

The existence property among set theories

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Abstract

Delineating the complexity of witnesses for existential statements is a crucial activity in definability theory. In classical descriptive set theory this is manifested by the role of uniformization theorems. Theories with underlying intuitionistic logic lend itself to extracting additional information from proofs. A hallmark of many an intuitionistic theory is the existence property, EP, i.e., if the theory proves an existential statement then there is a provably definable witness for it. However, there are well known exceptions. It was proved by H. Friedman and A. Scedrov that full intuitionistic Zermelo-Fraenkel set theory, IZF, formulated with Collection does not have the EP. As the corresponding theory formulated with Replacement has the EP, one might be tempted to put the blame for the failure of the existence properties squarely on Collection. However, recently it was shown that several well known intuitionistic set theories with Collection have the existence property. Among these theories are constructive Zermelo-Fraenkel set theory, CZF formulated with Exponentiation and also CZF augmented by the Power Set axiom (basically IZF with only Bounded Separation). The talk will present the proof-technology for the latter results and also expound on the failure of EP for IZF.