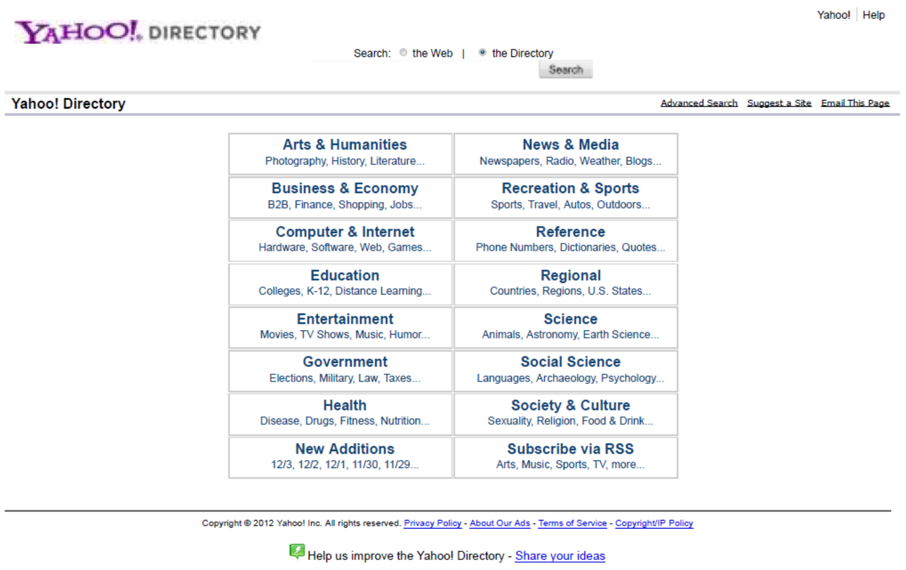
Online Social Networks and Media

Link Analysis and Web Search

How to Organize the Web

First try: Human curated Web directories

Yahoo, DMOZ, LookSmart



Yahoo! | Help

Search: ○ the Web | ● the Directory

Search

Yahoo! Directory

Advanced Search Suggest a Site Email This Page

Arts & Humanities Photography, History, Literature...	**News & Media** Newspapers, Radio, Weather, Blogs...
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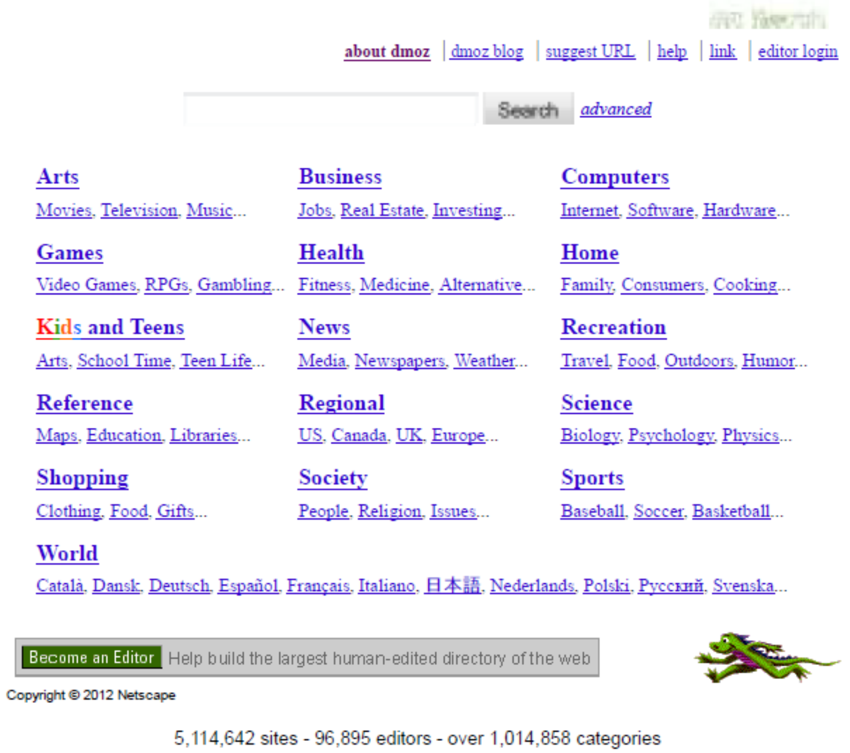

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Kids and Teens Arts, School Time, Teen Life...	**News** Media, Newspapers, Weather...	**Recreation** Travel, Food, Outdoors, Humor...
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World
Català, Dansk, Deutsch, Español, Français, Italiano, 日本語, Nederlands, Polski, Русский, Svenska...

Become an Editor Help build the largest human-edited directory of the web

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5,114,642 sites - 96,895 editors - over 1,014,858 categories

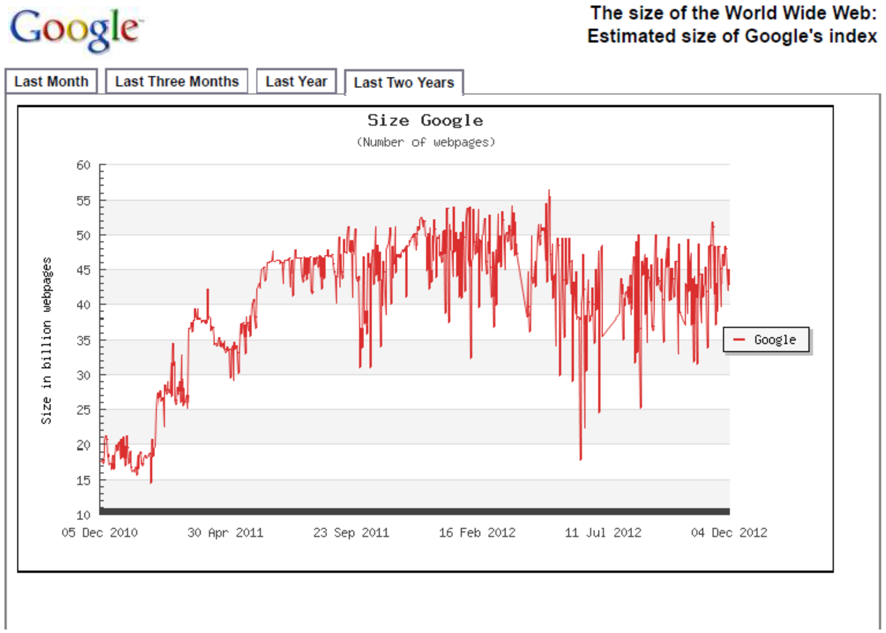
How to organize the web

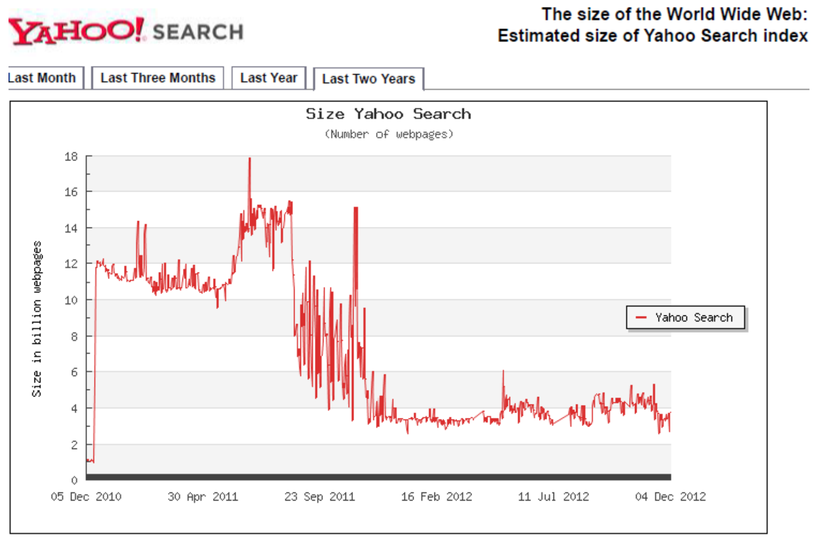
- Second try: Web Search
  - Information Retrieval investigates:
    - Find relevant docs in a small and trusted set e.g., Newspaper articles, Patents, etc. ("needle-in-a-haystack")
    - Limitation of keywords (synonyms, polysemy, etc)

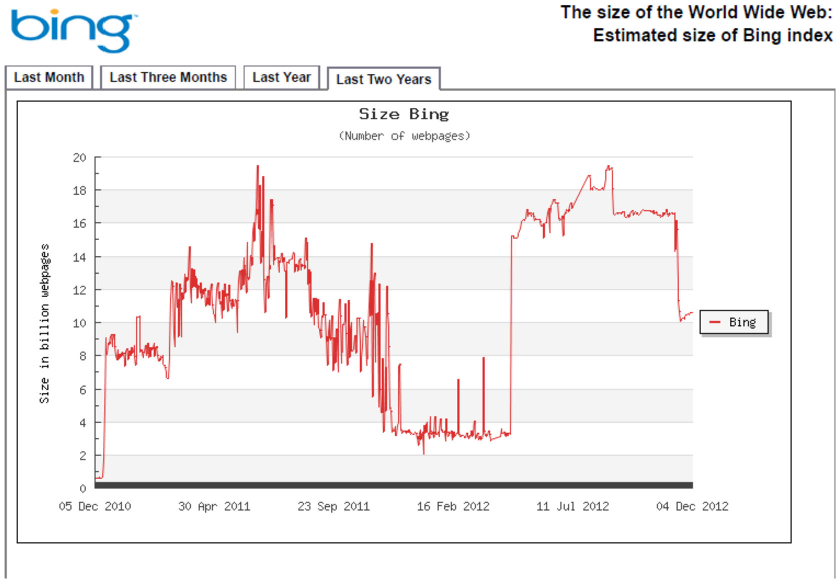
But: Web is huge, full of untrusted documents, random things, web spam, etc.

- Everyone can create a web page of high production value
- Rich diversity of people issuing queries
- Dynamic and constantly-changing nature of web content

Size of the Search Index







http://www.worldwidewebsize.com/

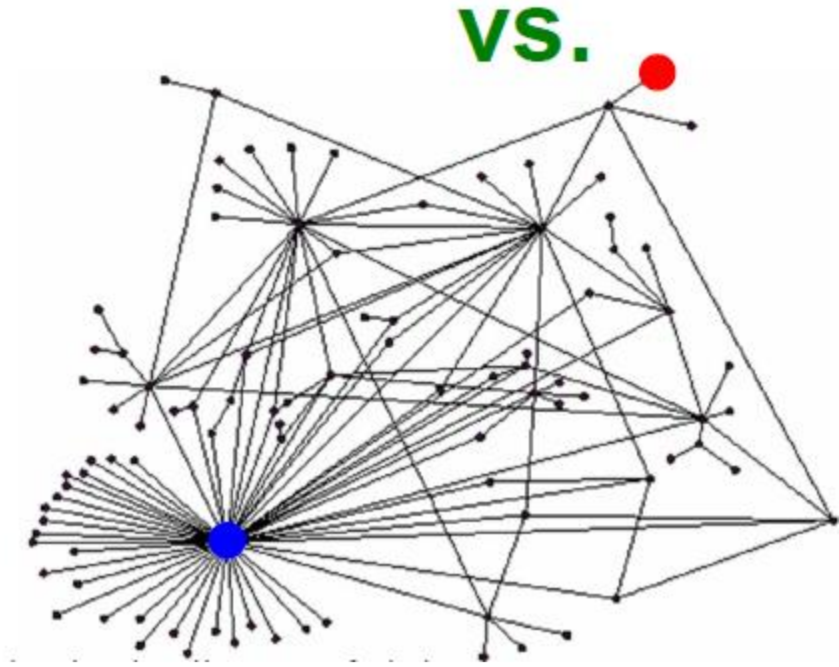
How to organize the web

- Third try (the Google era): using the web graph
  - Swift from relevance to *authoritativeness*
  - It is not only important that a page is relevant, but that it is also important on the web
- For example, what kind of results would we like to get for the query “greek newspapers”?

Link Analysis

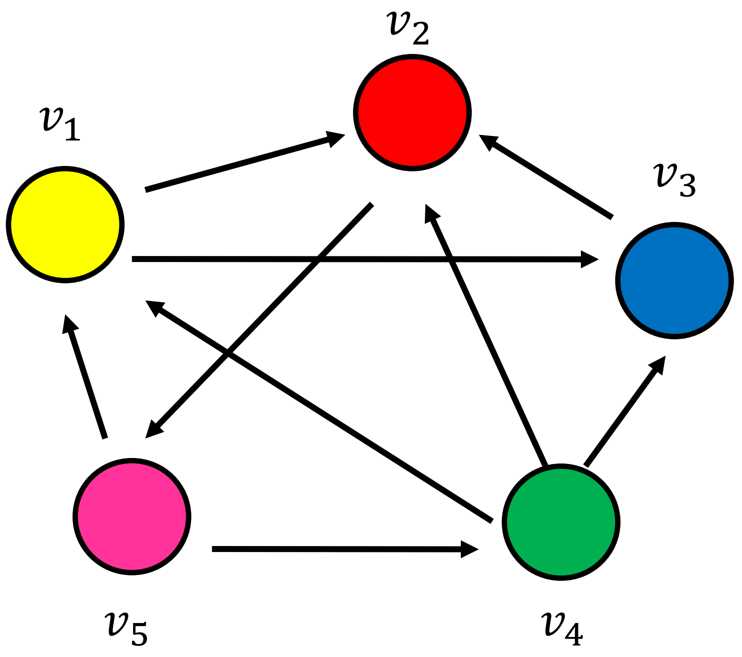
- Not all web pages are equal on the web
- The links act as endorsements:
  - When page *p* links to *q* it endorses the content of the content of *q*

What is the simplest way to measure importance of a page on the web?



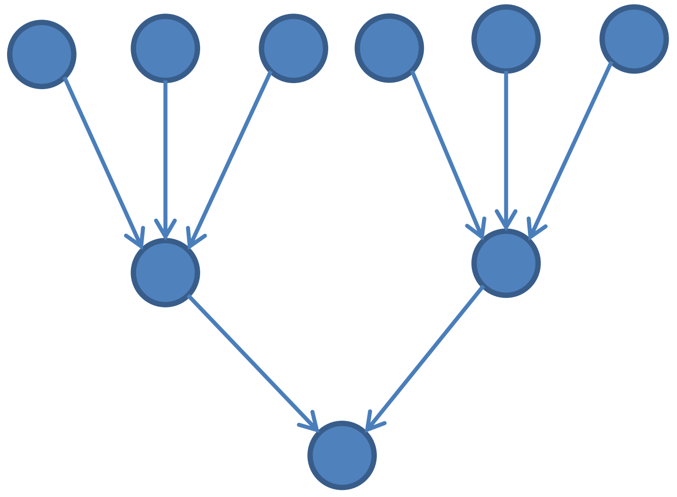
Rank by Popularity

- Rank pages according to the number of incoming edges (in-degree, degree centrality)



1. **Red Page**
2. **Yellow Page**
3. **Blue Page**
4. **Purple Page**
5. **Green Page**

Popularity



- It is not important only how many link to you, but also how important are the people that link to you.
- Good authorities are pointed by good authorities
  - Recursive definition of importance

THE PAGERANK ALGORITHM

PageRank

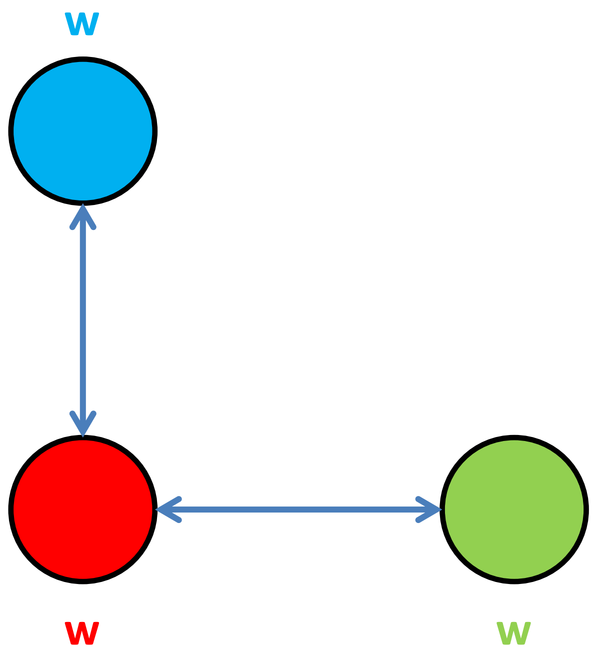
- Good authorities should be pointed by good authorities
  - The value of a node is the value of the nodes that point to it.
- How do we implement that?
  - Assume that we have a unit of authority to distribute to all nodes.
    - Initially each node gets $\frac{1}{n}$ amount of authority
  - Each node distributes the authority value they have to their neighbors
  - The authority value of each node is the sum of the authority fractions it collects from its neighbors.

$$w_v = \sum_{u \to v} \frac{1}{d_{out}(u)} w_u$$

w_v: the PageRank value of node v

Recursive definition

A simple example



w + w + w = 1

w = w + w

w = ½ w

w = ½ w

- Solving the system of equations we get the authority values for the nodes
  - w = ½ w = ¼ w = ¼

A more complex example

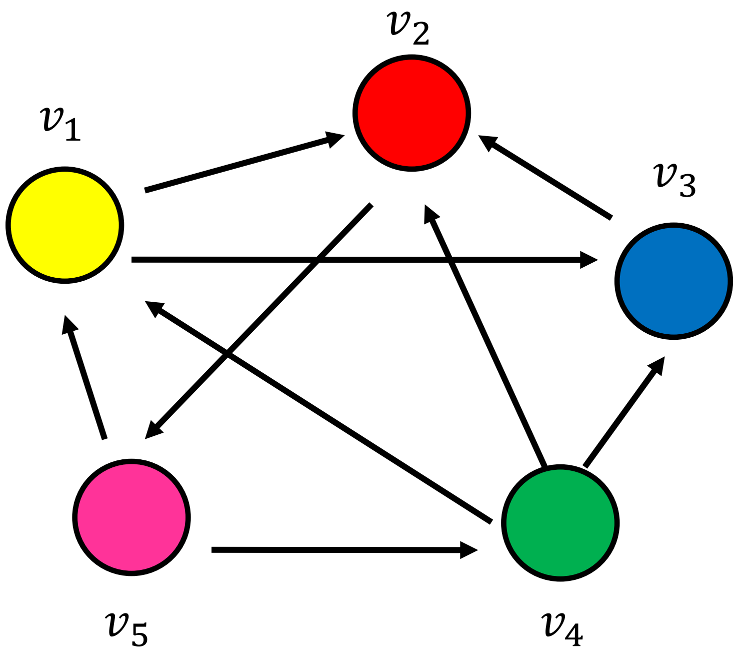
$w_1 = 1/3\, w_4 + 1/2\, w_5$

$w_2 = 1/2\, w_1 + w_3 + 1/3\, w_4$

$w_3 = 1/2\, w_1 + 1/3\, w_4$

$w_4 = 1/2\, w_5$

$w_5 = w_2$



$$w_v = \sum_{u \to v} \frac{1}{d_{out}(u)} w_u$$

Computing PageRank weights

- A simple way to compute the weights is by iteratively updating the weights
- PageRank Algorithm

```
Initialize all PageRank weights to $\frac{1}{n}$
Repeat:
        $w_v = \sum_{u \to v} \frac{1}{d_{out}(u)} w_u$
Until the weights do not change
```

- This process converges

Example

$$w_v = \sum_{u \to v} \frac{1}{d_{out}(u)} w_u$$

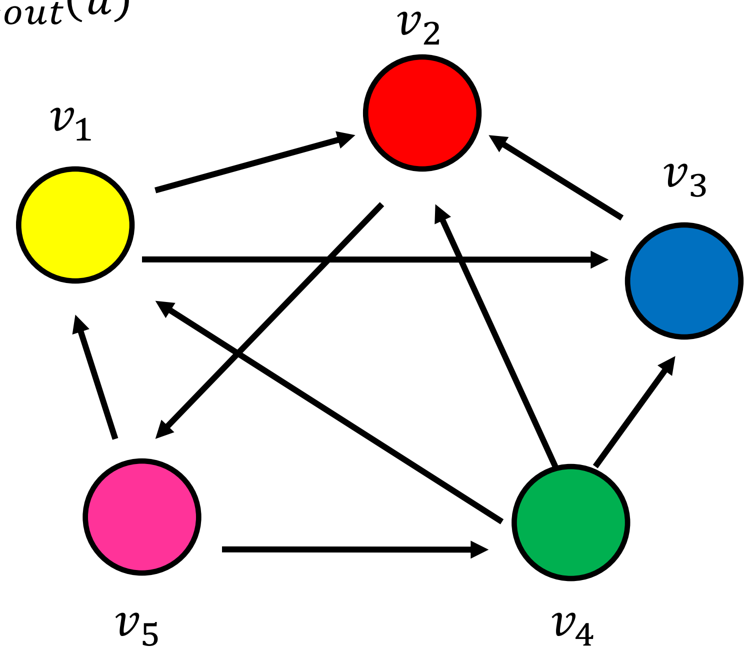
$w_1 = 1/3\ w_4 + 1/2\ w_5$

$w_2 = 1/2\ w_1 + w_3 + 1/3\ w_4$

$w_3 = 1/2\ w_1 + 1/3\ w_4$

$w_4 = 1/2\ w_5$

$w_5 = w_2$



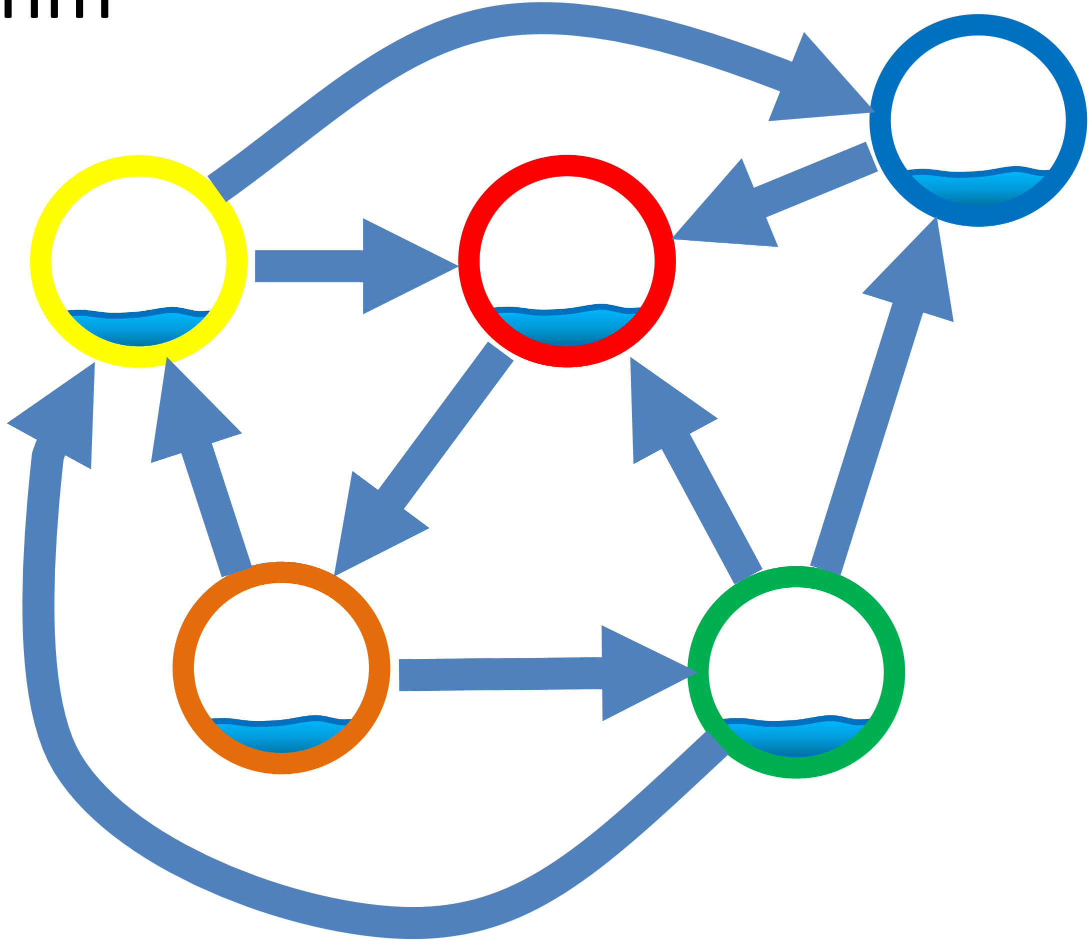
	w_1	w_2	w_3	w_4	w_5
t=0	0.2	0.2	0.2	0.2	0.2
t=1	0.16	0.36	0.16	0.1	0.2
t=2	0.13	0.28	0.11	0.1	0.36
t=3	0.22	0.22	0.1	0.18	0.28
t=4	0.2	0.27	0.17	0.14	0.22

Think of the weight as a fluid: there is constant amount of it in the graph, but it moves around until it stabilizes

The PageRank algorithm

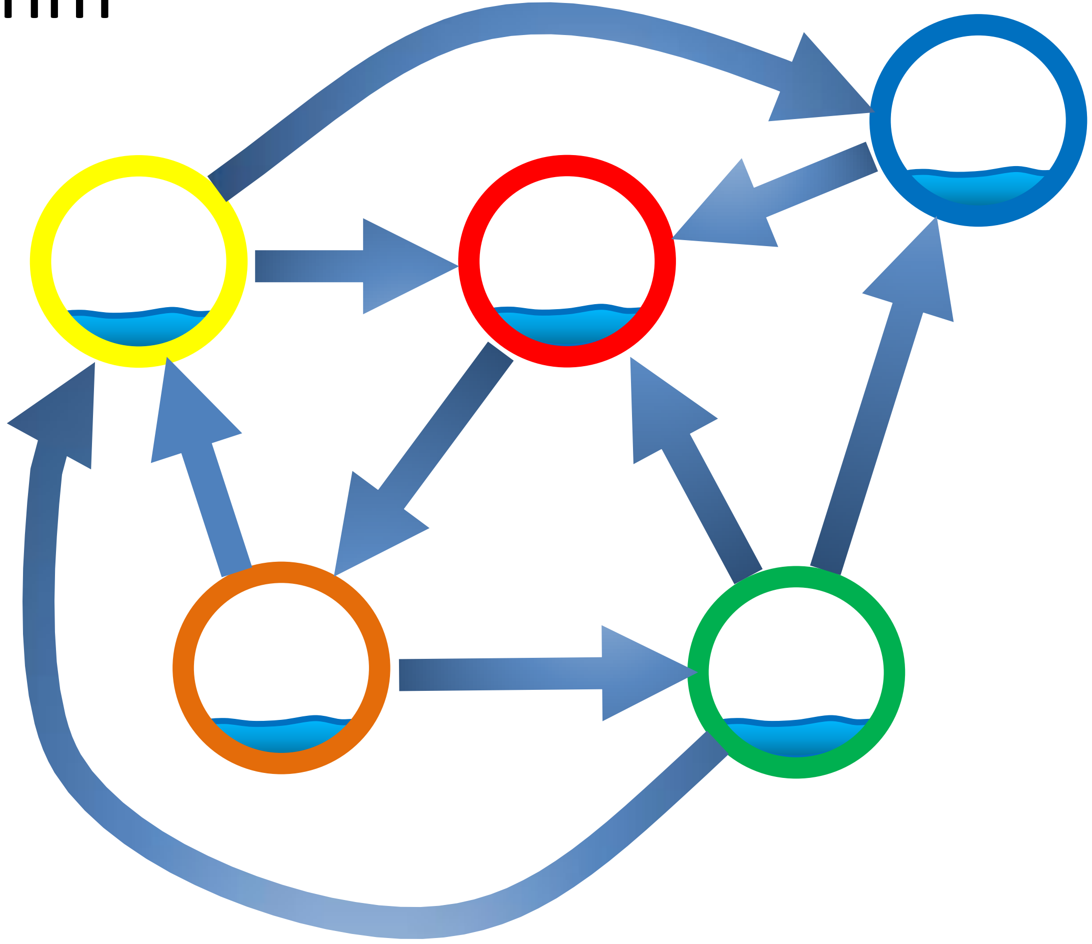
Think of the nodes in the graph as containers of capacity of 1 liter.

We distribute a liter of liquid equally to all containers



The PageRank algorithm

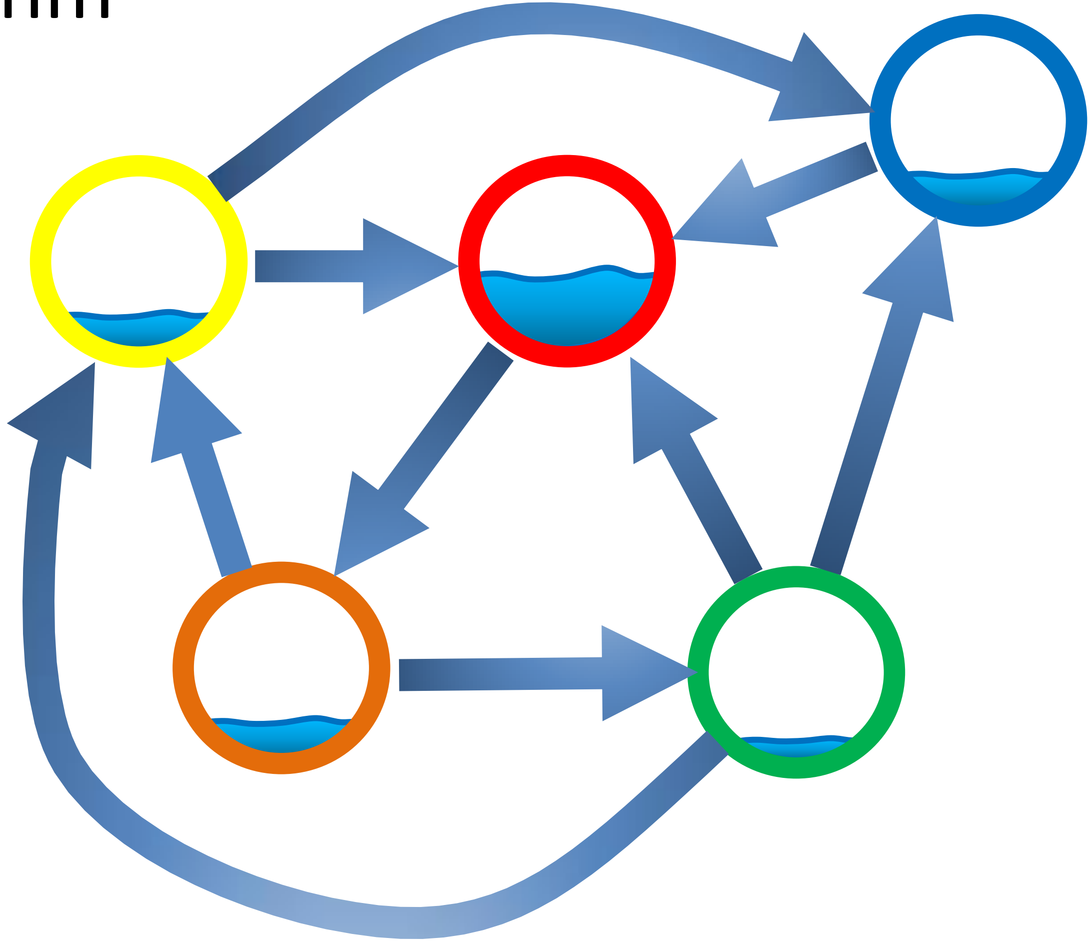
The edges act like pipes that transfer liquid between nodes.



The PageRank algorithm

The edges act like pipes that transfer liquid between nodes.

The contents of each node are distributed to its neighbors.



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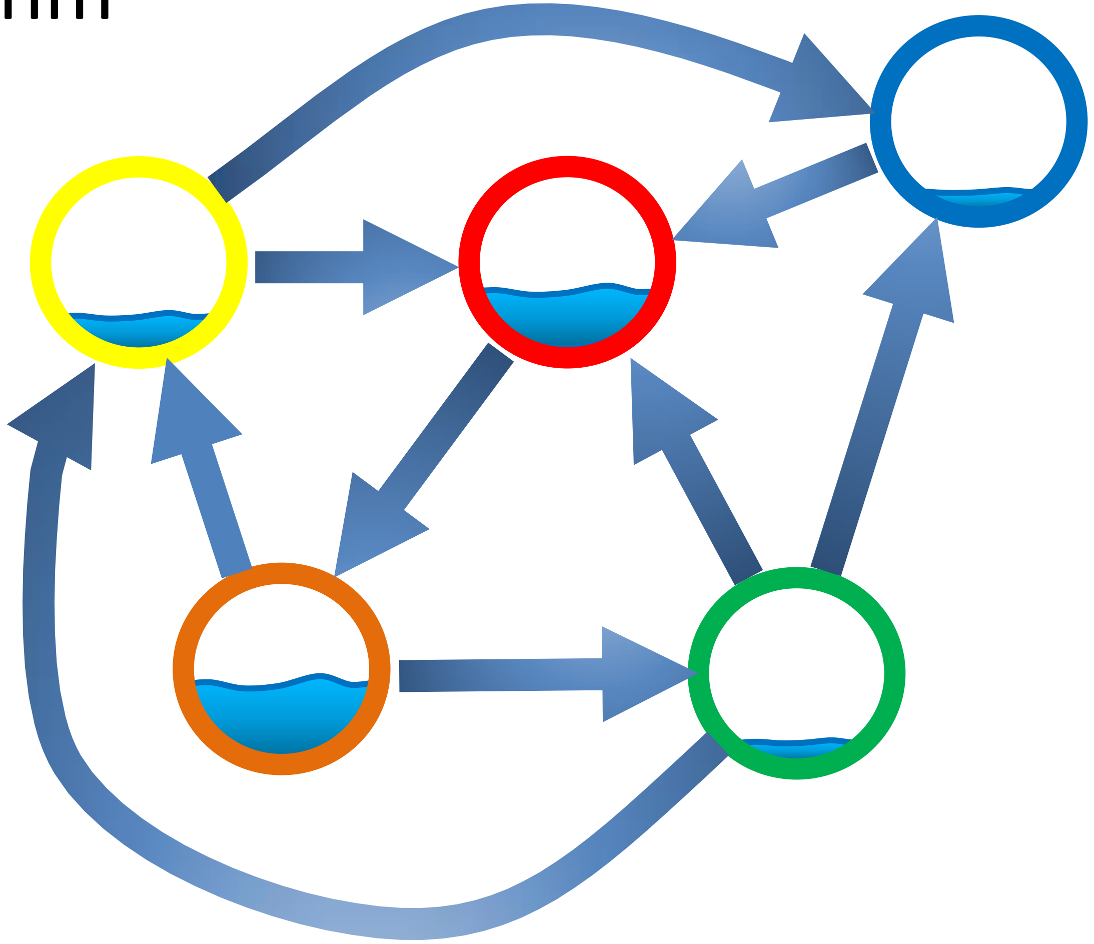

The PageRank algorithm

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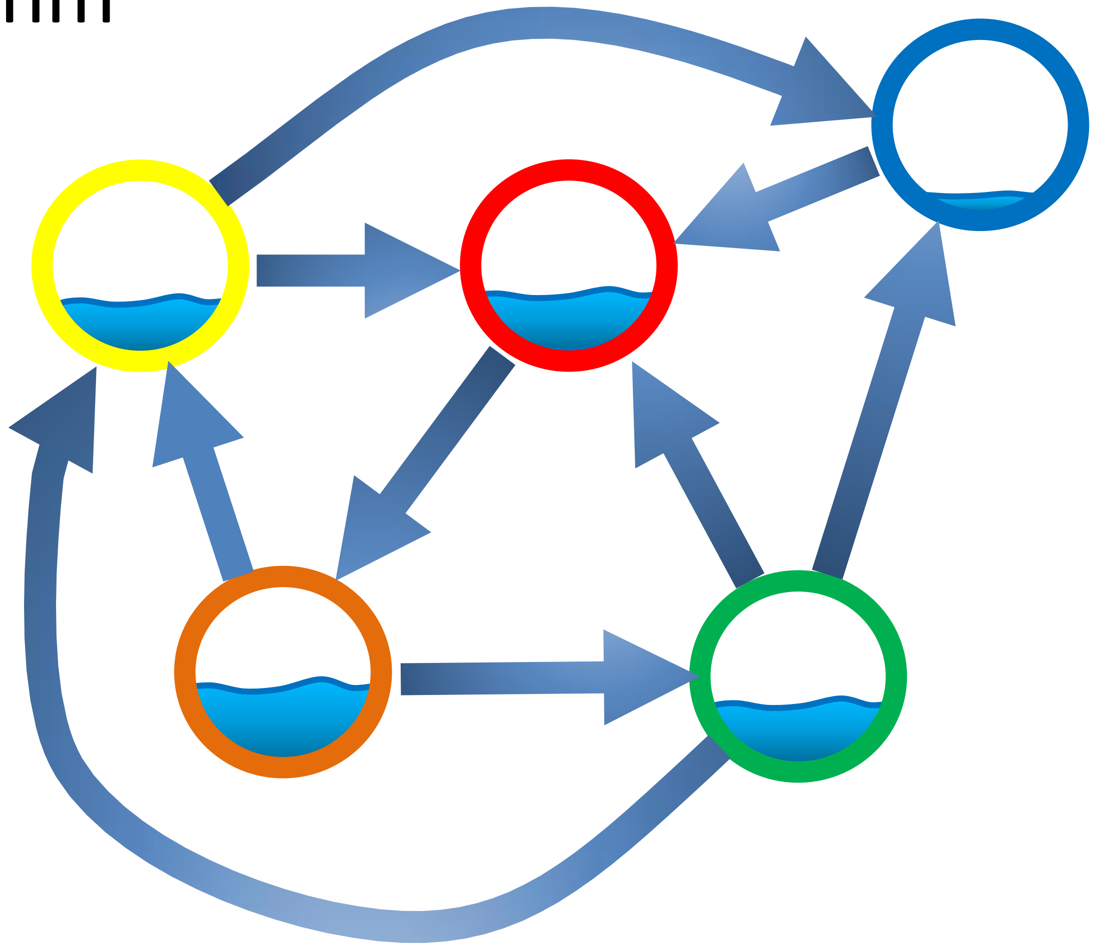
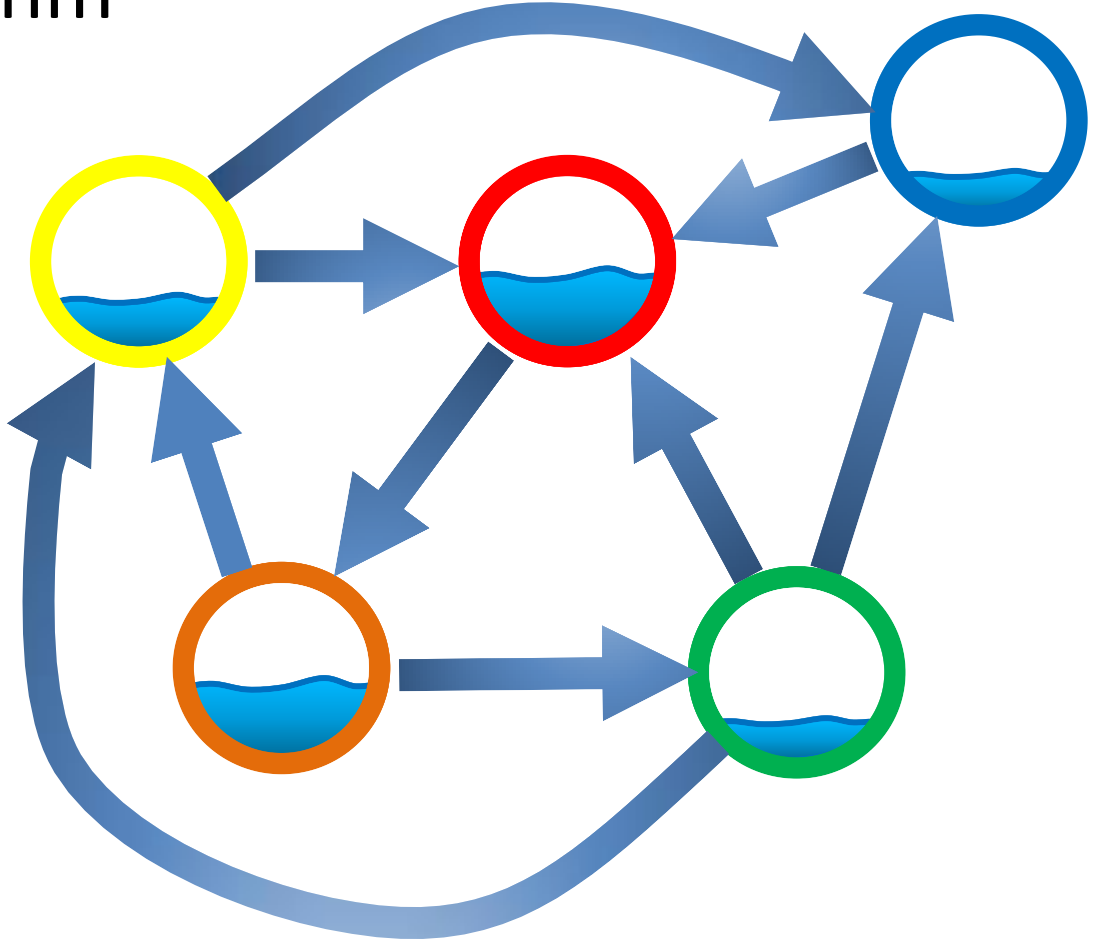


The PageRank algorithm

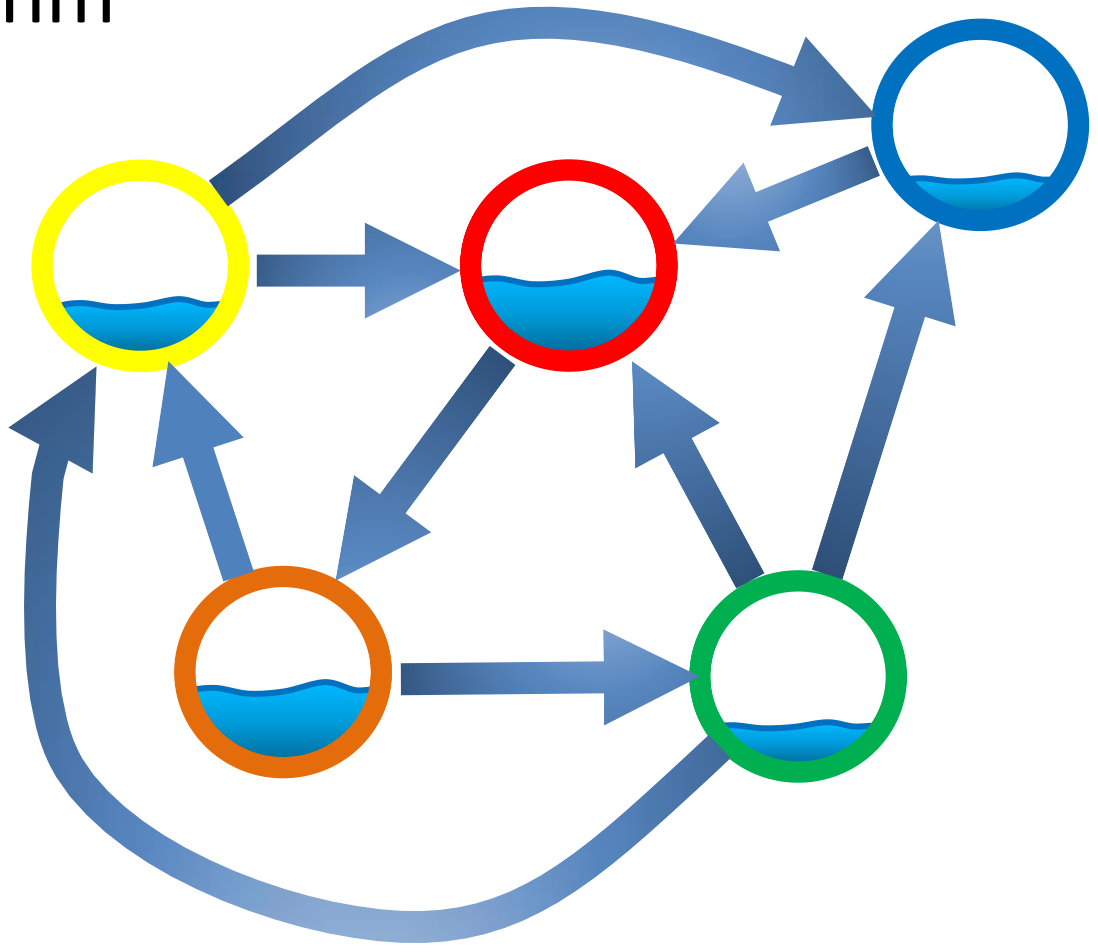
The system will reach an equilibrium state where the amount of liquid in each node remains constant.



The PageRank algorithm

The amount of liquid in each node determines the importance of the node.

Large quantity means large incoming flow from nodes with large quantity of liquid.



Example

$$w_v = \sum_{u \to v} \frac{1}{d_{out}(u)} w_u$$

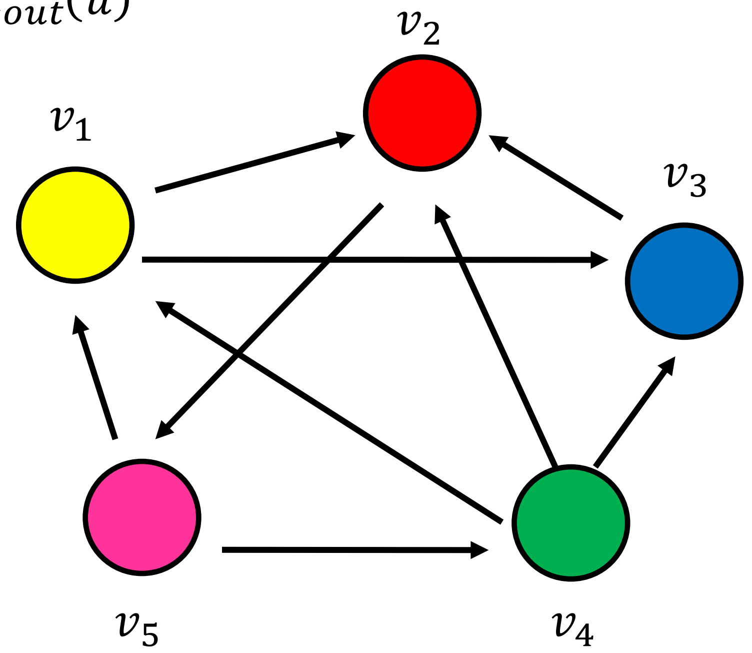
$w_1 = 1/3\ w_4 + 1/2\ w_5$

$w_2 = 1/2\ w_1 + w_3 + 1/3\ w_4$

$w_3 = 1/2\ w_1 + 1/3\ w_4$

$w_4 = 1/2\ w_5$

$w_5 = w_2$



	w_1	w_2	w_3	w_4	w_5
t=25	0.18	0.27	0.13	0.13	0.27

Think of the weight as a fluid: there is constant amount of it in the graph, but it moves around until it stabilizes

Random Walks on Graphs

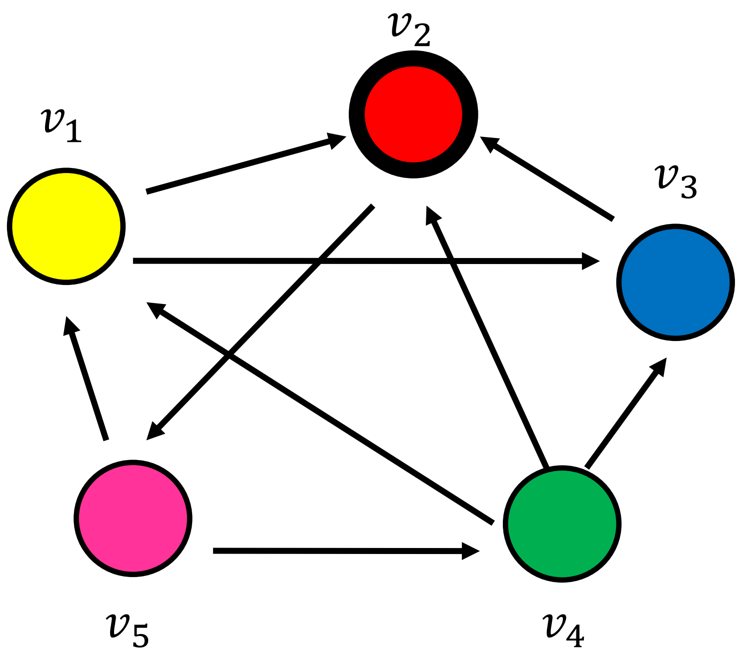
- The algorithm defines a random walk on the graph

- Random walk:
  - Start from a node chosen uniformly at random with probability $\frac{1}{n}$.
  - Pick one of the outgoing edges uniformly at random
  - Move to the destination of the edge
  - Repeat.

- The PageRank of node v is the probability that the random walk is at node v after a very large number of steps.
- The Random Surfer model
  - Users wander on the web, following links.

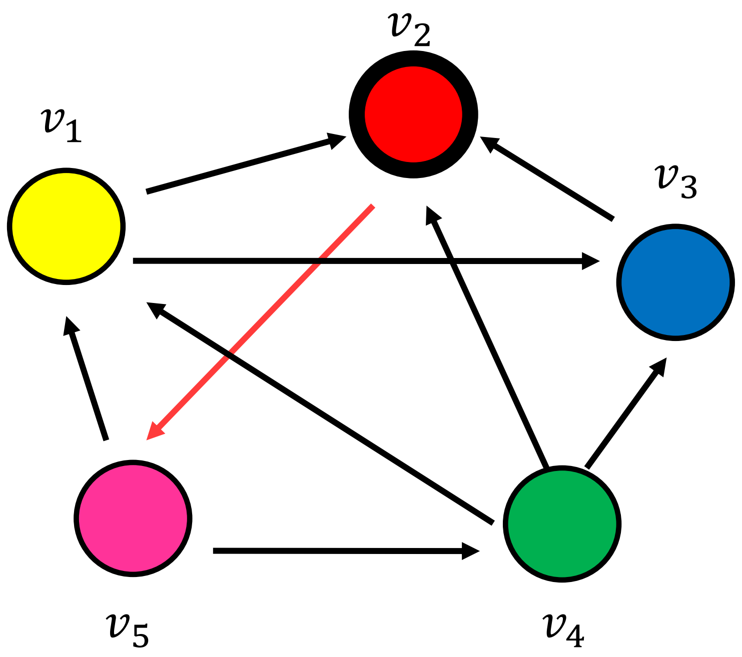
Example

- Step 0



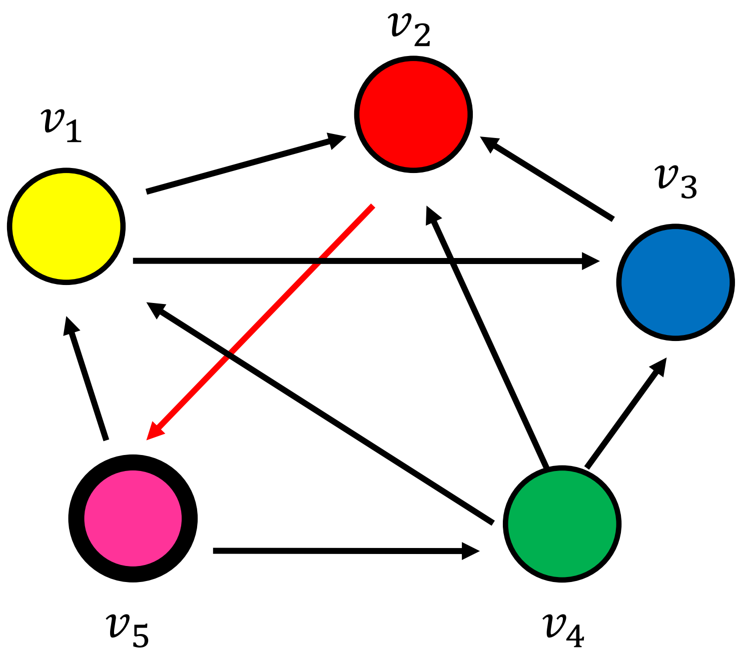
Example

- Step 0



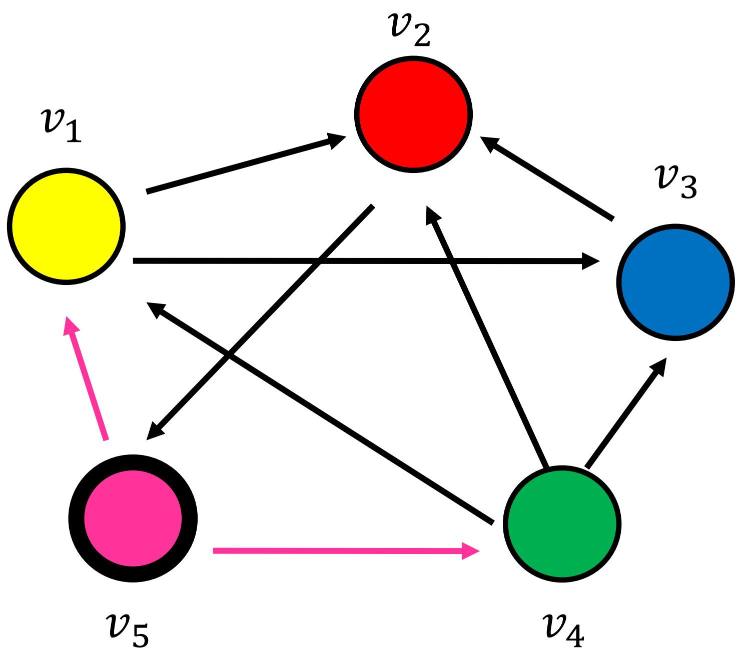
Example

- Step 1



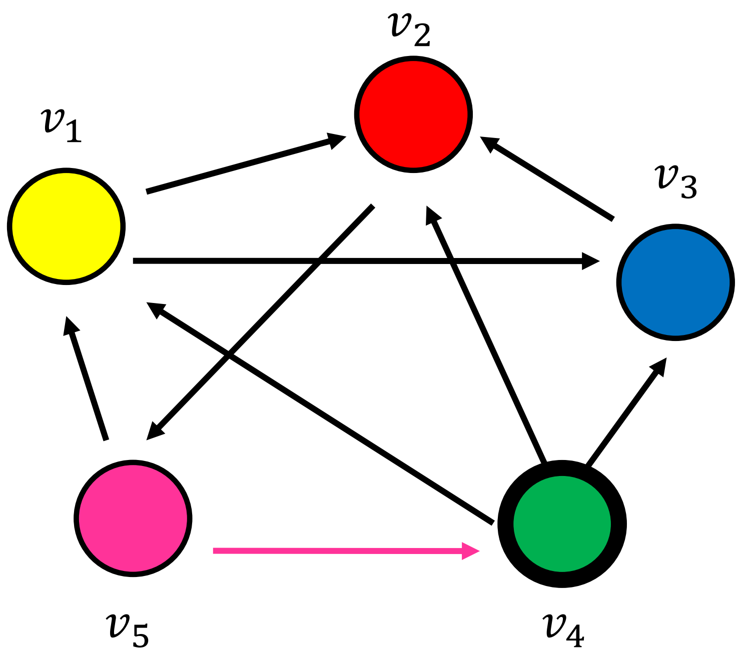
Example

- Step 1



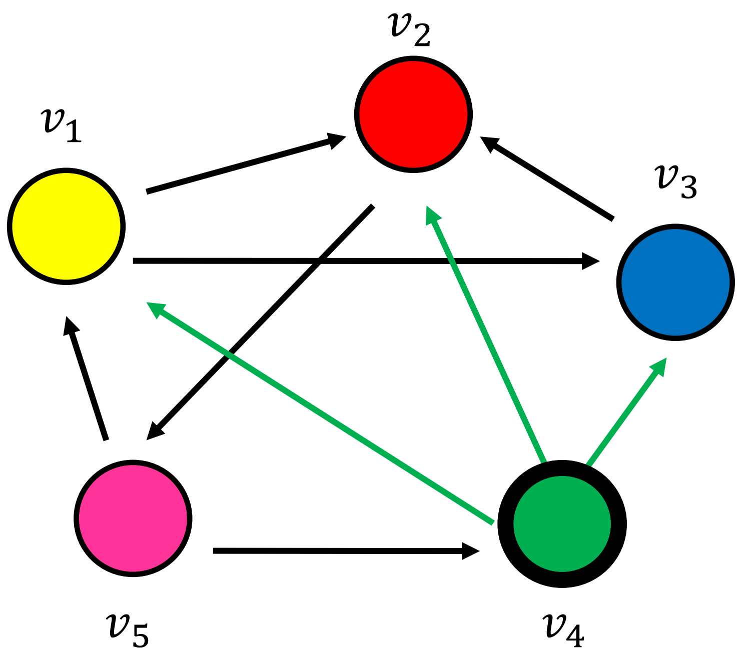
Example

- Step 2



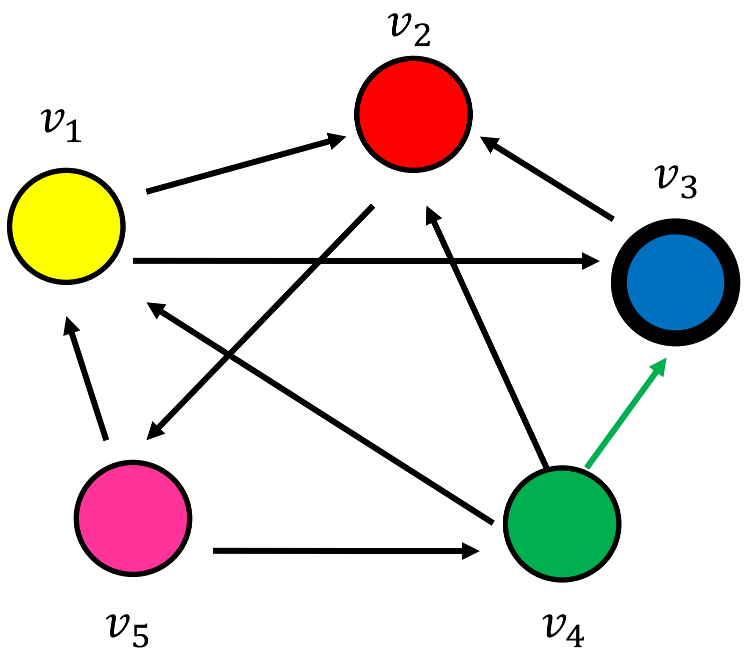
Example

- Step 2



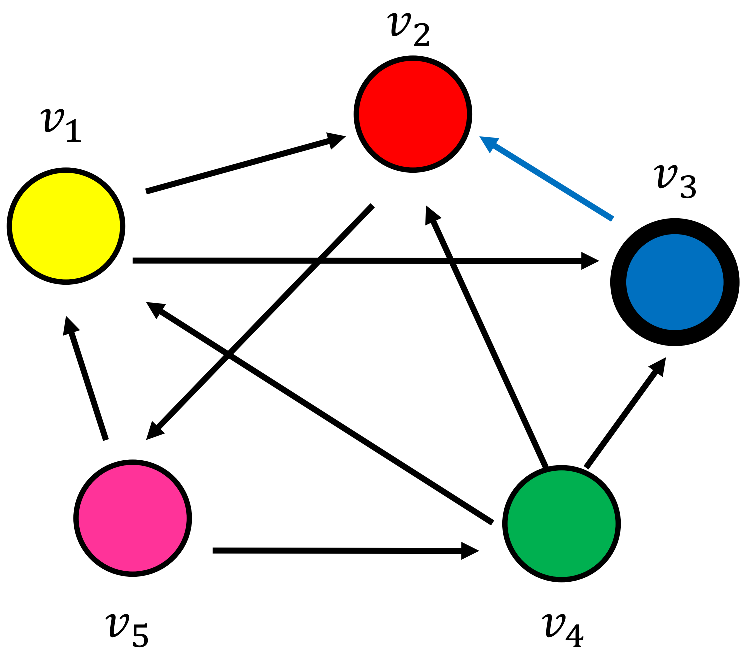
Example

- Step 3



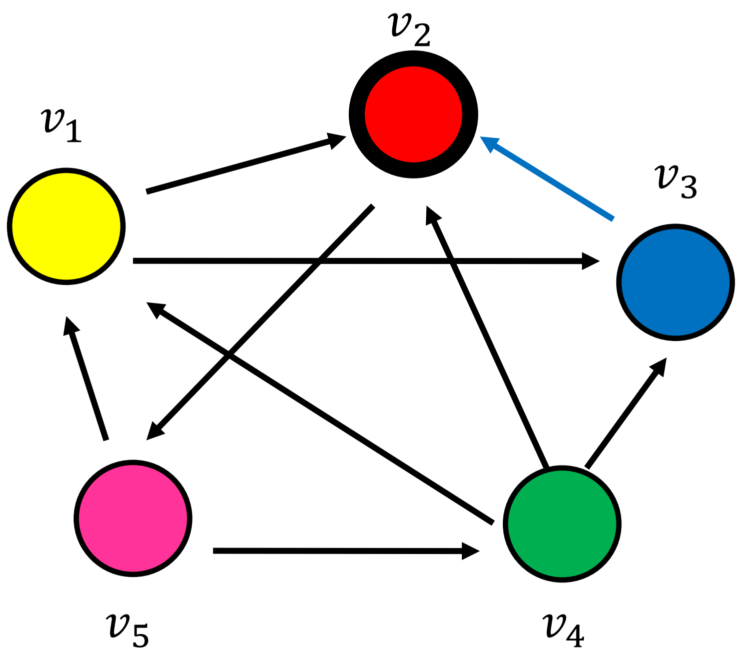
Example

- Step 3



Example

- Step 4…



Random walk

- Question: what is the probability p_i^t of being at node i after t steps?

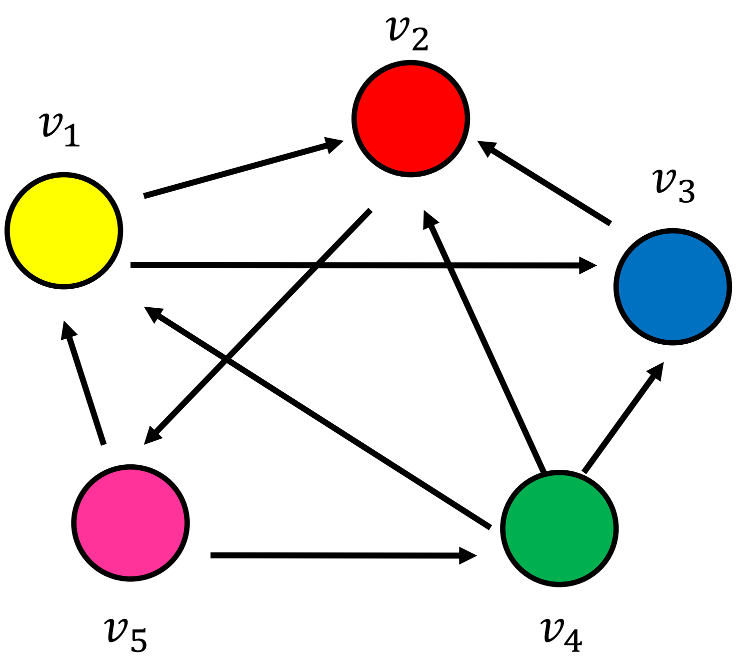
$$p_1^0 = \frac{1}{5} \qquad p_1^t = \frac{1}{3}p_4^{t-1} + \frac{1}{2}p_5^{t-1}$$

$$p_2^0 = \frac{1}{5} \qquad p_2^t = \frac{1}{2}p_1^{t-1} + p_3^{t-1} + \frac{1}{3}p_4^{t-1}$$

$$p_3^0 = \frac{1}{5} \qquad p_3^t = \frac{1}{2}p_1^{t-1} + \frac{1}{3}p_4^{t-1}$$

$$p_4^0 = \frac{1}{5} \qquad p_4^t = \frac{1}{2}p_5^{t-1}$$

$$p_5^0 = \frac{1}{5} \qquad p_5^t = p_2^{t-1}$$



Markov chains

- A Markov chain describes a discrete time stochastic process over a set of states
$$S = \{s_1, s_2, \dots, s_n\}$$
according to a transition probability matrix $P = \{P_{ij}\}$
  - P_{ij} = probability of moving to state j when at state i

- Matrix P has the property that the entries of all rows sum to 1
$$\sum_j P[i,j] = 1$$
A matrix with this property is called stochastic

- State probability distribution: The vector $p^t = (p_1^t, p_2^t, \dots, p_n^t)$ that stores the probability of being at state s_i after t steps

- Memorylessness property: The next state of the chain depends only at the current state and not on the past of the process (first order MC)
  - Higher order MCs are also possible

- Markov Chain Theory: After infinite steps the state probability vector converges to a unique distribution if the chain is irreducible (possible to get from any state to any other state) and aperiodic
  - These are the PageRank values

Random walks

- Random walks on graphs correspond to Markov Chains
  - The set of states S is the set of nodes of the graph G
  - The transition probability matrix is the probability that we follow an edge from one node to another

$$P[i,j] = \frac{1}{\mathrm{d}_{out}(i)}$$

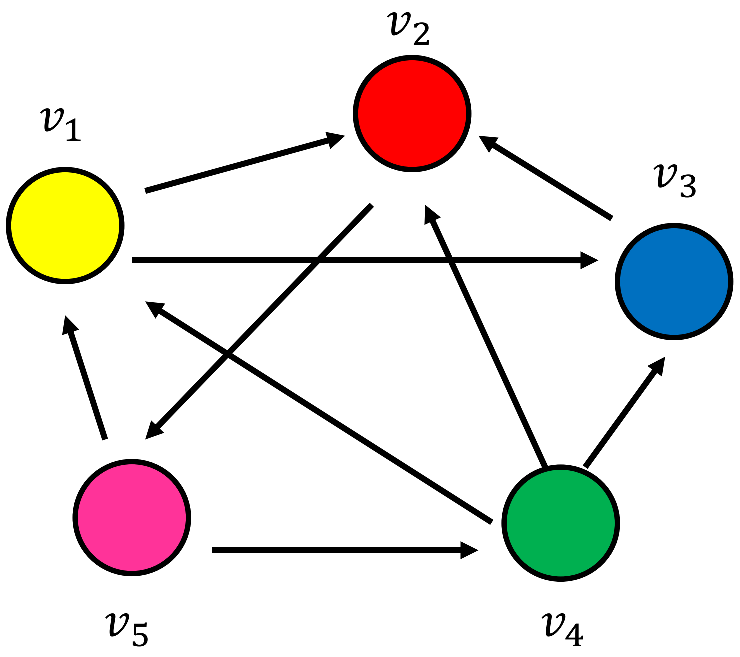
- We can compute the vector p^t at step t using a vector-matrix multiplication

$$p^{t+1} = p^t P$$

An example

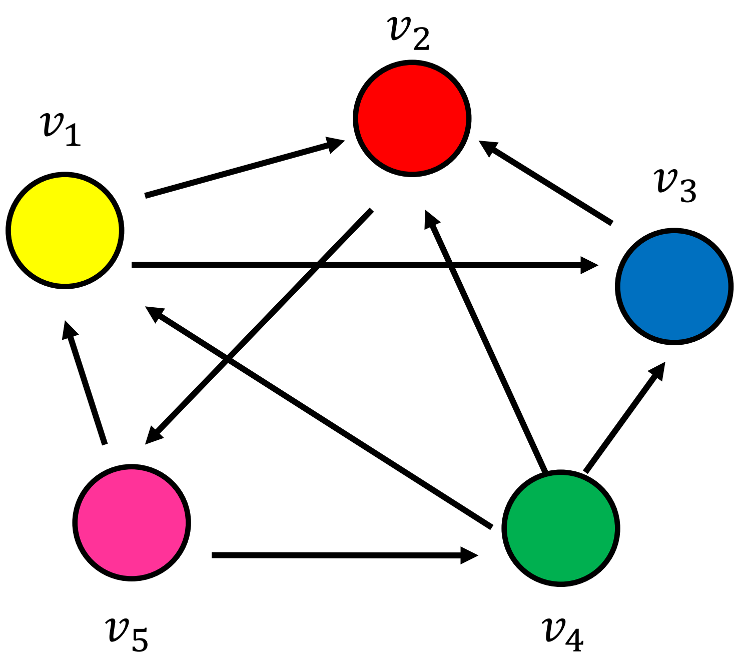
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$$



An example

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$$



$$p_1^t = \frac{1}{3} p_4^{t-1} + \frac{1}{2} p_5^{t-1}$$

$$p_2^t = \frac{1}{2} p_1^{t-1} + p_3^{t-1} + \frac{1}{3} p_4^{t-1}$$

$$p_3^t = \frac{1}{2} p_1^{t-1} + \frac{1}{3} p_4^{t-1}$$

$$p_4^t = \frac{1}{2} p_5^{t-1}$$

$$p_5^t = p_2^{t-1}$$

Stationary distribution

- The stationary distribution of a random walk with transition matrix P, is a probability distribution π, such that $\pi = \pi P$

- The stationary distribution is an eigenvector of matrix P
  – the principal left eigenvector of P – stochastic matrices have maximum eigenvalue 1

- Markov Chain Theory: The random walk converges to a unique stationary distribution independent of the initial vector if the graph is strongly connected, and not bipartite.
  – In our case these are the PageRank values.

Computing the stationary distribution

- The Power Method

```
Initialize $p^0$ to some distribution
Repeat
    $p^t = p^{t-1}P$
Until convergence
```

- After many iterations $p^t \to \pi$ regardless of the initial vector p^0
- Power method because it computes $p^t = p^0 P^t$
- Rate of convergence
  - determined by the second eigenvalue $\frac{|\lambda_2|}{|\lambda_1|}$

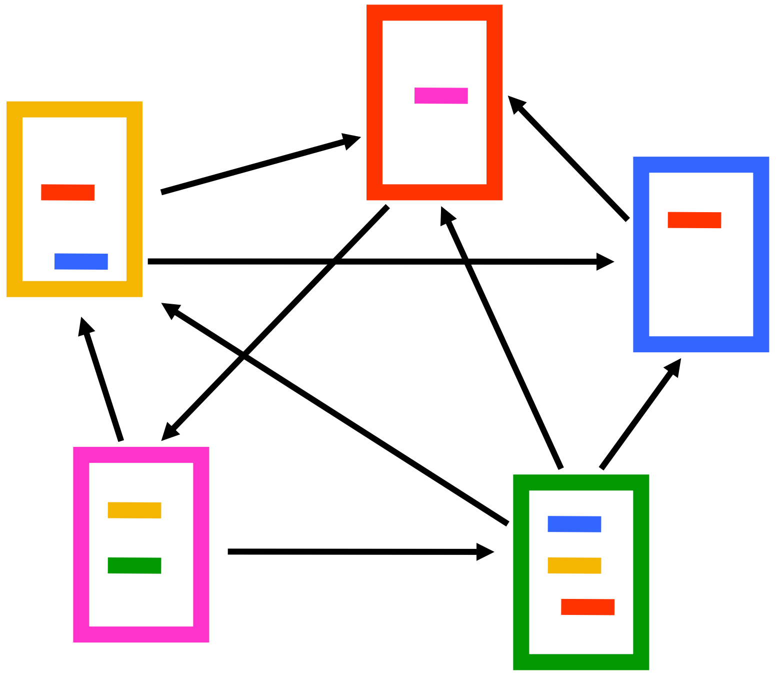
The stationary distribution

- What is the meaning of the stationary distribution π of a random walk?
- $\pi(i)$: the probability of being at node i after very large (infinite) number of steps, or, the the fraction of times that we visited state i as $t \rightarrow \infty$
- $\pi = p_0 P^{\infty}$, where P is the transition matrix, p_0 the original vector
  - $P(i,j)$: probability of going from i to j in one step
  - $P^2(i,j)$: probability of going from i to j in two steps (probability of all paths of length 2)
  - $P^{\infty}(i,j) = \pi(j)$: probability of going from i to j in infinite steps – starting point does not matter.

The PageRank random walk

- Vanilla random walk
  - make the adjacency matrix stochastic and run a random walk

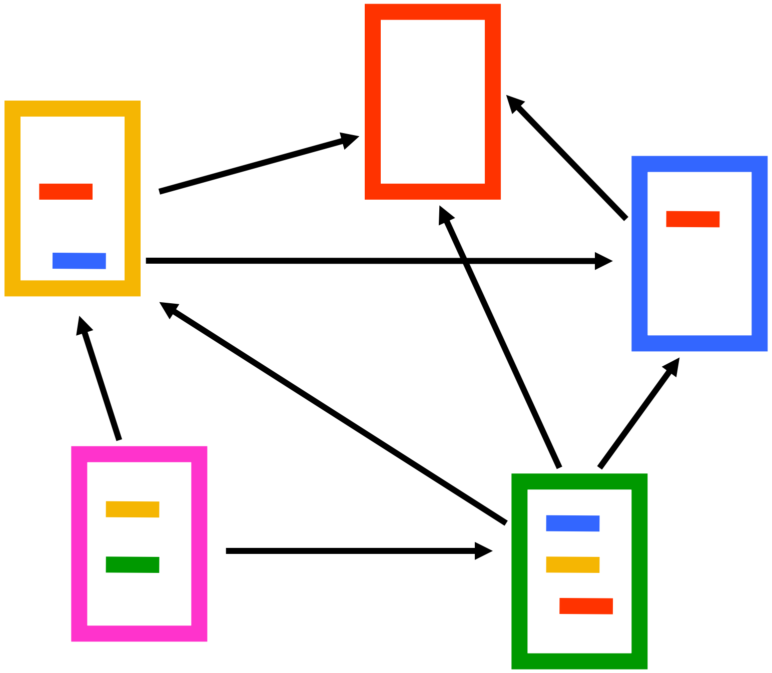
$$P = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$$



The PageRank random walk

- What about sink nodes?
  - what happens when the random walk moves to a node without any outgoing inks?

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$$



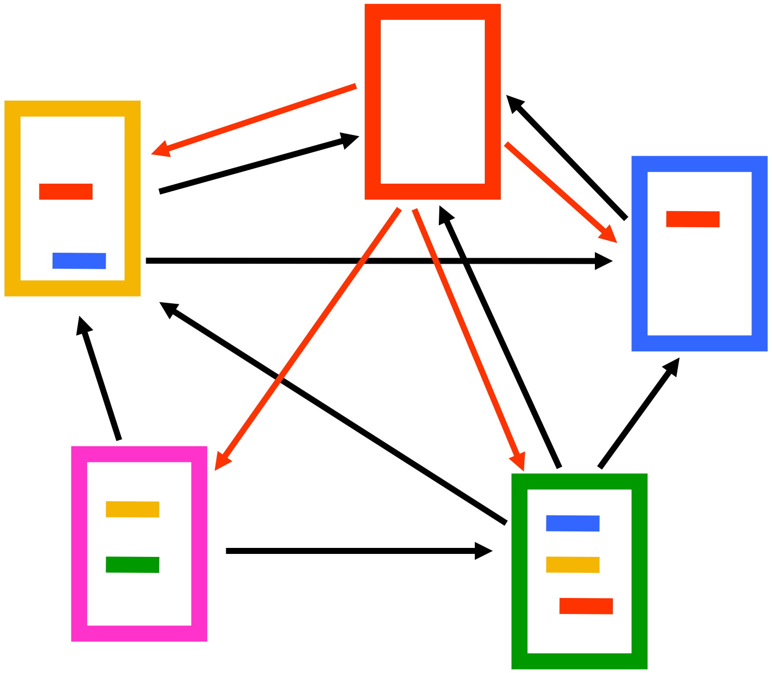
The PageRank random walk

- Replace these row vectors with a vector v
  - typically, the uniform vector

$$P' = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$$

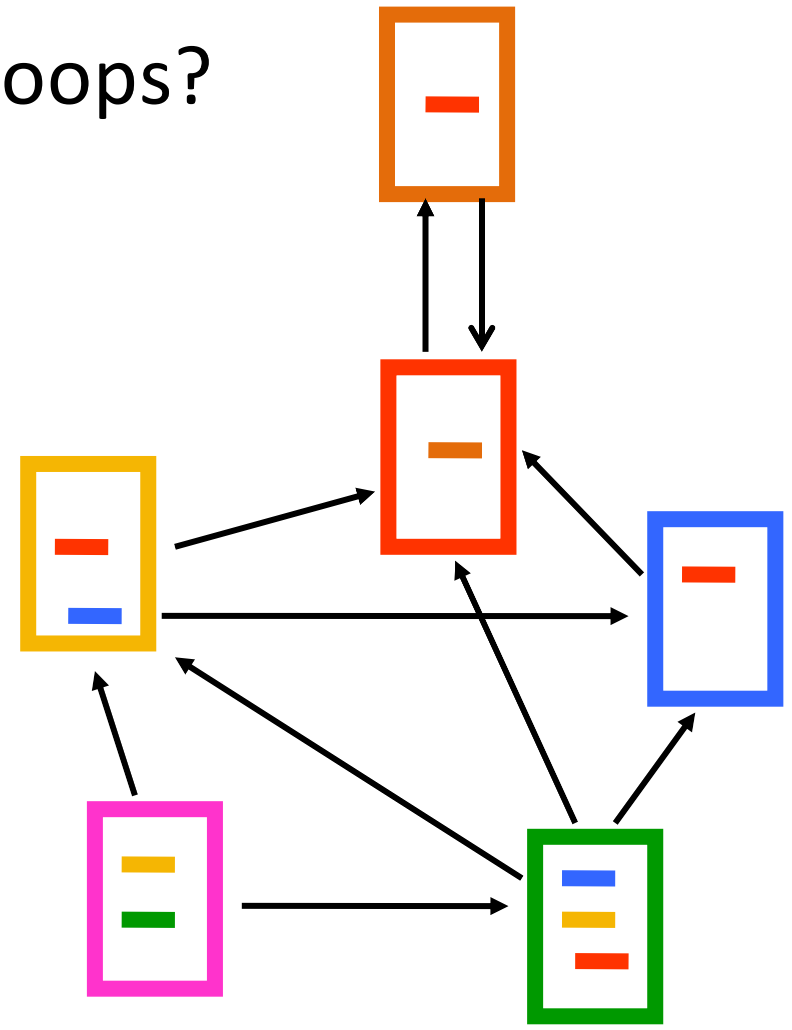
$P' = P + dv^T$

$$d = \begin{cases} 1 & \text{if i is sink} \\ 0 & \text{otherwise} \end{cases}$$



The PageRank random walk

- What about loops?
  - Spider traps



The PageRank random walk

- Add a random jump to vector v with prob $1 - \alpha$
  - typically, to a uniform vector
- Restarts after $1/(1-\alpha)$ steps in expectation
  - Guarantees irreducibility, convergence

$$P'' = \alpha \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 \end{bmatrix} + (1-\alpha) \begin{bmatrix} 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \end{bmatrix}$$

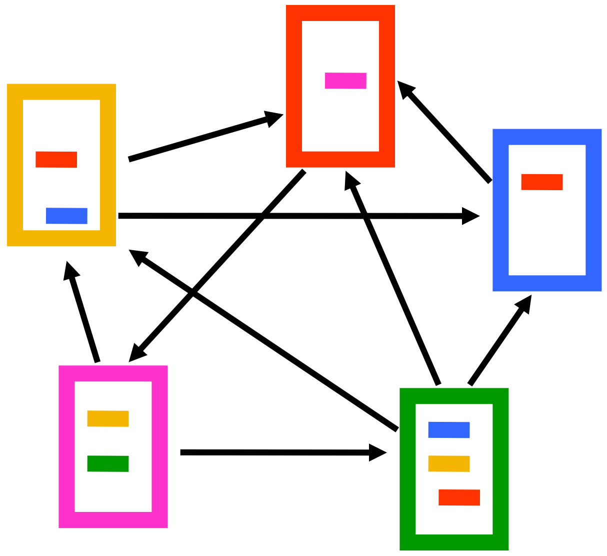
$P'' = \alpha P' + (1-\alpha)uv^T$, where u is the vector of all 1s

Random walk with restarts

PageRank algorithm [BP98]

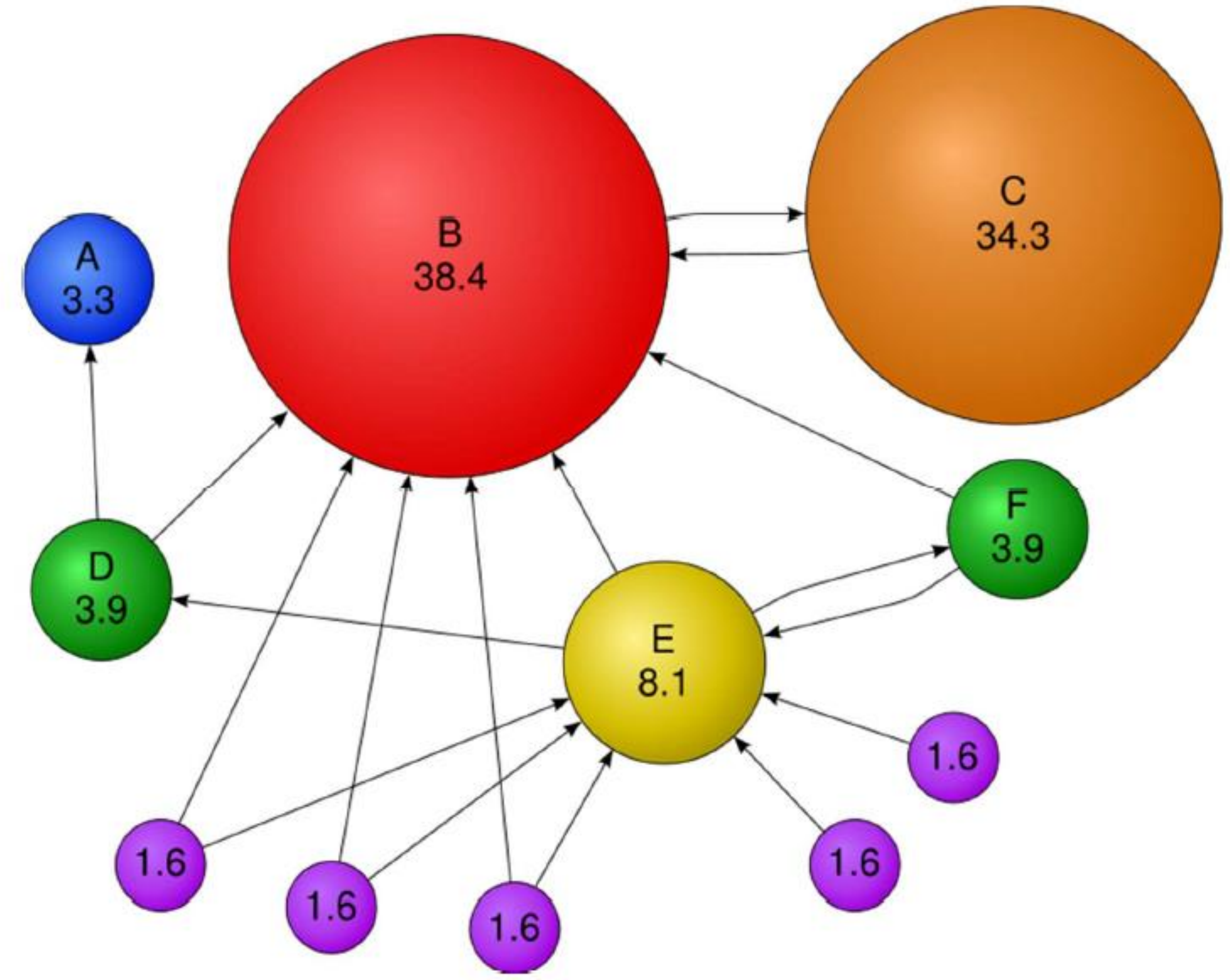
- The Random Surfer model
  - pick a page at random
  - with probability $1 - \alpha$ jump to a random page
  - with probability α follow a random outgoing link
- Rank according to the stationary distribution
- $w_v = \alpha \sum_{u \to v} \frac{1}{d_{out}(u)} w_u + (1 - \alpha) \frac{1}{n}$

  $\alpha = 0.85$ in most cases
- We repeat this computation until convergece



1. **Red Page**
2. **Purple Page**
3. **Yellow Page**
4. **Blue Page**
5. **Green Page**

PageRank: Example



Stationary distribution with random jump

- If v is the jump vector

$$
\begin{aligned}
p^0 &= v \\
p^1 &= \alpha p^0 P' + (1-\alpha)v = \alpha v P' + (1-\alpha)v \\
p^2 &= \alpha p^1 P' + (1-\alpha)v \\
&= \alpha^2 v P'^2 + (1-\alpha) v \alpha P' + (1-\alpha)v \\
&\vdots \\
p^\infty &= (1-\alpha)v + (1-\alpha)v\alpha P' + (1-\alpha)v\alpha^2 P'^2 + \cdots \\
&= (1-\alpha)(I-\alpha P')^{-1}
\end{aligned}
$$

- With the random jump the shorter paths are more important, since the weight decreases exponentially
  - makes sense when thought of as a restart

Random walks with restarts

- If v is not uniform, we can bias the random walk towards the nodes that are close to v
- Personalized PageRank:
  - Restart the random walk from a specific node x
  - All nodes are ranked according to their closeness to x
- Topic-Specific Pagerank.
  - Restart the random walk from a specific set of nodes (e.g., nodes about a topic)
  - All nodes are ranked according to their closeness to the topic.
- Random Walks with restarts is a general technique for measuring closeness on graphs.

Effects of random jump

- Guarantees convergence to unique distribution
- Motivated by the concept of random surfer
- Offers additional flexibility
  - personalization
  - anti-spam
- Controls the rate of convergence
  - the second eigenvalue of matrix P'' is α

Random walks on undirected graphs

- For undirected graphs, the stationary distribution of a random walk is proportional to the degrees of the nodes
  - Thus in this case a random walk is the same as degree popularity
- This is not longer true if we do random jumps
  - Now the short paths play a greater role, and the previous distribution does not hold.
  - Random walks with restarts to a single node are commonly used on undirected graphs for measuring similarity between nodes

PageRank implementation

- Store the graph as a list of edges
- Keep current pagerank values and new pagerank values
- Go through edges and update the values of the destination nodes.
- Repeat until the difference between the pagerank vectors (L_1 or L_∞ difference) is below some small value ε.

A (Matlab-friendly) PageRank algorithm

- Performing vanilla power method is now too expensive – the matrix is not sparse

```
$q^0 = v$
$t = 1$
repeat
    $q^t = (P'')^T q^{t-1}$
    $\delta = \|q^t - q^{t-1}\|$
    $t = t + 1$
until $\delta < \varepsilon$
```

Efficient computation of $y = (P'')^T x$

$$y = \alpha P^T x$$
$$\beta = \|x\|_1 - \|y\|_1$$
$$y = y + \beta v$$

P = normalized adjacency matrix

$P' = P + dv^T$, where d_i is 1 if i is sink and 0 o.w.

$P'' = \alpha P' + (1-\alpha)uv^T$, where u is the vector of all 1s

PageRank history

- Huge advantage for Google in the early days
  - It gave a way to get an idea for the value of a page, which was useful in many different ways
    - Put an order to the web.
  - After a while it became clear that the anchor text was probably more important for ranking
  - Also, link spam became a new (dark) art
- Flood of research
  - Numerical analysis got rejuvenated
  - Huge number of variations
  - Efficiency became a great issue.
  - Huge number of applications in different fields
    - Random walk is often referred to as PageRank.

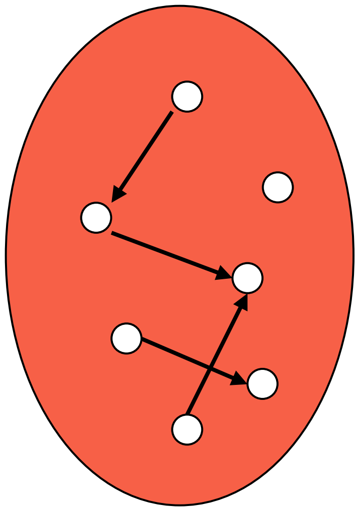
THE HITS ALGORITHM

The HITS algorithm

- Another algorithm proposed around the same time as PageRank for using the hyperlinks to rank pages
  - Kleinberg: then an intern at IBM Almaden
  - IBM never made anything out of it

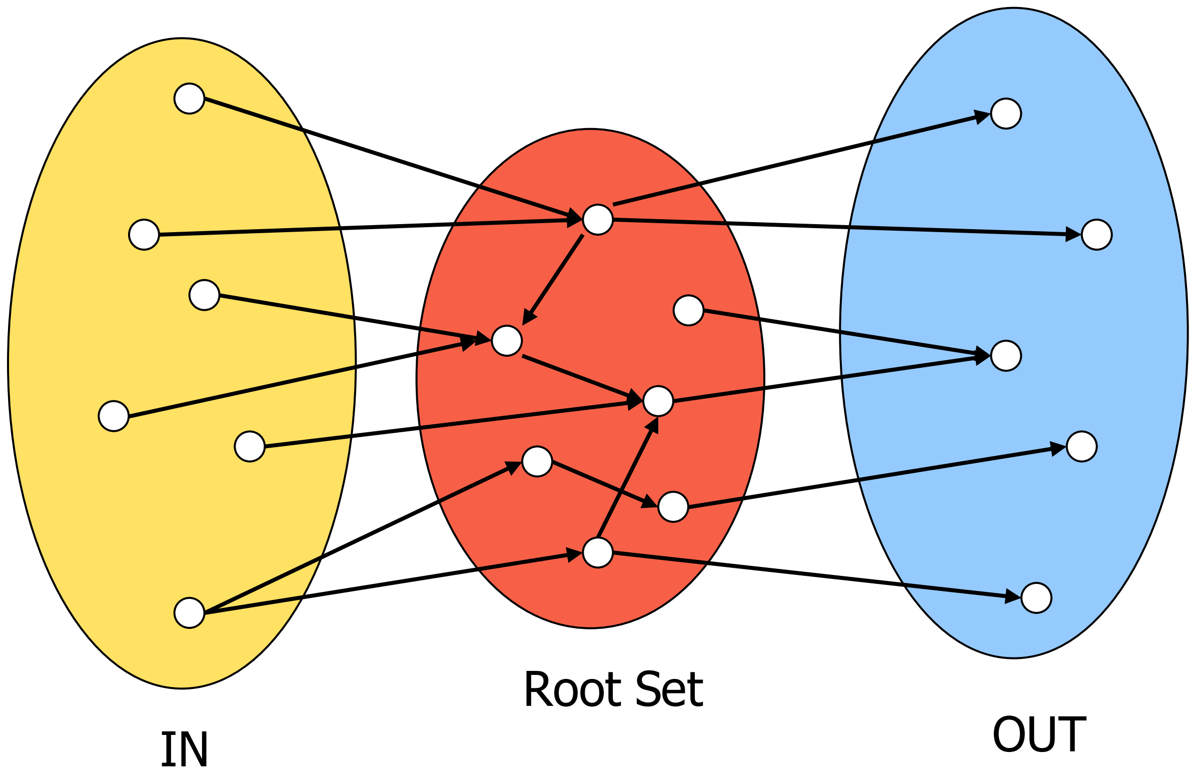
Query dependent input

Root set obtained from a text-only search engine

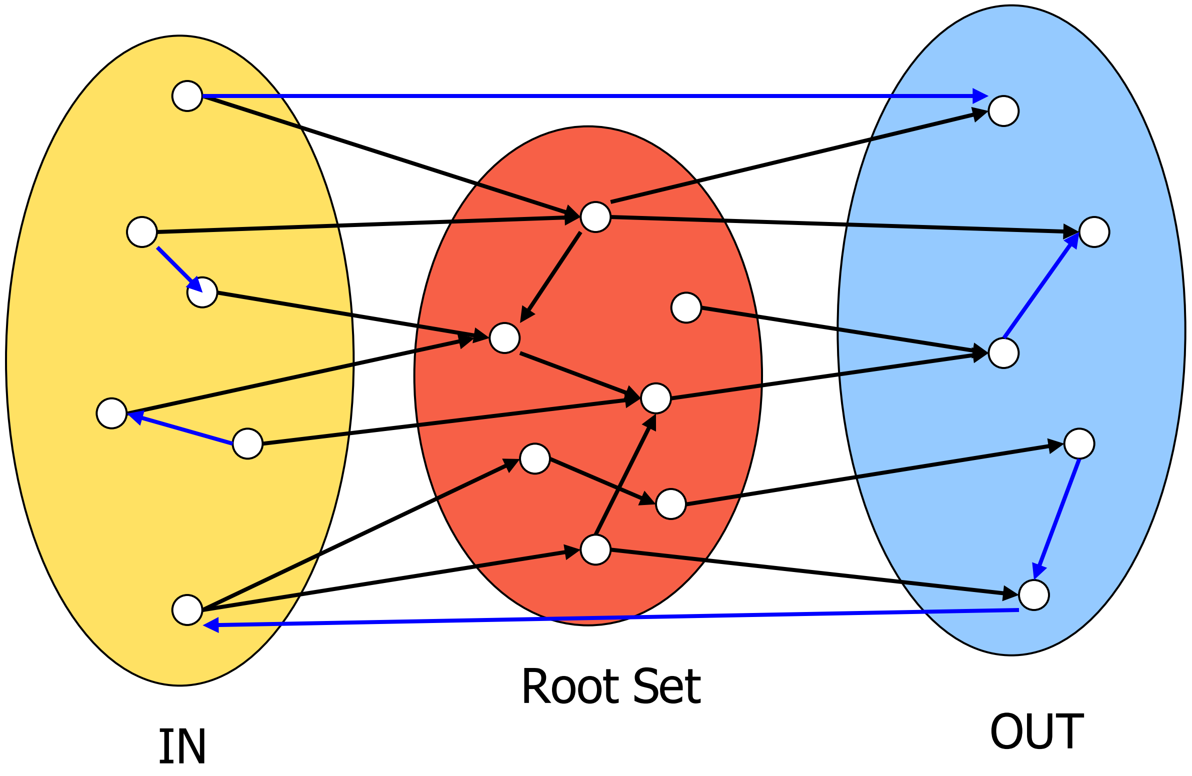


Root Set

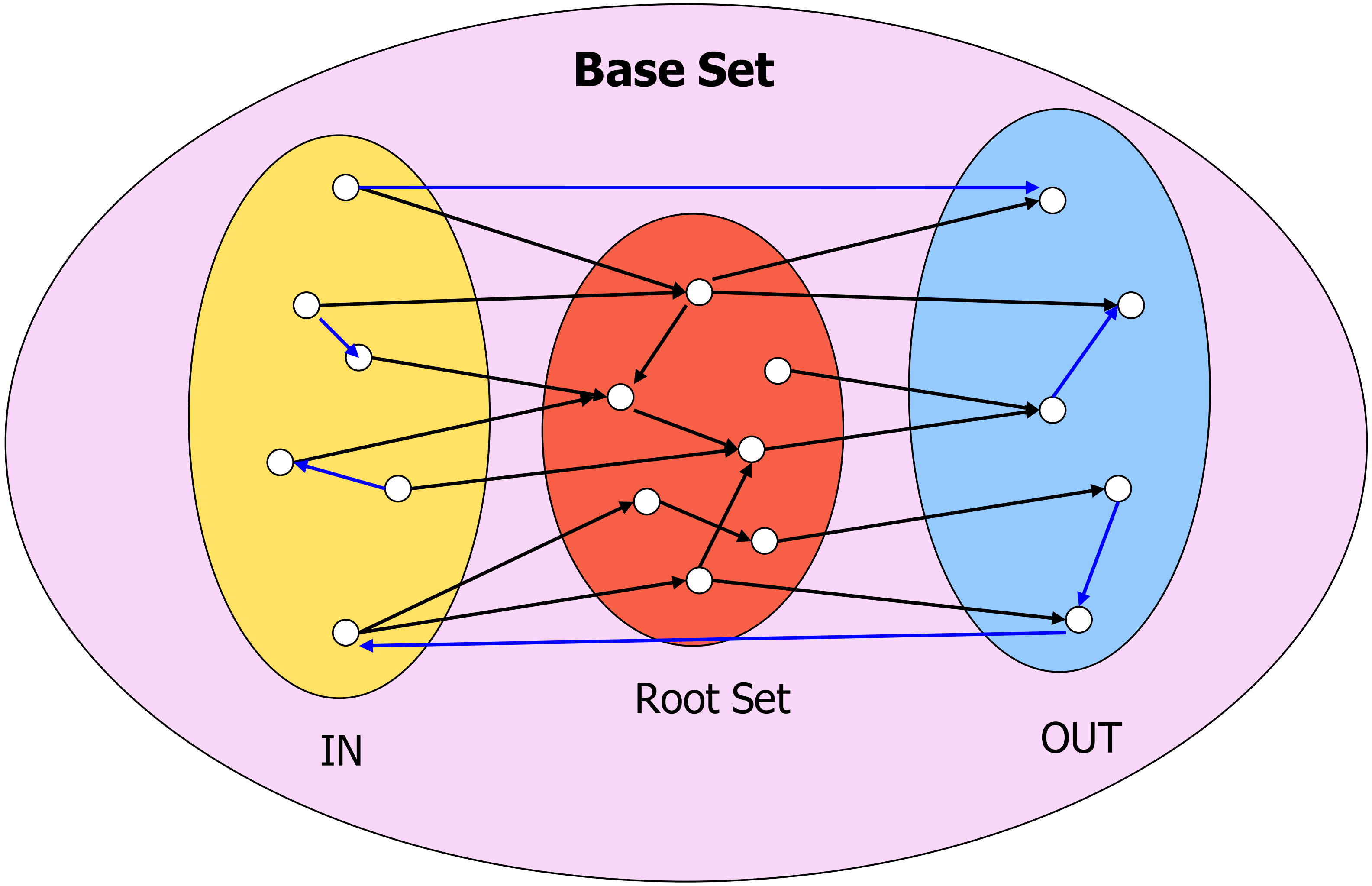
Query dependent input



Query dependent input

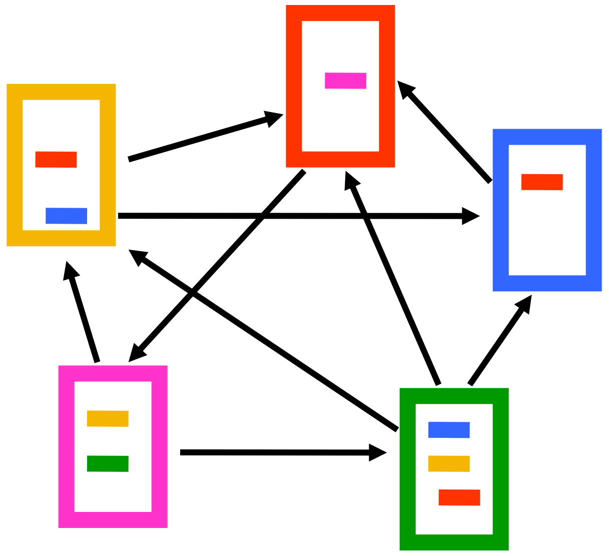


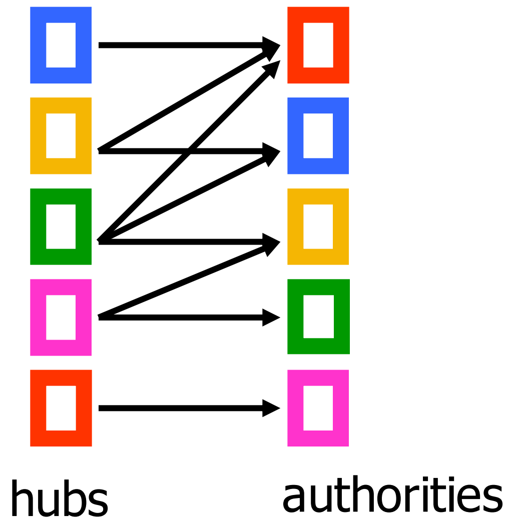
Query dependent input



Hubs and Authorities [K98]

- Authority is not necessarily transferred directly between authorities
- Pages have double identity
  - hub identity
  - authority identity
- Good hubs point to good authorities
- Good authorities are pointed by good hubs





Hubs and Authorities

- Two kind of weights:
  - Hub weight
  - Authority weight

- The hub weight is the sum of the authority weights of the authorities pointed to by the hub

- The authority weight is the sum of the hub weights that point to this authority.

HITS Algorithm

- Initialize all weights to 1.
- Repeat until convergence
  - O operation : hubs collect the weight of the authorities

$$h_i = \sum_{j:i \to j} a_j$$

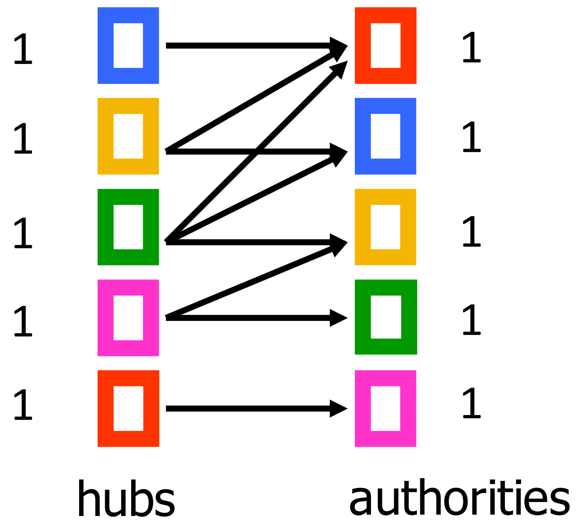
  - I operation: authorities collect the weight of the hubs

$$a_i = \sum_{j:j \to i} h_j$$

  - Normalize weights under some norm

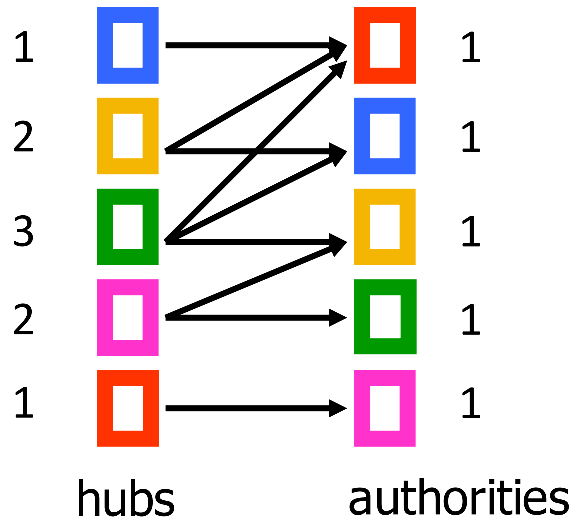
Example

Initialize



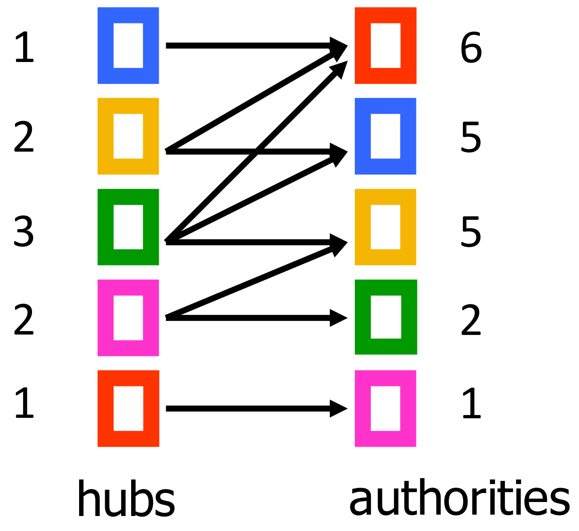
Example

Step 1: O operation



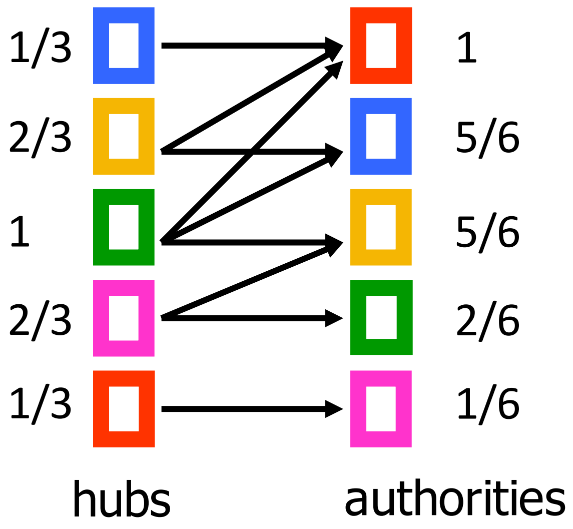
Example

Step 1: I operation



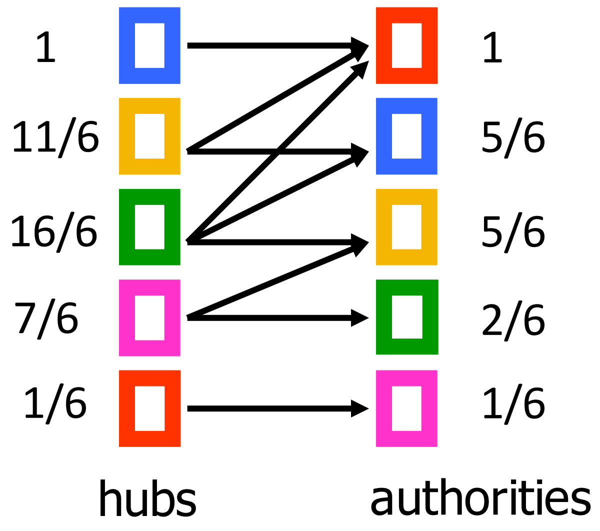
Example

Step 1: Normalization (Max norm)



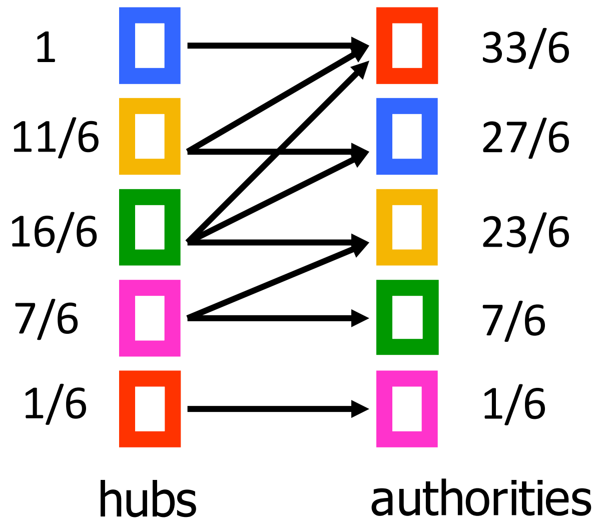
Example

Step 2: O step



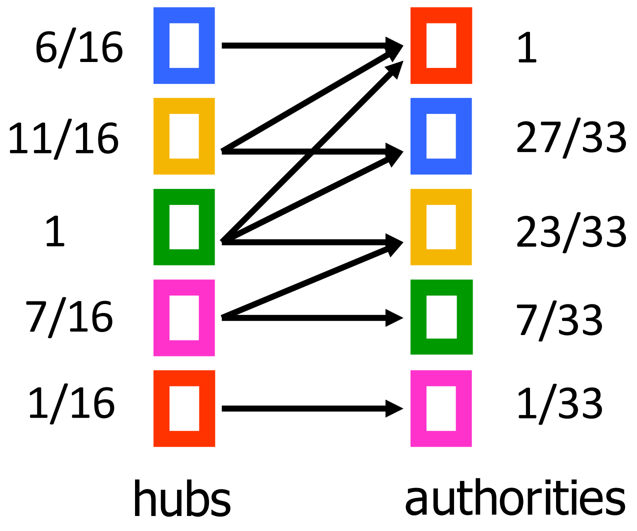
Example

Step 2: I step



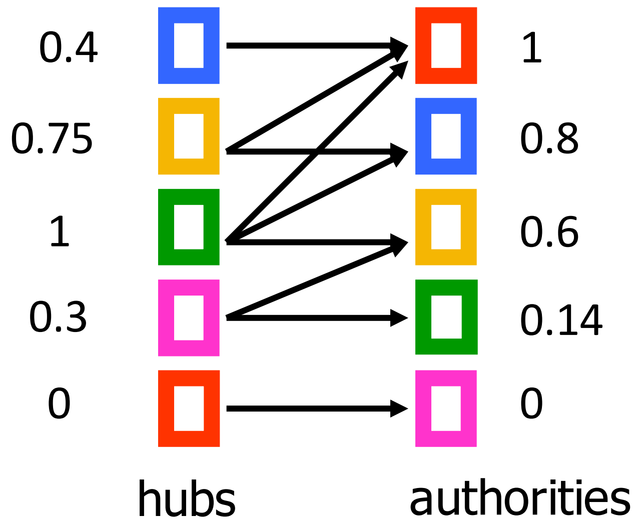
Example

Step 2: Normalization



Example

Convergence



HITS and eigenvectors

- The HITS algorithm is a power-method eigenvector computation
- In vector terms
  - $a^t = A^T h^{t-1}$ and $h^t = A a^{t-1}$
  - $a^t = A^T A a^{t-1}$ and $h^t = A A^T h^{t-1}$
  - Repeated iterations will converge to the eigenvectors
- The authority weight vector a is the eigenvector of $A^T A$ and the hub weight vector h is the eigenvector of $A A^T$

- The vectors a and h are called the singular vectors of the matrix A

Singular Value Decomposition

$$\underset{[n\times r]}{A = U}\ \ \underset{[r\times r]}{\Sigma}\ \ \underset{[r\times n]}{V^T} = \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \cdots & \vec{u}_r \end{bmatrix} \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_r \end{bmatrix} \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vdots \\ \vec{v}_r \end{bmatrix}$$

- **r** : rank of matrix A
- $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r$: singular values (square roots of eig-vals AA^T, A^TA)
- $\vec{u}_1, \vec{u}_2, \cdots, \vec{u}_r$: left singular vectors (eig-vectors of AA^T)
- $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_r$: right singular vectors (eig-vectors of A^TA)

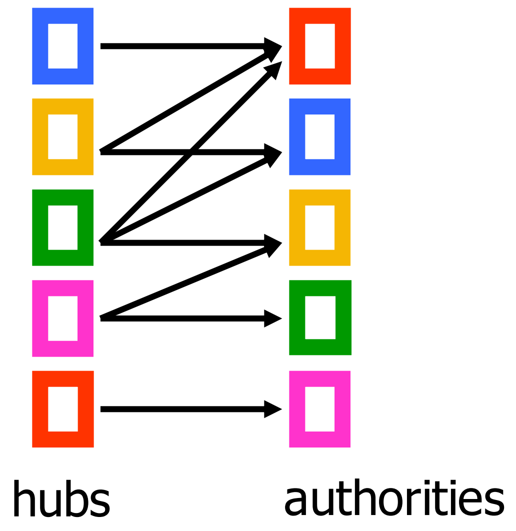
$$A = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T + \cdots + \sigma_r \vec{u}_r \vec{v}_r^T$$

Why does the Power Method work?

- If a matrix R is real and symmetric, it has real eigenvalues and eigenvectors: $(\lambda_1, w_1), (\lambda_2, w_2), \dots, (\lambda_r, w_r)$
  - r is the rank of the matrix
  - $|\lambda_1| \geq |\lambda_2| \geq \cdots \geq |\lambda_r|$
- For any matrix R, the eigenvectors $w_1, w_2, \dots, w_r$ of R define a basis of the vector space
  - For any vector x, $Rx = \alpha_1 w_1 + a_2 w_2 + \cdots + a_r w_r$
- After t multiplications we have:
  - $R^t x = \lambda_1^{t-1} \alpha_1 w_1 + \lambda_2^{t-1} a_2 w_2 + \cdots + \lambda_r^{t-1} a_r w_r$
- Normalizing (divide by λ_1^{t-1}) leaves only the term w_1.

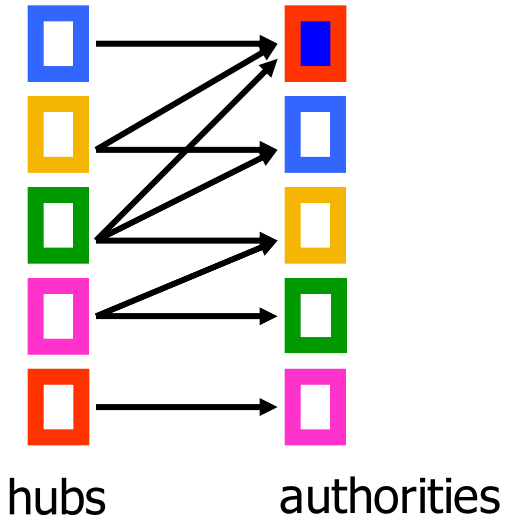
The SALSA algorithm

- Perform a random walk on the bipartite graph of hubs and authorities alternating between the two



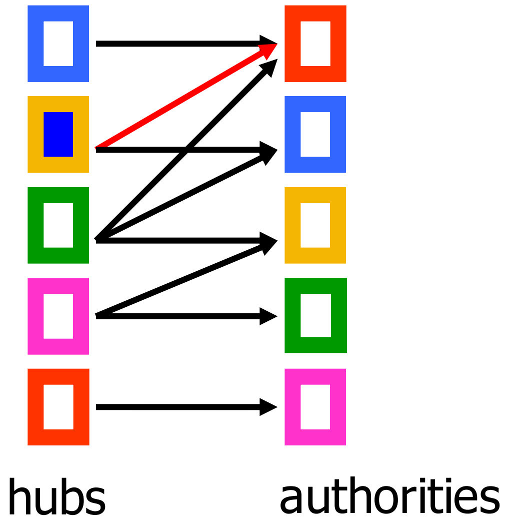
The SALSA algorithm

- Start from an authority chosen uniformly at random
  - e.g. the red authority



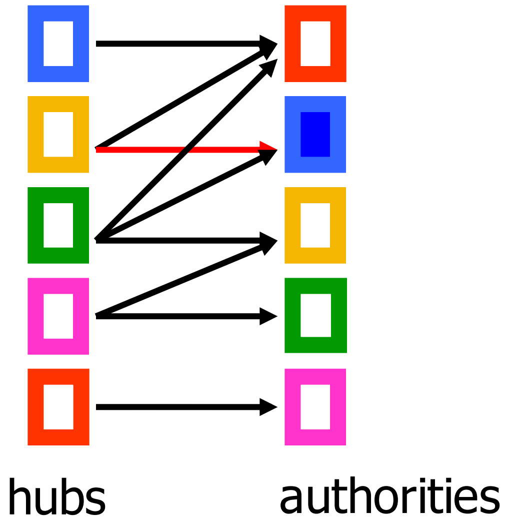
The SALSA algorithm

- Start from an authority chosen uniformly at random
  - e.g. the red authority
- Choose one of the in-coming links uniformly at random and move to a hub
  - e.g. move to the yellow authority with probability 1/3



The SALSA algorithm

- Start from an authority chosen uniformly at random
  - e.g. the red authority
- Choose one of the in-coming links uniformly at random and move to a hub
  - e.g. move to the yellow authority with probability 1/3
- Choose one of the out-going links uniformly at random and move to an authority
  - e.g. move to the blue authority with probability 1/2



The SALSA algorithm

- Formally we have probabilities:
  - a_i: probability of being at authority i
  - h_j: probability of being at hub j
- The probability of being at authority i is computed as:

$$a_i = \sum_{j \in N_{in}(i)} \frac{1}{d_{out}(j)} h_j$$

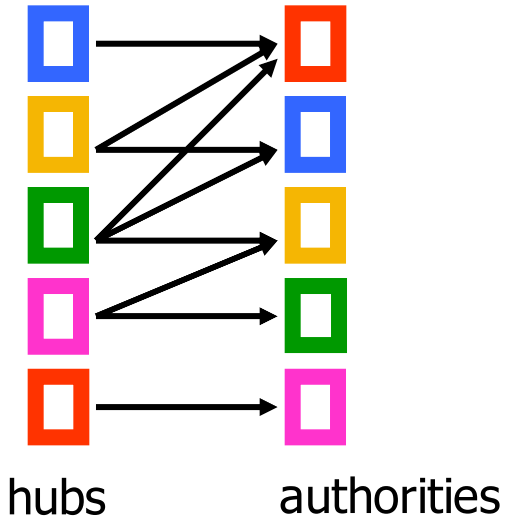
- The probability of being at hub j is computed as

$$h_j = \sum_{i \in N_{out}(j)} \frac{1}{d_{in}(i)} a_i$$

- Repeated computation converges

The SALSA algorithm [LM00]

- In matrix terms
  - A_c = the matrix A where columns are normalized to sum to 1
  - A_r = the matrix A where rows are normalized to sum to 1
- The hub computation
  - $h = A_c a$
- The authority computation
  - $a = A_r^T h = A_r^T A_c a$
- In MC terms the transition matrix
  - $P = A_r A_c^T$



$$h_2 = 1/3\, a_1 + 1/2\, a_2$$

$$a_1 = h_1 + 1/2\, h_2 + 1/3\, h_3$$