DATA MINING
LECTURE 5

Similarity and Distance

Sketching, Locality Sensitive Hashing

SIMILARITY AND DISTANCE

Thanks to:

Tan, Steinbach, and Kumar, “Introduction to Data Mining”

Rajaraman and Ullman, “Mining Massive Datasets”

Similarity and Distance

- For many different problems we need to quantify how close two objects are.
- Examples:
  - For an item bought by a customer, find other similar items
  - Group together the customers of site so that similar customers are shown the same ad.
  - Group together web documents so that you can separate the ones that talk about politics and the ones that talk about sports.
  - Find all the near-duplicate mirrored web documents.
  - Find credit card transactions that are very different from previous transactions.
- To solve these problems we need a definition of similarity, or distance.
  - The definition depends on the type of data that we have

Similarity

- Numerical measure of how alike two data objects are.
  - A function that maps pairs of objects to real values
  - Higher when objects are more alike.
- Often falls in the range [0,1], sometimes in [-1,1]

- Desirable properties for similarity
  1. s(p, q) = 1 (or maximum similarity) only if p = q. (Identity)
  2. s(p, q) = s(q, p) for all p and q. (Symmetry)

Similarity between sets

- Consider the following documents

apple releases new ipod

apple releases new ipad

new apple pie recipe

- Which ones are more similar?

- How would you quantify their similarity?

Similarity: Intersection

- Number of words in common

apple releases new ipod

apple releases new ipad

new apple pie recipe

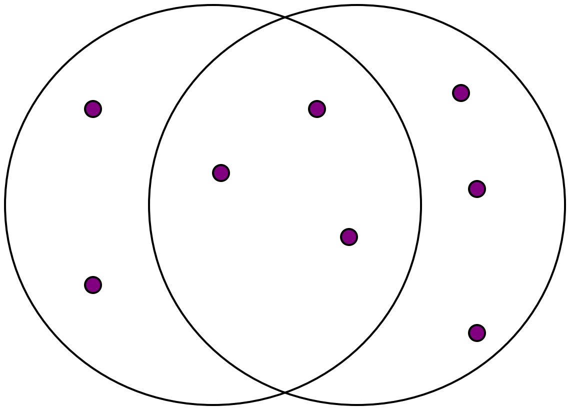
- Sim(D,D) = 3, Sim(D,D) = Sim(D,D) =2
- What about this document?

Vefa rereases new book with apple pie recipes

- Sim(D,D) = Sim(D,D) = 3

Jaccard Similarity

- The Jaccard similarity (Jaccard coefficient) of two sets S_1, S_2 is the size of their intersection divided by the size of their union.
  - JSim $(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$.



3 in intersection.
8 in union.
Jaccard similarity
= 3/8

- Extreme behavior:
  - Jsim(X,Y) = 1, iff X = Y
  - Jsim(X,Y) = 0 iff X,Y have not elements in common
- JSim is symmetric

Similarity: Intersection

- Number of words in common

apple releases new ipod

apple releases new ipad

new apple pie recipe

Vefa rereases new book with apple pie recipes

- JSim(D,D) = 3/5
- JSim(D,D) = JSim(D,D) = 2/6
- JSim(D,D) = JSim(D,D) = 3/9

Similarity between vectors

Documents (and sets in general) can also be represented as vectors

document	Apple	Microsoft	Obama	Election
D1	10	20	0	0
D2	30	60	0	0
D2	0	0	10	20

How do we measure the similarity of two vectors?

How well are the two vectors aligned?

Example

document	Apple	Microsoft	Obama	Election
D1	1/3	2/3	0	0
D2	1/3	2/3	0	0
D2	0	0	1/3	2/3

Documents D1, D2 are in the “same direction”
Document D3 is orthogonal to these two

Cosine Similarity

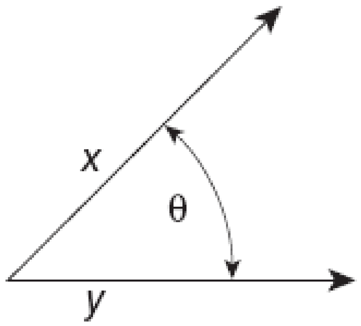


Figure 2.16. Geometric illustration of the cosine measure.

- Sim(X,Y) = cos(X,Y)
  - The cosine of the angle between X and Y

- If the vectors are aligned (correlated) angle is zero degrees and cos(X,Y)=1
- If the vectors are orthogonal (no common coordinates) angle is 90 degrees and cos(X,Y) = 0

- Cosine is commonly used for comparing documents, where we assume that the vectors are normalized by the document length.

Cosine Similarity - math

- If d_1 and d_2 are two vectors, then

$$\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1|| \, ||d_2|| ,$$

where • indicates vector dot product and || *d* || is the length of vector *d*.

- Example:

d_1 = **3 2 0 5 0 0 0 2 0 0**
d_2 = **1 0 0 0 0 0 0 1 0 2**

$d_1 \bullet d_2$ = 3*1 + 2*0 + 0*0 + 5*0 + 0*0 + 0*0 + 0*0 + 2*1 + 0*0 + 0*2 = 5

$||d_1|| = (3*3+2*2+0*0+5*5+0*0+0*0+0*0+2*2+0*0+0*0)^{0.5} = (42)^{0.5} = 6.481$

$||d_2|| = (1*1+0*0+0*0+0*0+0*0+0*0+0*0+1*1+0*0+2*2)^{0.5} = (6)^{0.5} = 2.245$

$\cos(d_1, d_2) = .3150$

Similarity between vectors

document	Apple	Microsoft	Obama	Election
D1	10	20	0	0
D2	30	60	0	0
D2	0	0	10	20

cos(D1,D2) = 1

cos(D1,D3) = cos(D2,D3) = 0

Distance

- Numerical measure of how different two data objects are
  - A function that maps pairs of objects to real values
  - Lower when objects are more alike
- Minimum distance is 0, when comparing an object with itself.
- Upper limit varies

Distance Metric

- A distance function d is a distance metric if it is a function from pairs of objects to real numbers such that:
  1. $d(x,y) \geq 0$. (non-negativity)
  2. $d(x,y) = 0$ iff $x = y$. (identity)
  3. $d(x,y) = d(y,x)$. (symmetry)
  4. $d(x,y) \leq d(x,z) + d(z,y)$ (triangle inequality).

Triangle Inequality

- Triangle inequality guarantees that the distance function is well-behaved.
  - The direct connection is the shortest distance

- It is useful also for proving properties about the data
  - For example, suppose I want to find an object that minimizes the sum of distances to all points in my dataset
  - If I select the best point from my dataset, the sum of distances I get is at most twice that of the optimal point.

Distances for real vectors

- Vectors $x = (x_1, \dots, x_d)$ and $y = (y_1, \dots, y_d)$

- **L_p** norms or Minkowski distance:

$$L_p(x, y) = \left[|x_1 - y_1|^p + \cdots + |x_d - y_d|^p\right]^{1/p}$$

- **L_2** norm: Euclidean distance:

$$L_2(x, y) = \sqrt{|x_1 - y_1|^2 + \cdots + |x_d - y_d|^2}$$

- **L_1** norm: Manhattan distance:

$$L_1(x, y) = |x_1 - y_1| + \cdots + |x_d - y_d|$$

L_p norms are known to be distance metrics

- **L_∞** norm:

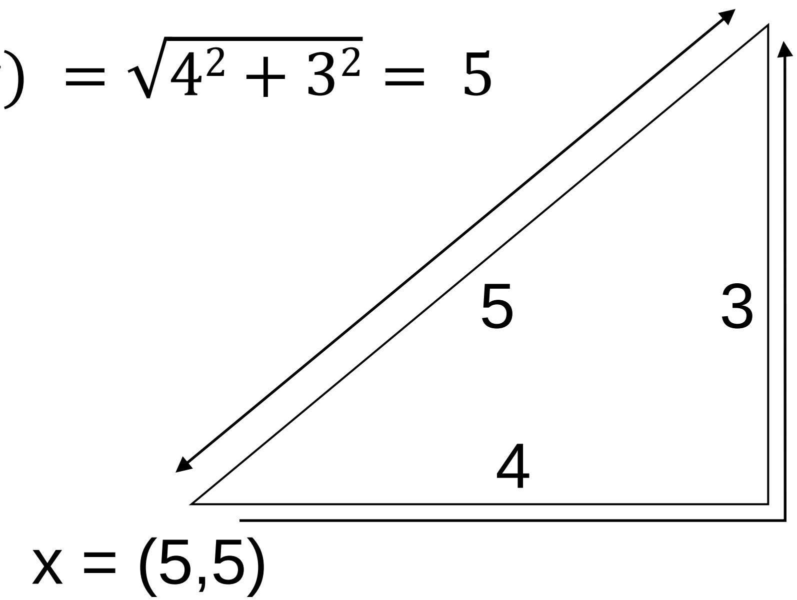
$$L_\infty(x, y) = \max\{|x_1 - y_1|, \dots, |x_d - y_d|\}$$

  - The limit of **L_p** as p goes to infinity.

Example of Distances

L$_2$-norm:

$$dist(x, y) = \sqrt{4^2 + 3^2} = 5$$



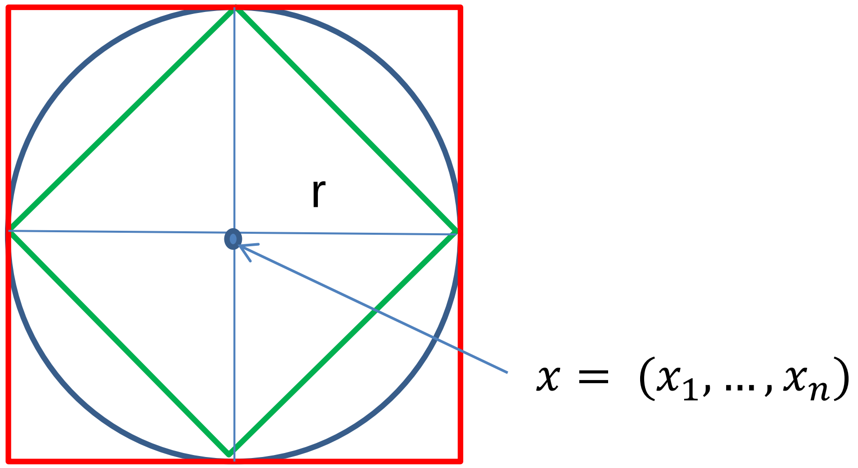
L$_1$-norm:

$$dist(x, y) = 4 + 3 = 7$$

L$_\infty$-norm:

$$dist(x, y) = \max\{3,4\} = 4$$

Example



Green: All points y at distance $L_1(x,y) = r$ from point x

Blue: All points y at distance $L_2(x,y) = r$ from point x

Red: All points y at distance $L_\infty(x,y) = r$ from point x

L_p distances for sets

- We can apply all the L_p distances to the cases of sets of attributes, with or without counts, if we represent the sets as vectors
  - E.g., a transaction is a 0/1 vector
  - E.g., a document is a vector of counts.

Similarities into distances

- Jaccard distance:

$$JDist(X,Y) = 1 - JSim(X,Y)$$

- Jaccard Distance is a metric

- Cosine distance:

$$Dist(X,Y) = 1 - \cos(X,Y)$$

- Cosine distance is a metric

Why Jaccard Distance Is a Distance Metric

- JDist(x,x) = 0
  - since JSim(x,x) = 1
- JDist(x,y) = JDist(y,x)
  - by symmetry of intersection
- JDist(x,y) $\geq$ 0
  - since intersection of X,Y cannot be bigger than the union.
- Triangle inequality:
  - Follows from the fact that JSim(X,Y) is the probability of randomly selected element from the union of X and Y to belong to the intersection

Hamming Distance

- Hamming distance is the number of positions in which bit-vectors differ.
  - Example: p_1 = 10101
    p_2 = 10011.
    - d(p_1, p_2) = 2 because the bit-vectors differ in the 3rd and 4th positions.
    - The L_1 norm for the binary vectors
- Hamming distance between two vectors of categorical attributes is the number of positions in which they differ.
  - Example: x = (married, low income, cheat),
    y = (single, low income, not cheat)
  - d(x,y) = 2

Why Hamming Distance Is a Distance Metric

- d(x,x) = 0 since no positions differ.
- d(x,y) = d(y,x) by symmetry of "different from."
- d(x,y) $\geq$ 0 since strings cannot differ in a negative number of positions.
- Triangle inequality: changing *x* to *z* and then to *y* is one way to change *x* to *y*.

- For binary vectors if follows from the fact that L_1 norm is a metric

Distance between strings

- How do we define similarity between strings?

weird	wierd
intelligent	unintelligent
Athena	Athina

- Important for recognizing and correcting typing errors and analyzing DNA sequences.

Edit Distance for strings

- The edit distance of two strings is the number of inserts and deletes of characters needed to turn one into the other.
- Example: x = abcde ; y = bcduve.
  - Turn *x* into *y* by deleting a, then inserting u and v after d.
  - Edit distance = 3.
- Minimum number of operations can be computed using dynamic programming
- Common distance measure for comparing DNA sequences

Why Edit Distance Is a Distance Metric

- d(x,x) = 0 because 0 edits suffice.
- d(x,y) = d(y,x) because insert/delete are inverses of each other.
- d(x,y) $\geq$ 0: no notion of negative edits.
- Triangle inequality: changing *x* to *z* and then to *y* is one way to change *x* to *y*. The minimum is no more than that

Variant Edit Distances

- Allow insert, delete, and mutate.
  - Change one character into another.
- Minimum number of inserts, deletes, and mutates also forms a distance measure.

- Same for any set of operations on strings.
  - Example: substring reversal or block transposition OK for DNA sequences
  - Example: character transposition is used for spelling

Distances between distributions

- We can view a document as a distribution over the words

document	Apple	Microsoft	Obama	Election
D1	0.35	0.5	0.1	0.05
D2	0.4	0.4	0.1	0.1
D2	0.05	0.05	0.6	0.3

- KL-divergence (Kullback-Leibler) for distributions P,Q

$$D_{KL}(P\|Q) = \sum_x p(x) \log \frac{p(x)}{q(x)}$$

- KL-divergence is asymmetric. We can make it symmetric by taking the average of both sides
- JS-divergence (Jensen-Shannon)

$$JS(P,Q) = \frac{1}{2} D_{KL}(P\|Q) + \frac{1}{2} D_{KL}(Q\|P)$$

SKETCHING AND LOCALITY SENSITIVE HASHING

Thanks to:

Rajaraman and Ullman, “Mining Massive Datasets”

Evimaria Terzi, slides for Data Mining Course.

Why is similarity important?

- We saw many definitions of similarity and distance
- How do we make use of similarity in practice?
- What issues do we have to deal with?

An important problem

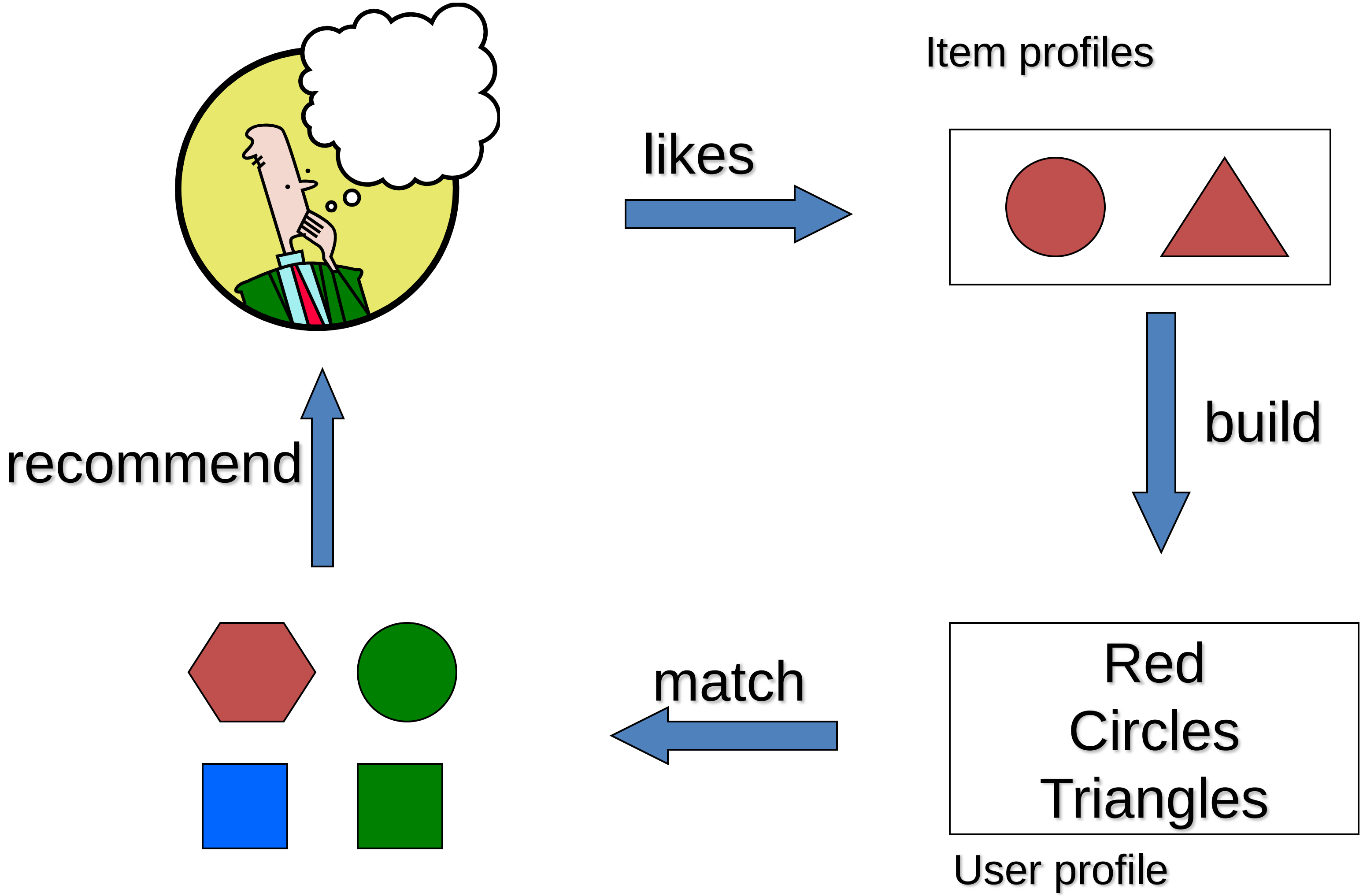
- Recommendation systems
  - When a user buys an item (initially books) we want to recommend other items that the user may like
  - When a user rates a movie, we want to recommend movies that the user may like
  - When a user likes a song, we want to recommend other songs that they may like

- A big success of data mining
- Exploits the long tail

Recommendation Systems

- Content-based:
  - Represent the items into a feature space and recommend items to customer C similar to previous items rated highly by C
    - Movie recommendations: recommend movies with same actor(s), director, genre, …
    - Websites, blogs, news: recommend other sites with “similar” content

Plan of action



Limitations of content-based approach

- Finding the appropriate features
  - e.g., images, movies, music
- Overspecialization
  - Never recommends items outside user’s content profile
  - People might have multiple interests
- Recommendations for new users
  - How to build a profile?

Recommendation Systems (II)

- Collaborative Filtering (user –user)
  - Consider user c
  - Find set D of other users whose ratings are "similar" to c's ratings
  - Estimate user's ratings based on ratings of users in D

Recommendation Systems (III)

- Collaborative filtering (item-item)
  - For item s, find other similar items
  - Estimate rating for item based on ratings for similar items
  - Can use same similarity metrics and prediction functions as in user-user model
- In practice, it has been observed that item-item often works better than user-user

Pros and cons of collaborative filtering

- Works for any kind of item
  - No feature selection needed
- New user problem
- New item problem
- Sparsity of rating matrix
  - Cluster-based smoothing?

Another important problem

- Find duplicate and near-duplicate documents from a web crawl.
- Why is it important:
  - Identify mirrored web pages, and avoid indexing them, or serving them multiple times
  - Find replicated news stories and cluster them under a single story.
  - Identify plagiarism
- What if we wanted exact duplicates?

Finding similar items

- Both the problems we described have a common component
  - We need a quick way to find highly similar items to a query item
  - OR, we need a method for finding all pairs of items that are highly similar.
- Also known as the Nearest Neighbor problem, or the All Nearest Neighbors problem

- We will examine it for the case of near-duplicate web documents.

Main issues

- What is the right representation of the document when we check for similarity?
  - E.g., representing a document as a set of characters will not do (why?)
- When we have billions of documents, keeping the full text in memory is not an option.
  - We need to find a shorter representation
- How do we do pairwise comparisons of billions of documents?
  - If exact match was the issue it would be ok, can we replicate this idea?

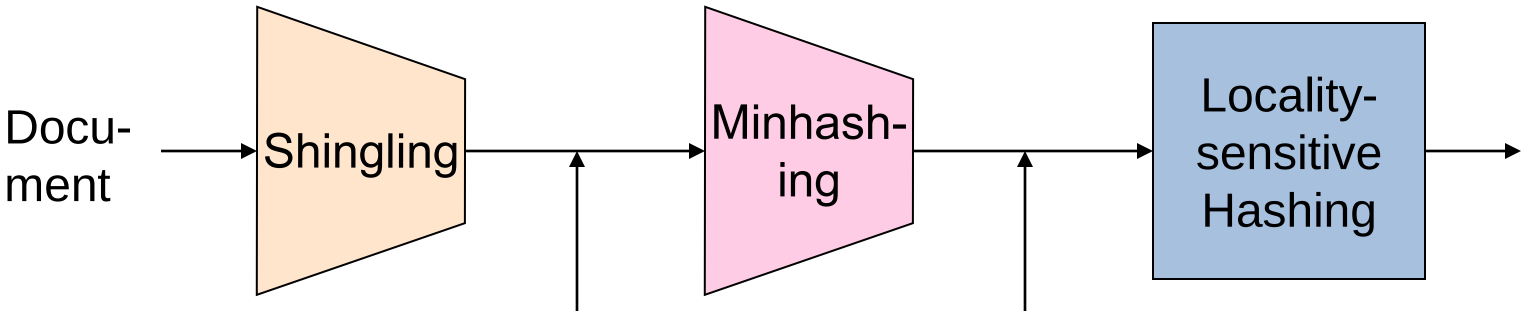
Three Essential Techniques for Similar Documents

1. Shingling : convert documents, emails, etc., to sets.

2. Minhashing : convert large sets to short signatures, while preserving similarity.

3. Locality-Sensitive Hashing (LSH): focus on pairs of signatures likely to be similar.

The Big Picture



The set of strings of length *k* that appear in the doc-ument

Signatures : short integer vectors that represent the sets, and reflect their similarity

Candidate pairs : those pairs of signatures that we need to test for similarity.

Shingles

- A k -shingle (or k -gram) for a document is a sequence of k characters that appears in the document.
- Example: document = abcab. k=2
  - Set of 2-shingles = {ab, bc, ca}.
  - Option: regard shingles as a bag, and count ab twice.

- Represent a document by its set of k-shingles.

Shingling

- Shingle: a sequence of k contiguous characters

```
a rose is a rose is a rose
a rose is 
 rose is a
  rose is a 
   ose is a r
    se is a ro
     e is a ros
       is a rose
       is a rose 
        s a rose i
          a rose is
          a rose is 
```

Working Assumption

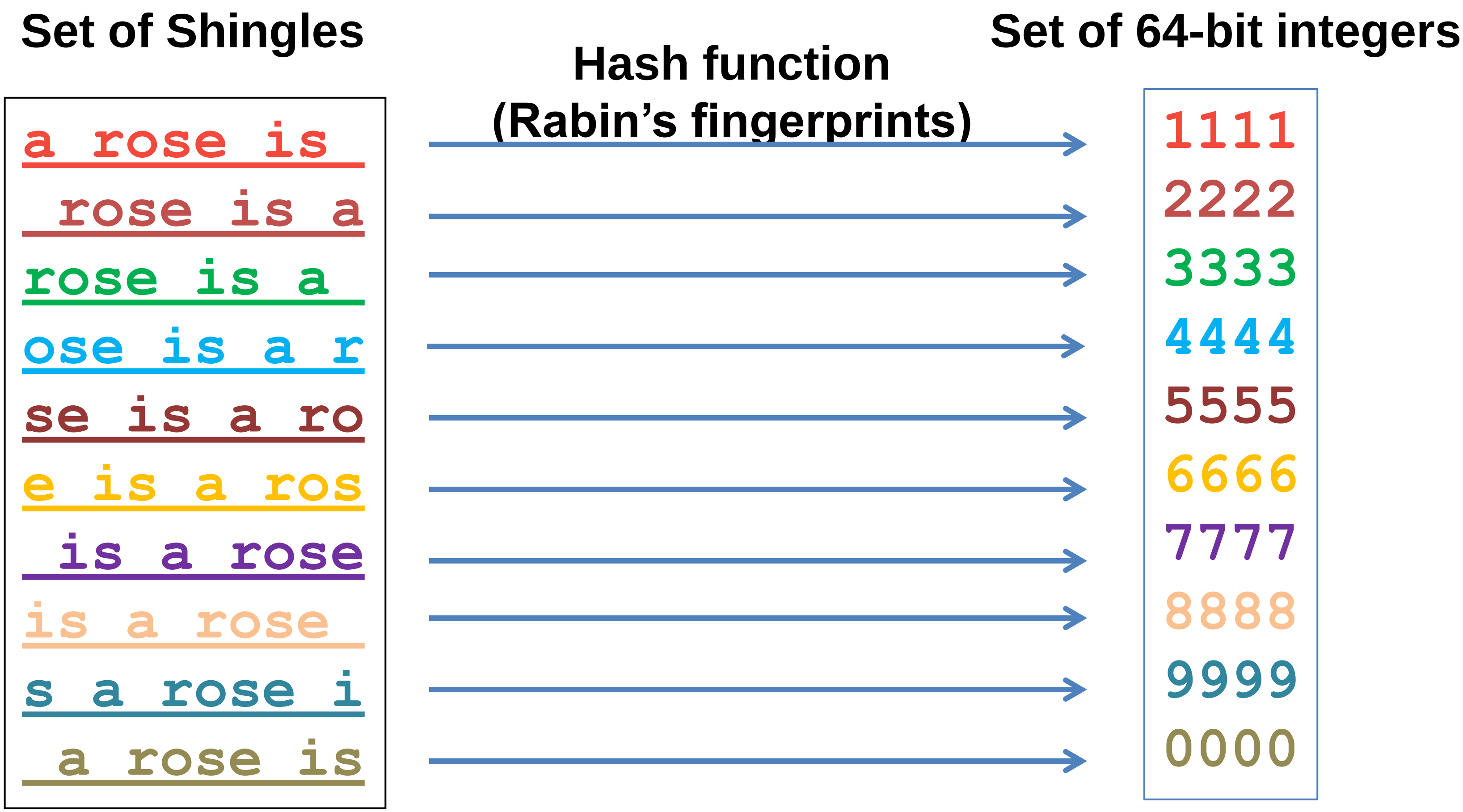
- Documents that have lots of shingles in common have similar text, even if the text appears in different order.
- Careful: you must pick k large enough, or most documents will have most shingles.
  - Extreme case $k = 1$: all documents are the same
  - $k = 5$ is OK for short documents; $k = 10$ is better for long documents.
- Alternative ways to define shingles:
  - Use words instead of characters
  - Anchor on stop words (to avoid templates)

Shingles: Compression Option

- To compress long shingles, we can hash them to (say) 4 bytes.
- Represent a doc by the set of hash values of its *k*-shingles.
- From now on we will assume that shingles are integers
  - Collisions are possible, but very rare

Fingerprinting

- Hash shingles to 64-bit integers



Basic Data Model: Sets

- Document: A document is represented as a set shingles (more accurately, hashes of shingles)

- Document similarity: Jaccard similarity of the sets of shingles.
  - Common shingles over the union of shingles
  - *Sim* $(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$.

- Although we use the documents as our driving example the techniques we will describe apply to any kind of sets.
  - E.g., similar customers or items.

Signatures

- Problem: shingle sets are too large to be kept in memory.
- Key idea: "hash" each set S to a small signature Sig (S), such that:

1. Sig (S) is small enough that we can fit a signature in main memory for each set.
2. Sim (S_1, S_2) is (almost) the same as the "similarity" of Sig (S_1) and Sig (S_2). (signature preserves similarity).

- Warning: This method can produce false negatives, and false positives (if an additional check is not made).
  - False negatives: Similar items deemed as non-similar
  - False positives: Non-similar items deemed as similar

From Sets to Boolean Matrices

- Represent the data as a boolean matrix M
  - Rows = the universe of all possible set elements
    - In our case, shingle fingerprints take values in $[0\ldots2^{64}-1]$
  - Columns = the sets
    - In our case, documents, sets of shingle fingerprints
  - M(r,S) = 1 in row r and column S if and only if r is a member of S.
- Typical matrix is sparse.
  - We do not really materialize the matrix

Example

- Universe: **U = {A,B,C,D,E,F,G}**

- X = {A,B,F,G}
- Y = {A,E,F,G}

- Sim(X,Y) = $\frac{3}{5}$

	X	Y
A	1	1
B	1	0
C	0	0
D	0	0
E	0	1
F	1	1
G	1	1

Example

- Universire: **U = {A,B,C,D,E,F,G}**

- X = {A,B,F,G}
- Y = {A,E,F,G}

- Sim(X,Y) = $\frac{3}{5}$

At least one of the columns has value 1

	X	Y
A	1	1
B	1	0
C	0	0
D	0	0
E	0	1
F	1	1
G	1	1

Example

- Universire: **U = {A,B,C,D,E,F,G}**

- X = {A,B,F,G}
- Y = {A,E,F,G}

- Sim(X,Y) = $\frac{3}{5}$

Both columns have value 1

	X	Y
A	1	1
B	1	0
C	0	0
D	0	0
E	0	1
F	1	1
G	1	1

Minhashing

- Pick a random permutation of the rows (the universe U).
- Define "hash" function for set S
  - h(S) = the index of the first row (in the permuted order) in which column S has 1.
  - OR
  - h(S) = the index of the first element of S in the permuted order.
- Use k (e.g., k = 100) independent random permutations to create a signature.

Example of minhash signatures

- Input matrix

	S_1	S_2	S_3	S_4
A	1	0	1	0
B	1	0	0	1
C	0	1	0	1
D	0	1	0	1
E	0	1	0	1
F	1	0	1	0
G	1	0	1	0

→

A
C
G
F
B
E
D

→

		S_1	S_2	S_3	S_4
1	**A**	1	0	1	0
2	**C**	0	1	0	1
3	**G**	1	0	1	0
4	**F**	1	0	1	0
5	**B**	1	0	0	1
6	**E**	0	1	0	1
7	**D**	0	1	0	1
		1	**2**	**1**	**2**

Example of minhash signatures

- Input matrix

	S_1	S_2	S_3	S_4
A	1	0	1	0
B	1	0	0	1
C	0	1	0	1
D	0	1	0	1
E	0	1	0	1
F	1	0	1	0
G	1	0	1	0

D
B
A
C
F
G
E

		S_1	S_2	S_3	S_4
1	**D**	0	1	0	1
2	**B**	1	0	0	1
3	**A**	1	0	1	0
4	**C**	0	1	0	1
5	**F**	1	0	1	0
6	**G**	1	0	1	0
7	**E**	0	1	0	1
		2	**1**	**3**	**1**

Example of minhash signatures

- Input matrix

	S_1	S_2	S_3	S_4
A	1	0	1	0
B	1	0	0	1
C	0	1	0	1
D	0	1	0	1
E	0	1	0	1
F	1	0	1	0
G	1	0	1	0

C
D
G
F
A
B
E

		S_1	S_2	S_3	S_4
1	**C**	0	1	0	1
2	**D**	0	1	0	1
3	**G**	1	0	1	0
4	**F**	1	0	1	0
5	**A**	1	0	1	0
6	**B**	1	0	0	1
7	**E**	0	1	0	1
		3	**1**	**3**	**1**

Example of minhash signatures

- Input matrix

	S_1	S_2	S_3	S_4
A	1	0	1	0
B	1	0	0	1
C	0	1	0	1
D	0	1	0	1
E	0	1	0	1
F	1	0	1	0
G	1	0	1	0

≈

Signature matrix

	S_1	S_2	S_3	S_4
h_1	1	2	1	2
h_2	2	1	3	1
h_3	3	1	3	1

- Sig(S) = vector of hash values
  - e.g., $Sig(S_2) = [2,1,1]$
- Sig(S,i) = value of the i-th hash function for set S
  - E.g., $Sig(S_2,3) = 1$

Hash function Property

$$\Pr(h(S_1) = h(S_2)) = Sim(S_1,S_2)$$

- where the probability is over all choices of permutations.
- Why?
  - The first row where one of the two sets has value 1 belongs to the union.
    - Recall that union contains rows with at least one 1.
  - We have equality if both sets have value 1, and this row belongs to the intersection

Example

- Universe: **U = {A,B,C,D,E,F,G}**
- X = {A,B,F,G}
- Y = {A,E,F,G}

Rows C,D could be anywhere
they do not affect the probability

- Union =
  {A,B,E,F,G}
- Intersection =
  {A,F,G}

	X	Y
A	1	1
B	1	0
C	0	0
D	0	0
E	0	1
F	1	1
G	1	1

D
*
*
C
*
*
*

	X	Y
D	0	0
C	0	0

Example

- Universe: **U = {A,B,C,D,E,F,G}**
- X = {A,B,F,G}
- Y = {A,E,F,G}

The * rows belong to the union

- Union =

  {A,B,E,F,G}
- Intersection =

  {A,F,G}

	X	Y
A	1	1
B	1	0
C	0	0
D	0	0
E	0	1
F	1	1
G	1	1

D
*
*
C
*
*
*

	X	Y
D	0	0
C	0	0

Example

- Universire: **U = {A,B,C,D,E,F,G}**
- X = {A,B,F,G}
- Y = {A,E,F,G}

The question is what is the value of the **first *** element

- Union =
  {A,B,E,F,G}
- Intersection =
  {A,F,G}

	X	Y
A	1	1
B	1	0
C	0	0
D	0	0
E	0	1
F	1	1
G	1	1

D
*
*
C
*
*
*

	X	Y
D	0	0
C	0	0

Example

- Universire: **U = {A,B,C,D,E,F,G}**
- X = {A,B,F,G}
- Y = {A,E,F,G}

If it belongs to the intersection then h(X) = h(Y)

- Union =

  {A,B,E,F,G}

- Intersection =

  {A,F,G}

	X	Y
A	1	1
B	1	0
C	0	0
D	0	0
E	0	1
F	1	1
G	1	1

D
*
*
C
*
*
*

	X	Y
D	0	0
C	0	0

Example

- Universire: **U = {A,B,C,D,E,F,G}**
- X = {A,B,F,G}
- Y = {A,E,F,G}

Every element of the union is equally likely to be the * element

$$\Pr(h(X) = h(Y)) = \frac{|\{A,F,G\}|}{|\{A,B,E,F,G\}|} = \frac{3}{5} = \mathrm{Sim}(X,Y)$$

- Union = {A,B,E,F,G}
- Intersection = {A,F,G}

	X	Y
A	1	1
B	1	0
C	0	0
D	0	0
E	0	1
F	1	1
G	1	1

D
*
*
C
*
*
*

	X	Y
D	0	0
C	0	0

Similarity for Signatures

- The similarity of signatures is the fraction of the hash functions in which they agree.

	S_1	S_2	S_3	S_4
A	1	0	1	0
B	1	0	0	1
C	0	1	0	1
D	0	1	0	1
E	0	1	0	1
F	1	0	1	0
G	1	0	1	0

≈

Signature matrix

S_1	S_2	S_3	S_4
1	2	1	2
2	1	3	1
3	1	3	1

	Actual	Sig
(S_1, S_2)	0	0
(S_1, S_3)	3/5	2/3
(S_1, S_4)	1/7	0
(S_2, S_3)	0	0
(S_2, S_4)	3/4	1
(S_3, S_4)	0	0

Zero similarity is preserved
High similarity is well approximated

- With multiple signatures we get a good approximation

Is it now feasible?

- Assume a billion rows
- Hard to pick a random permutation of 1…billion
- **Even representing a random permutation requires 1 billion entries!!!**
- How about accessing rows in permuted order?
- ☹

Being more practical

Approximating row permutations: pick k=100 hash functions $(h_1,\ldots,h_k)$

In practice this means selecting the function parameters

for each row r

In practice only the rows (shingles) that appear in the data

for each hash function h_i

compute h_i (r)

h_i (r) = index of shingle r in permutation

for each column S that has 1 in row r

S contains shingle r

if h_i (r) is a smaller value than Sig(S,i) **then**

Sig(S,i) = h_i (r);

Find the shingle r with minimum index

Sig(S,i) will become the smallest value of h_i(r) among all rows (shingles) for which column S has value 1 (shingle belongs in S); *i*.e., h_i (r) gives the min index for the i-th permutation

Example

x	Row	S1	S2
0	A	1	0
1	B	0	1
2	C	1	1
3	D	1	0
4	E	0	1

$h(x) = x+1 \bmod 5$
$g(x) = 2x+3 \bmod 5$

h(x)	Row	S1	S2
1	E	0	1
2	A	1	0
3	B	0	1
4	C	1	1
0	D	1	0

g(x)	Row	S1	S2
3	B	0	1
0	E	0	1
2	C	1	0
4	A	1	1
1	D	1	0

	Sig1	Sig2
$h(0) = 1$	1	-
$g(0) = 3$	3	-
$h(1) = 2$	1	2
$g(1) = 0$	3	0
$h(2) = 3$	1	2
$g(2) = 2$	2	0
$h(3) = 4$	1	2
$g(3) = 4$	2	0
$h(4) = 0$	1	0
$g(4) = 1$	2	0

Implementation – (4)

- Often, data is given by column, not row.
  - E.g., columns = documents, rows = shingles.
- If so, sort matrix once so it is by row.
- And always compute $h_i(r)$ only once for each row.