Algebraic Closure in Pseudofinite Fields

Özlem Beyarslan
Boğaziçi University

Abstract

A pseudofinite field is perfect pseudo-algebraically closed (PAC) field which has $\hat{\mathbb{Z}}$, the profinite cyclic group, as absolute Galois group. Pseudofinite fields exist and they can be realized for example as ultraproducts of finite fields. A tournament on a set X is an irreflexive binary relation $R \subset X \times X$ such that for every $x \neq y$ in X exactly one of $R(x, y)$ and $R(y, x)$ holds. A pseudofinite field F interprets a tournament by the formula $\exists z : (x - y) = z^2$. The automorphism group of any field interpreting a 0-definable tournament can not have any involutions. We generalize this simple observation to get the following result: For almost all completions of the theory of pseudofinite fields, we show that over a substructure A, algebraic closure agrees with definable closure, as soon as A contains the relative algebraic closure of the prime field. This is joint work with Ehud Hrushovski.