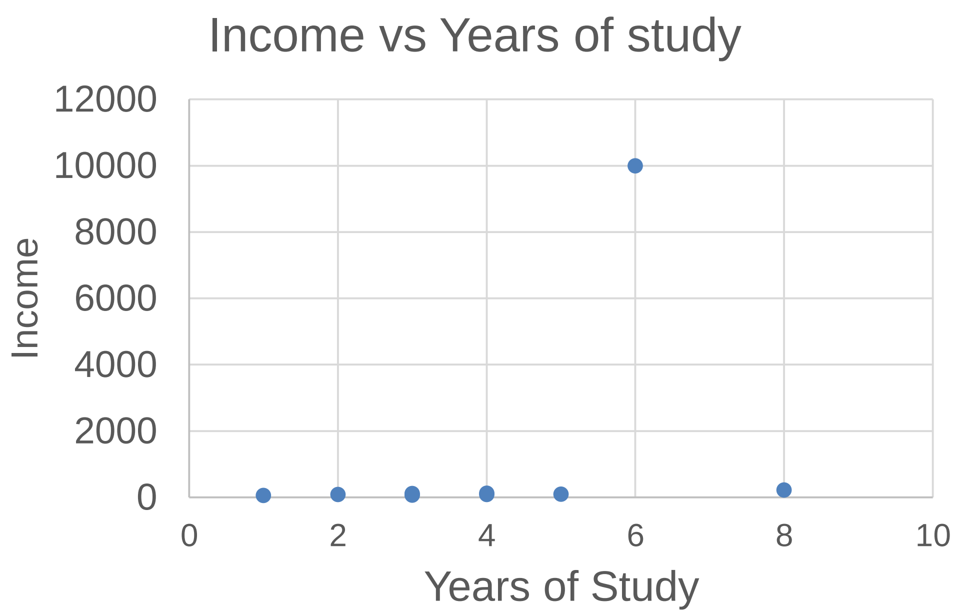
Tid	Refund	Marital Status	Taxable Income	Years of Study
1	Yes	Single	125K	4
2	No	Married	100K	5
3	No	Single	70K	3
4	Yes	Married	120K	3
5	No	Divorced	10000K	6
6	No	NULL	60K	1
7	Yes	Divorced	220K	8
8	No	Single	85K	3
9	No	Married	90K	2
10	No	Single	90K	4

Scatter plot:

X axis is one attribute, Y axis is the other

For each entry we have two values

Plot the entries as two-dimensional points



Plotting attributes against each other

Tid	Refund	Marital Status	Taxable Income	Years of Study
1	Yes	Single	125K	4
2	No	Married	100K	5
3	No	Single	70K	3
4	Yes	Married	120K	3
5	No	Divorced	10000K	6
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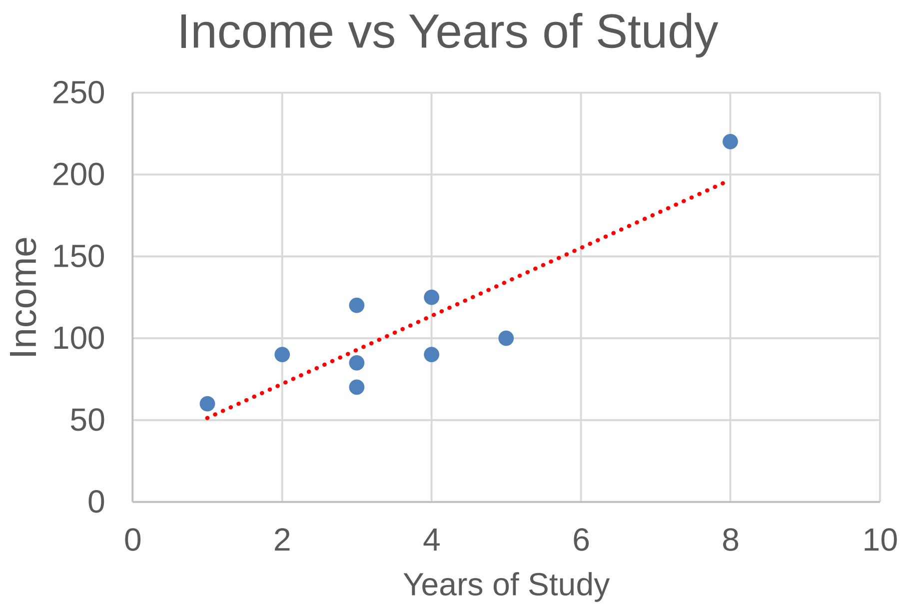
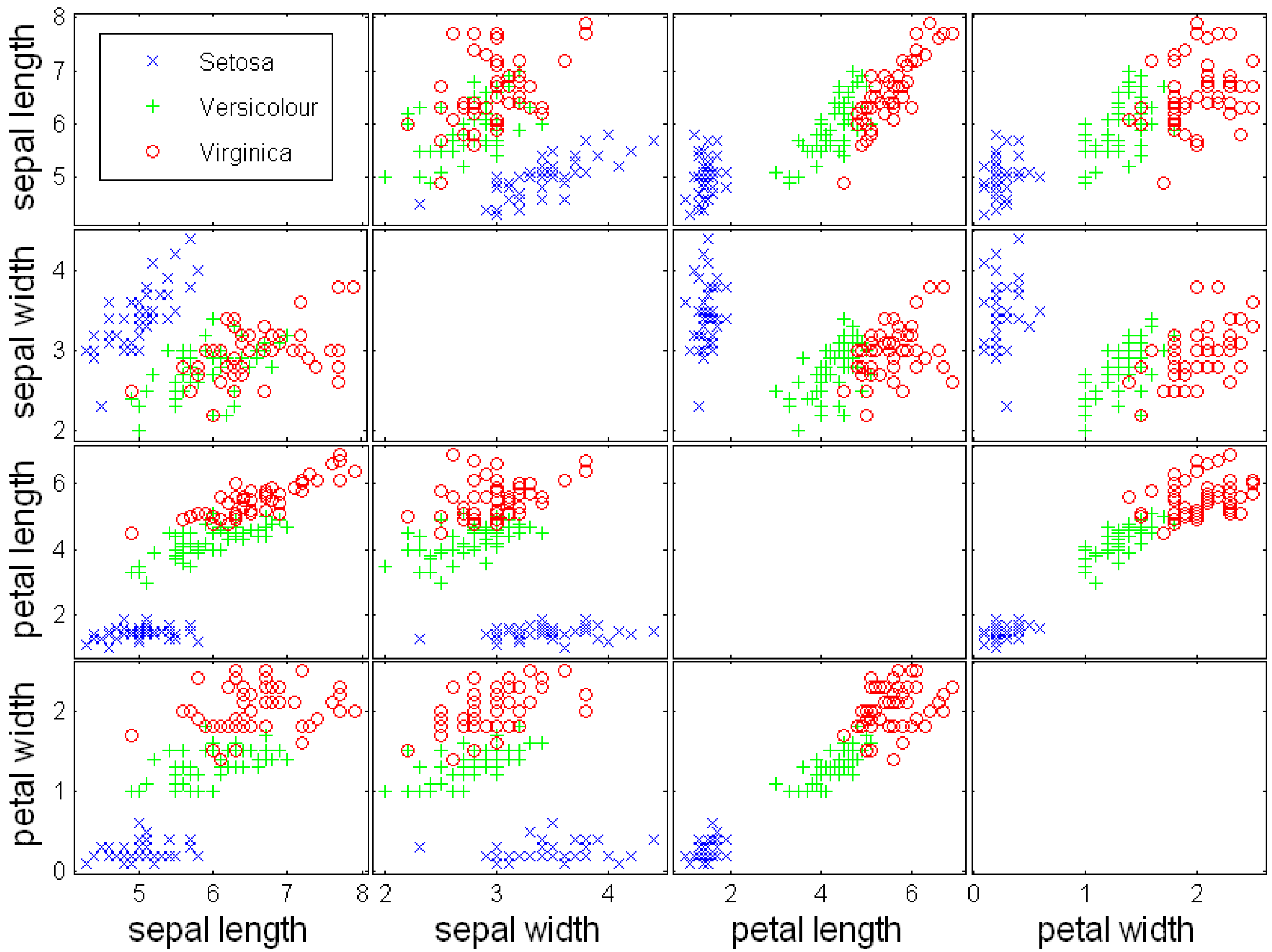
9	No	Married	90K	2
10	No	Single	90K	4

Scatter plot:
X axis is one attribute, Y axis is the other
For each entry we have two values
Plot the entries as two-dimensional points

After removing the outlier value there is a clear correlation



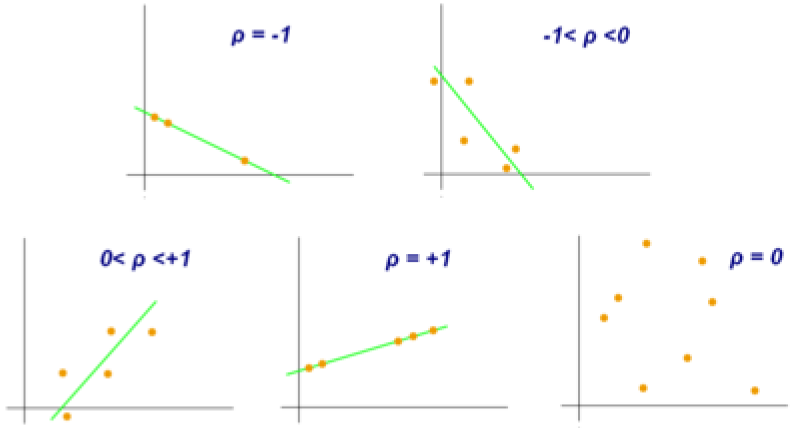
Scatter Plot Array of Iris Attributes



Measuring correlation

- Pearson correlation coefficient: measures the extent to which two variables are linearly correlated
  - $X = \{x_1, \ldots, x_n\}$
  - $Y = \{y_1, \ldots, y_n\}$
  - $corr(X,Y) = \frac{\sum_i (x_i - \mu_X)(y_i - \mu_Y)}{\sqrt{\sum_i (x_i - \mu_X)^2}\sqrt{\sum_i (y_i - \mu_Y)^2}}$
  - It comes with a p-value
    - The p-value is the probability that the correlation was by chance.
  - Assumes no outliers and that the variables are normally distributed

Must have pairs of observations



- Spearman rank correlation coefficient: tells us if two variable are rank-correlated
  - They place items in the same order – Pearson correlation of the rank vectors
  - For ranking without ties it looks at the differences between the ranks of the same items

Post-processing

- Visualization
  - The human eye is a powerful analytical tool
  - If we visualize the data properly, we can discover patterns and demonstrate trends
  - Visualization is the way to present the data so that patterns can be seen
    - E.g., histograms and plots are a form of visualization
    - There are multiple techniques (a field on its own)

Visualization on a map

- John Snow, London 1854

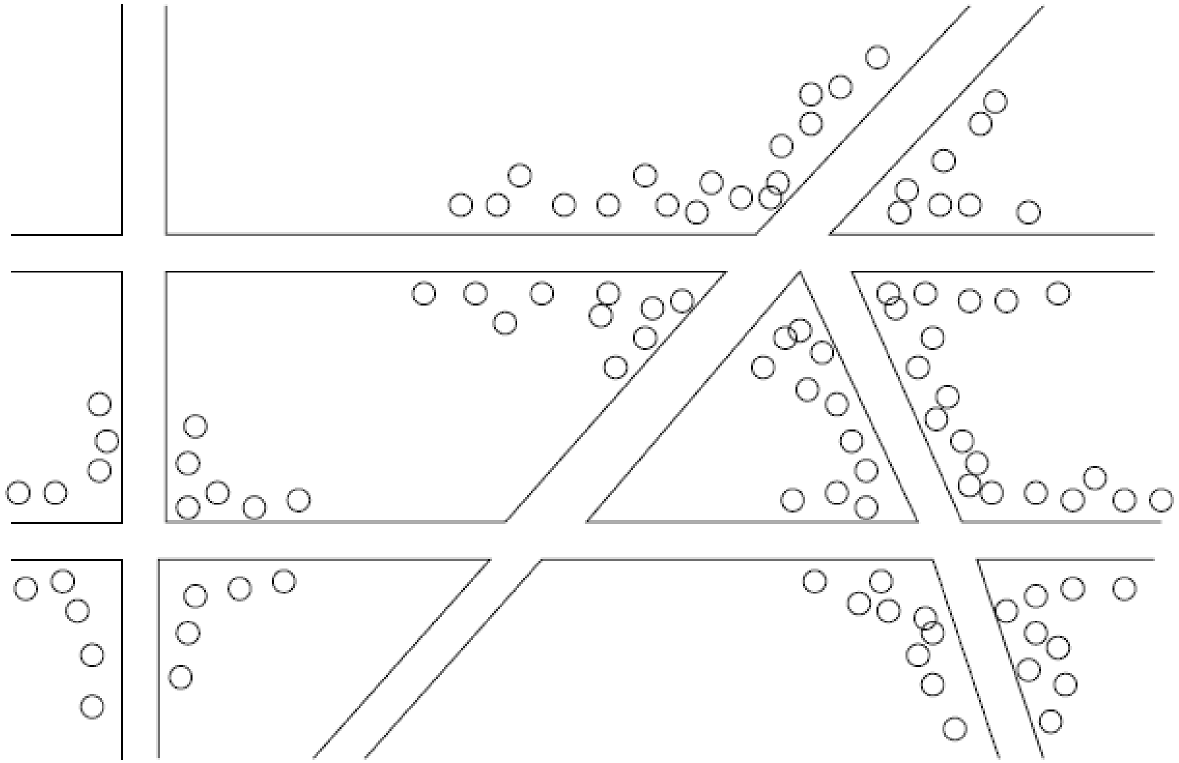
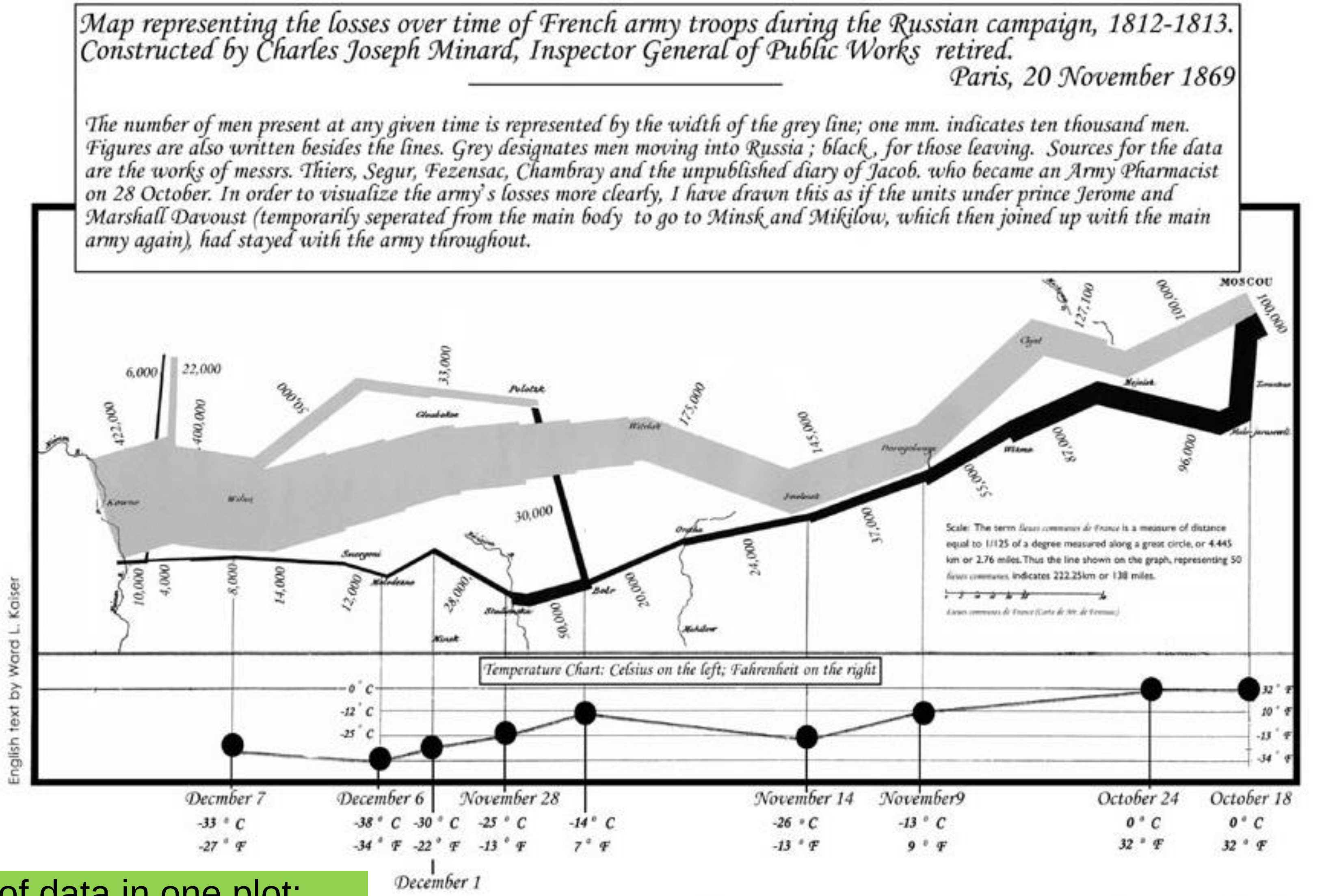


Figure 1.1: Plotting cholera cases on a map of London

Charles Minard map



Six types of data in one plot:
size of army, temperature,
direction, location, dates etc

Another interesting visualization

- China growth over the years

Dimensionality Reduction

- The human eye is limited to processing visualizations in two (at most three) dimensions
- One of the great challenges in visualization is to visualize high-dimensional data into a two-dimensional space
  - Dimensionality reduction
  - Distance preserving embeddings
- Dimensionality reduction is also a preprocessing technique:
  - Reduce the amount of data
  - Extract the useful information.

Example

- Consider the following 6-dimensional dataset

$$D = \begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 2 & 4 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 2 & 4 & 6 \\ 1 & 2 & 3 & 1 & 2 & 3 \\ 2 & 4 & 6 & 2 & 4 & 6 \end{bmatrix}$$

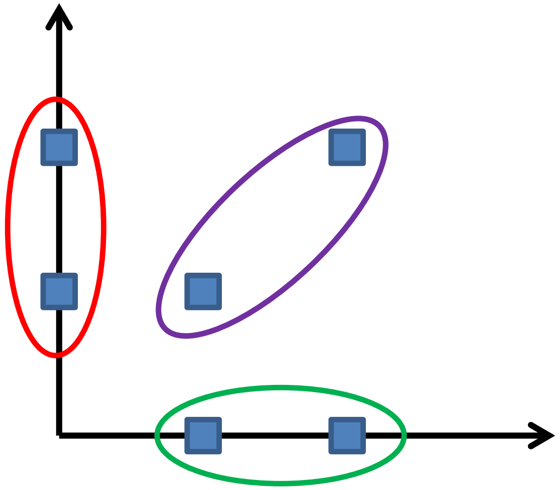
- What do you observe? Can we reduce the dimension of the data?

Example

$$D = \begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 2 & 4 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 2 & 4 & 6 \\ 1 & 2 & 3 & 1 & 2 & 3 \\ 2 & 4 & 6 & 2 & 4 & 6 \end{bmatrix}$$

- Each row is a multiple of two vectors
  - $x = [1, 2, 3, 0, 0, 0]$
  - $y = [0, 0, 0, 1, 2, 3]$
- We can rewrite D as

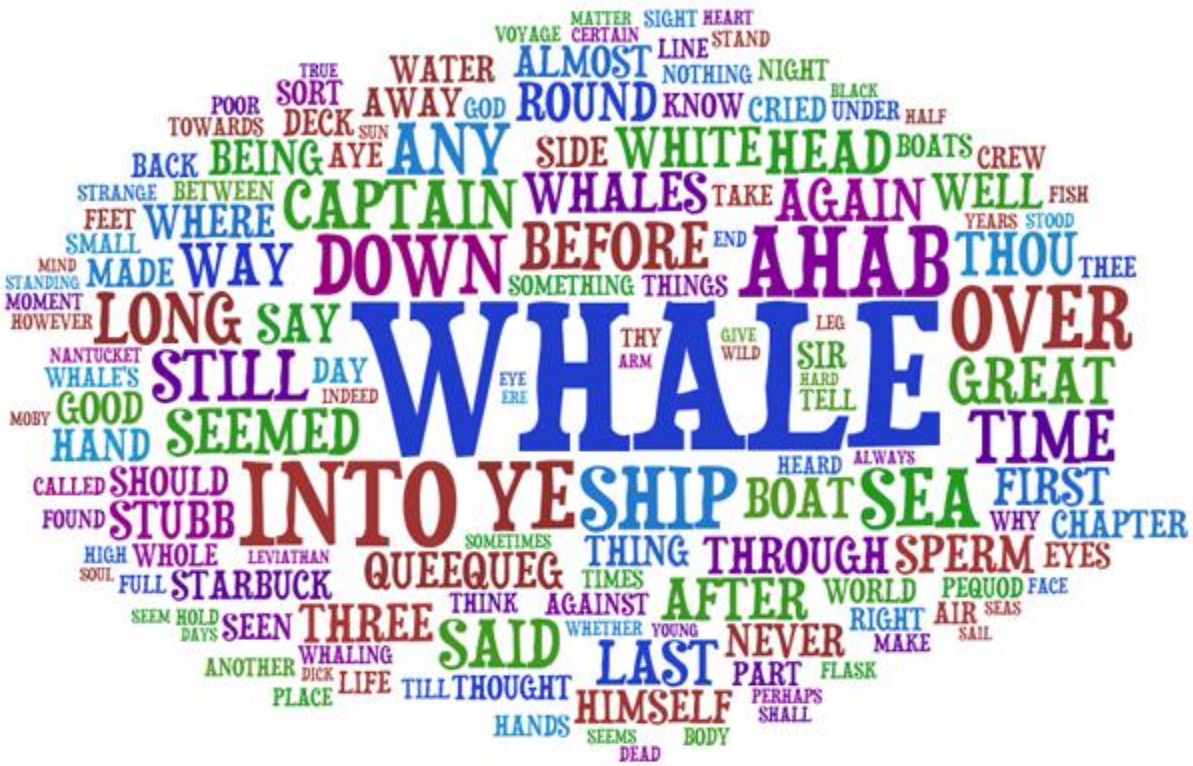
$$D = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 1 \\ 0 & 2 \\ 1 & 1 \\ 2 & 2 \end{bmatrix}$$



Three types of data points

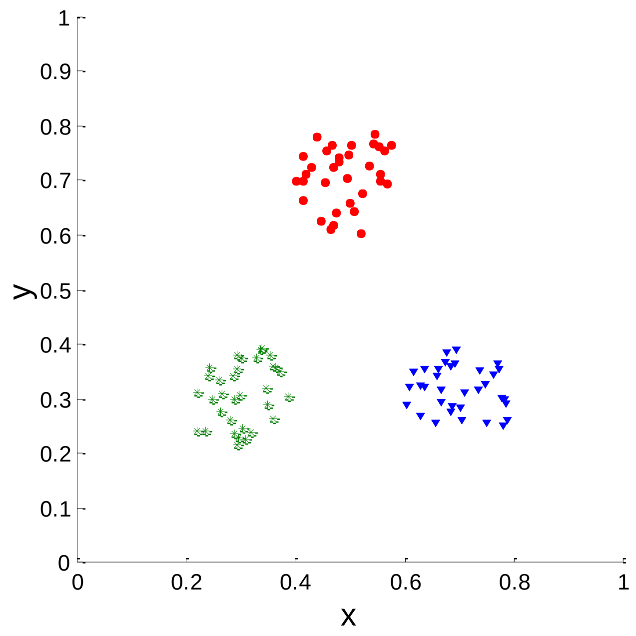
Word Clouds

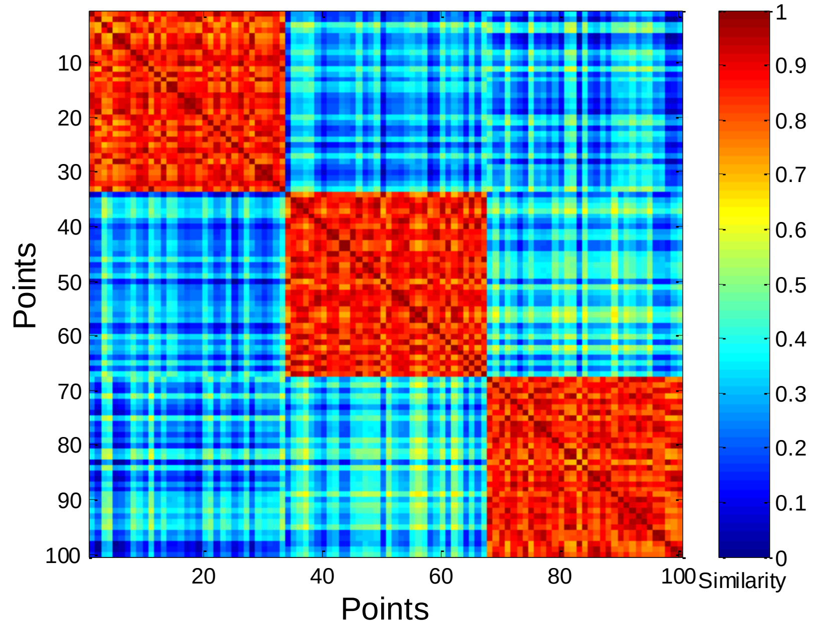
- A fancy way to visualize a document or collection of documents.



Heatmaps

- Plot a point-to-point similarity matrix using a heatmap:
  - Deep red = high values (hot)
  - Dark blue = low values (cold)

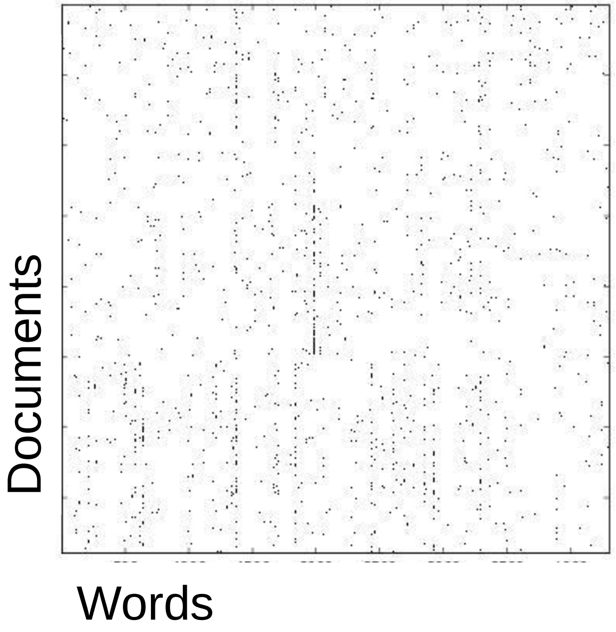




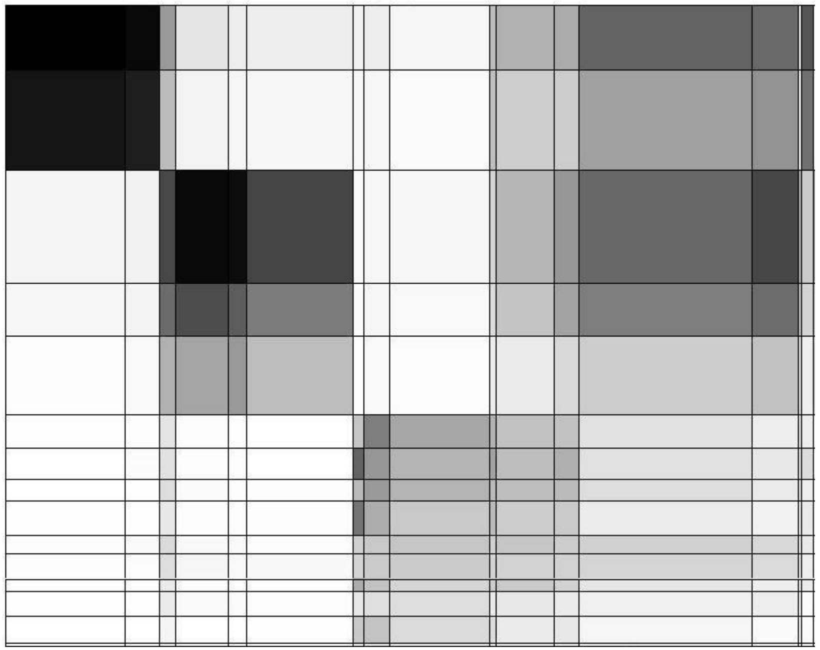
The clustering structure becomes clear in the heatmap

Heatmaps

- Heatmap (grey scale) of the data matrix
  - Document-word frequencies



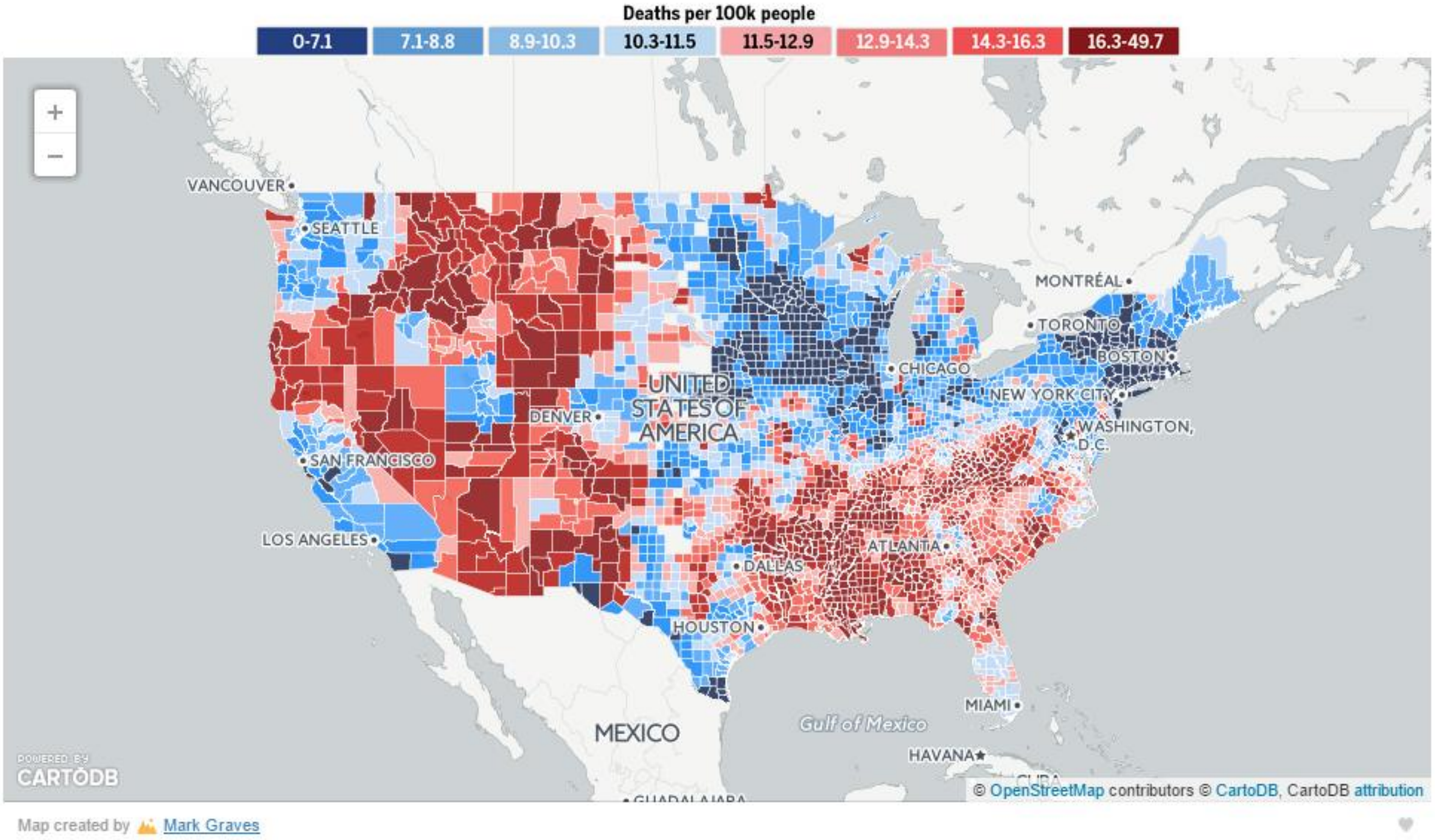
Before clustering



After clustering

Heatmaps

A very popular way to visualize data



http://projects.oregonlive.com/ucc-shooting/gun-deaths.php

Statistical Significance

- When we extract knowledge from a large dataset we need to make sure that what we found is not an artifact of randomness
  - E.g., we find that many people buy milk and toilet paper together.
  - But many (more) people buy milk and toilet paper independently
- Statistical tests compare the results of an experiment with those generated by a null hypothesis
  - E.g., a null hypothesis is that people select items independently.
- A result is interesting if it cannot be produced by randomness.
  - An important problem is to define the null hypothesis correctly: What is random?

Meaningfulness of Answers

- A big data-mining risk is that you will "discover" patterns that are meaningless.
- Statisticians call it Bonferroni's principle: (roughly) if you look in more places for interesting patterns than your amount of data will support, you are bound to find crap.
- The Rhine Paradox: a great example of how not to conduct scientific research.

Rhine Paradox – (1)

- Joseph Rhine was a parapsychologist in the 1950's who hypothesized that some people had Extra-Sensory Perception.
- He devised (something like) an experiment where subjects were asked to guess 10 hidden cards – red or blue.
- He discovered that almost 1 in 1000 had ESP – they were able to get all 10 right!

Rhine Paradox – (2)

- He told these people they had ESP and called them in for another test of the same type.
- Alas, he discovered that almost all of them had lost their ESP.
  - Why?
- What did he conclude?
  - Answer on next slide.

Rhine Paradox – (3)

- He concluded that you shouldn’t tell people they have ESP; it causes them to lose it.