

# MEMPSODE: Comparing Particle Swarm Optimization and Differential Evolution on a Hybrid Memetic Global Optimization Framework

Draft version \*

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## ABSTRACT

In this paper we present an experimental comparison between two well known population-based schemes, namely Particle Swarm Optimization (PSO) and Differential Evolution (DE), that are incorporated in a memetic global optimization framework. We use the recently published MEMPSODE software [16], that implements the memetic global optimization first described in [13] and incorporates Merlin optimization environment [10]. Since the original description of the algorithm in [13] involved only a PSO variant for the exploration phase, using MEMPSODE software we attempt an empirical assessment of the DE. The results based on the noiseless testbed, indicate that the usage of DE may lead to superior performance.

## Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, memetic algorithms, particle swarm optimization, differential evolution, local search*; G.4 [Mathematical Software]

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## Keywords

Memetic global optimization, Particle swarm optimization, Differential evolution, Benchmarking, Black-box optimization, Merlin optimization environment

## 1. INTRODUCTION

Evolutionary Algorithms (EAs) and Swarm Intelligence (SI) approaches have been established as powerful optimization tools for solving optimization problems [2, 8, 12]. They are based on models that draw their inspiration from physical systems. Based on natural selection and evolution schemes these algorithms exhibit remarkable capability of locating global solutions for optimization problems.

The rise of EA and SI algorithms has sparked the development of a closely related category, namely the *Memetic Algorithms* (MAs). MAs constitute a class of hybrid metaheuristics that combine population-based optimization algorithms with local search procedures [9]. The rationale behind their development was the necessity for powerful algorithms where the global exploration capability of EAs and SI approaches would be complemented with the efficiency and accuracy of classical local optimization techniques.

In this work we examine the performance of a memetic optimization software that incorporates Particle Swarm Optimization (PSO) or Differential Evolution (DE) schemes for exploration in addition to powerful local optimization algorithms for exploitation. Our primary target is to determine the impact of the choice between UPSO and DE in the algorithmic framework presented in [13]. By no means we are attempting an extensive benchmark of all MEMPSODE user defined parameters. Instead we use a default set of parameters both on PSO and on DE and the same local search procedure.

## 2. ALGORITHM PRESENTATION

The tested software follows closely the PSO-based memetic approaches reported in [13] and extends them also to the DE

framework. More specifically, the Unified PSO (UPSO) approach [11], which harnesses the strengths of standard local and global PSO variants is implemented. In both cases, direct calls to LS procedures are facilitated via the established *Merlin* optimization environment [10], providing the ability to develop a variety of MAs. In order to present the algorithm implemented by MEMSPODE we provide summary for all key algorithms incorporated. A pseudo-code summarising the methodology is presented in Algorithm 1.

## 2.1 Unified Particle Swarm Optimization

PSO was introduced by Eberhart and Kennedy [1]. The main concept of the method includes a population, also called *swarm*, of search points, also called *particles*, searching for optimal solutions within the search space, simultaneously. The particles move in the search space by assuming an adaptable position shift, called *velocity*, at each iteration.

Moreover, each particle retains in a memory the best position it has ever visited, i.e., the position with the lowest function value. To each particle is assigned a *neighborhood*, which determines the indices of its mates that will share its experience. Obviously, the neighborhood scheme affects the flow of information among the particles. Two well known neighborhood scheme have been used extensively. The local *lbest* scheme were each particle is assumed to communicate only with its mates with adjacent indices and the global *gbest* scheme were any new information (best position) is immediately communicated to every single particle in each iteration.

Putting our description in a mathematical framework, let us assume the  $n$ -dimensional continuous optimization problem:

$$\min_{x \in X \subset \mathbb{R}^n} f(x), \quad (1)$$

where the search space  $X$  is an orthogonal hyperbox in  $\mathbb{R}^n$ :

$$X \equiv [l_1, r_1] \times [l_2, r_2] \times \cdots \times [l_n, r_n].$$

A swarm of  $N$  particles is a set of search points:

$$S = \{x_1, x_2, \dots, x_N\},$$

where the  $i$ -th particle is defined as:

$$x_i = (x_{i1}, x_{i2}, \dots, x_{in})^\top \in X, \quad i = 1, 2, \dots, N.$$

The velocity (position shift) of  $x_i$  is denoted as:

$$v_i = (v_{i1}, v_{i2}, \dots, v_{in})^\top, \quad i = 1, 2, \dots, N,$$

and its best position as:

$$p_i = (p_{i1}, p_{i2}, \dots, p_{in})^\top \in X, \quad i = 1, 2, \dots, N.$$

Let  $g_i = \arg \min_{j \in \mathcal{N}_i} f(p_j)$ , and  $t$  denote the algorithm's iteration counter.

The classical PSO model can be generalized in the UPSO scheme [11]. Following this scheme and if we assume that  $G_i^{(t+1)}$  and  $L_i^{(t+1)}$  denote the velocity update of  $x_i$  in the gbest and lbest PSO model, respectively:

$$\begin{aligned} G_{ij}^{(t+1)} &= \chi \left[ v_{ij}^{(t)} + c_1 r_1 \left( p_{ij}^{(t)} - x_{ij}^{(t)} \right) + c_2 r_2 \left( p_{gj}^{(t)} - x_{ij}^{(t)} \right) \right] \textcircled{2} \\ L_{ij}^{(t+1)} &= \chi \left[ v_{ij}^{(t)} + c_1 r_1 \left( p_{ij}^{(t)} - x_{ij}^{(t)} \right) + c_2 r_2 \left( p_{gij}^{(t)} - x_{ij}^{(t)} \right) \right] \textcircled{3} \end{aligned}$$

where  $g$  is the index of the overall best particle, i.e.:

$$g = \arg \min_{j=1, \dots, N} f(p_j),$$

then the particle is updated as follows [11]:

$$U_{ij}^{(t+1)} = u G_{ij}^{(t+1)} + (1 - u) L_{ij}^{(t+1)}, \quad (4)$$

$$\begin{aligned} x_{ij}^{(t+1)} &= x_{ij}^{(t)} + U_{ij}^{(t+1)}, \quad (5) \\ i &= 1, 2, \dots, N, \quad j = 1, 2, \dots, n. \end{aligned}$$

The parameter  $u \in [0, 1]$  is called the *unification factor* and it balances the influence (trade-off) of the global and local velocity update. Obviously, the lbest PSO model is retrieved for  $u = 0$ , while for  $u = 1$  the gbest PSO model is obtained. All intermediate values produce combinations with diverse convergence properties.

## 2.2 Differential Evolution

The Differential Evolution (DE) algorithm was introduced by Storn and Price [14] as a population-based stochastic optimization algorithm for numerical optimization problems. DE is formulated similarly to PSO. A population:

$$P = \{x_1, x_2, \dots, x_N\},$$

of  $N$  individuals is utilized to probe the search space,  $X \subset \mathbb{R}^n$ . The population is randomly initialized, usually following a uniform distribution within the search space.

Each individual is an  $n$ -dimensional vector:

$$x_i = (x_{i1}, x_{i2}, \dots, x_{in})^\top \in X, \quad i = 1, 2, \dots, N,$$

servicing as a candidate solution of the problem at hand. The population is iteratively evolved by applying two operators, *mutation* and *recombination*, on each individual to produce new candidate solutions. Then, the new and the old individuals are merged and selection takes place to construct the new population consisting of the  $N$  best individuals. The procedure continues in the same manner until a termination criterion is satisfied.

The mutation operator produces a new vector,  $v_i$ , for each individual,  $x_i$ ,  $i = 1, 2, \dots, N$ , by combining some of the rest individuals of the population. There most common (but not the only) operator proposed to accomplish this task:

$$\text{OP1: } v_i^{(t+1)} = x_g^{(t)} + F \left( x_{r_1}^{(t)} - x_{r_2}^{(t)} \right), \quad (6)$$

where  $t$  denotes the iteration counter;  $F \in (0, 1]$  is a fixed user-defined parameter;  $g$  denotes the index of the best individual in the population, i.e., the one with the lowest function value; and  $r_j \in \{1, 2, \dots, N\}$ ,  $j = 1, 2, \dots, 5$ , are mutually different randomly selected indices that differ also from the index  $i$ . Thus, in order to be able to apply all mutation operators, it must hold that  $N > 5$ .

After the mutation, a recombination operator is applied producing a trial vector:

$$u_i = (u_{i1}, u_{i2}, \dots, u_{in}), \quad i = 1, 2, \dots, N,$$

for each individual. This vector is defined as follows:

$$u_{ij}^{(t+1)} = \begin{cases} v_{ij}^{(t+1)}, & \text{if } R_j \leq CR \text{ or } j = \text{RI}(i), \\ x_{ij}^{(t)}, & \text{if } R_j > CR \text{ and } j \neq \text{RI}(i), \end{cases} \quad (7)$$

where  $j = 1, 2, \dots, n$ ;  $R_j$  is a random variable uniformly distributed in the range  $[0, 1]$ ;  $CR \in [0, 1]$  is a user-defined crossover constant; and  $\text{RI}(i) \in \{1, 2, \dots, n\}$ , is a randomly selected index.

Finally, each trial vector is compared against the corresponding individual and the best between them comprise

the new individual in the next generation, i.e.:

$$x_i^{(t+1)} = \begin{cases} u_i^{(t+1)}, & \text{if } f(u_i^{(t+1)}) < f(x_i^{(t)}), \\ x_i^{(t)}, & \text{otherwise.} \end{cases} \quad (8)$$

### 2.3 Local search

A local solution to an optimization problem can be obtained by applying local optimization methods. The hybrid schemes implemented in MEMPSODE have a need for deterministic local search procedures that require a starting point,  $x_0$ , and generate a sequence of points,  $\{x_k\}_{k=0}^{\infty}$ , in order to determine a minimizer within a prescribed accuracy. The generation of a new point,  $x_{k+1}$ , in the sequence is based on information collected for the current iterate,  $x_k$ . Typically, this information includes the function value at  $x_k$ , as well as the first- and probably second-order derivatives of  $f(x)$  at  $x_k$ . In all cases, the aim is to find a new iterate with lower function value than the current one.

The *Merlin* optimization environment [10] is an efficient and robust general purpose optimization package. It is designed to solve multi-dimensional optimization problems. Merlin offers a variety of well established gradient-based and gradient-free optimization algorithms. Gradient-based algorithms include three methods from the conjugate gradient family, the method of Levenberg-Marquardt, the DFP and several variations of the BFGS algorithms (BFGS) [?]. The gradient-free algorithms include a pattern search and the nonlinear Simplex method.

### 2.4 Memetic Algorithm

The design of MPSO in [13] was based on three fundamental schemes, henceforth called the *memetic strategies*:

- Scheme 1:** LS is applied only on the overall best position,  $p_g$ , of the swarm.
- Scheme 2:** LS is applied on each locally best position,  $p_i$ ,  $i = 1, 2, \dots, N$ , with a prescribed fixed probability,  $\rho \in (0, 1]$ .
- Scheme 3:** LS is applied both on the best position,  $p_g$ , as well as on some randomly selected locally best positions,  $p_i$ ,  $i \in \{1, 2, \dots, N\}$ .

These schemes can be applied either at each iteration or whenever a specific number of consecutive iterations has been completed.

Of course, many other memetic strategies can be considered. For instance, a simple one would be the application of LS on every particle. However, such an approach would be costly in terms of function evaluations. In practice, only a small number of particles are considered as start points for LS, as pointed out in [6]. The memetic strategies proposed in [13] were also adopted in MEMPSODE for both PSO- and DE-based MAs.

## 3. EXPERIMENTAL PROCEDURE

We used the default restart mechanism provided by the testbed for a maximum number of  $100\,000 \times n$  function evaluations. The third memetic scheme was used with probability of local search set to  $p_i = 0.05$ . Both UPSO and DE used a swarm size  $N = 25$  particles. In UPSO the unification factor  $u$  was set to 1 and the initial velocity vector was restraint by a factor of 0.01. For the DE experiments we applied OP1 with default values  $F = 0.5$  and  $CR = 0.7$ .

Each local search has an upper limit of 4 000 function evaluations. Whenever derivatives were needed (eg. BFGS) we applied an  $O(h)$  finite differences formula where  $h$  is an adaptable step size (see [15]). For the local search we applied the BFGS method implemented in Merlin.

The experiments have been conducted on an Intel I7-2600 processor on 3.4 GHz with 8GB RAM.

## 4. RESULTS

Results from experiments according to [4] on the benchmark functions given in [3, 5] are presented in Figures 1, 2 and 3 and in Tables 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value,  $f_t = f_{\text{opt}} + \Delta f$ , and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach  $f_t$ , summed over all trials and divided by the number of trials that actually reached  $f_t$  [4].

A direct comparison between UPSO and DE variants of MEMPSODE memetic algorithm can be deduced by observing the scatter plots in figure 2 and the starred records in table 1. In the separable case DE variant outperforms UPSO especially as dimensionality increases. For the moderate category DE seems to outperform only on function 7 (step ellipsoid) and scores marginally better in all other cases. The same behaviour is repeated for the ill-conditioned cases where DE variant is slightly better. Finally, DE variant outperforms PSO variant in all multimodal functions but PSO variant seems to behave better for the weak structured cases.

DE variant superiority is also obvious by inspecting figure 3. In almost all cases (except the weak structured functions) the ECDF of DE variant lies higher than the corresponding ECDF of PSO variant and this pattern is repeated in all levels of accuracy.

It is also worth mentioning that the DE variant scored the best recorded ETF for some accuracy levels in the case of ill condition functions (10-14). From the corresponding lines of table 1 we can see that for relatively low levels of accuracy the achieved ERT scores are quite competitive.

As a general remark, MEMPSODE seems a very promising and competitive new algorithm that incorporates state of the art schemes of swarm intelligence algorithms with the robust and versatile Merlin optimization environment.

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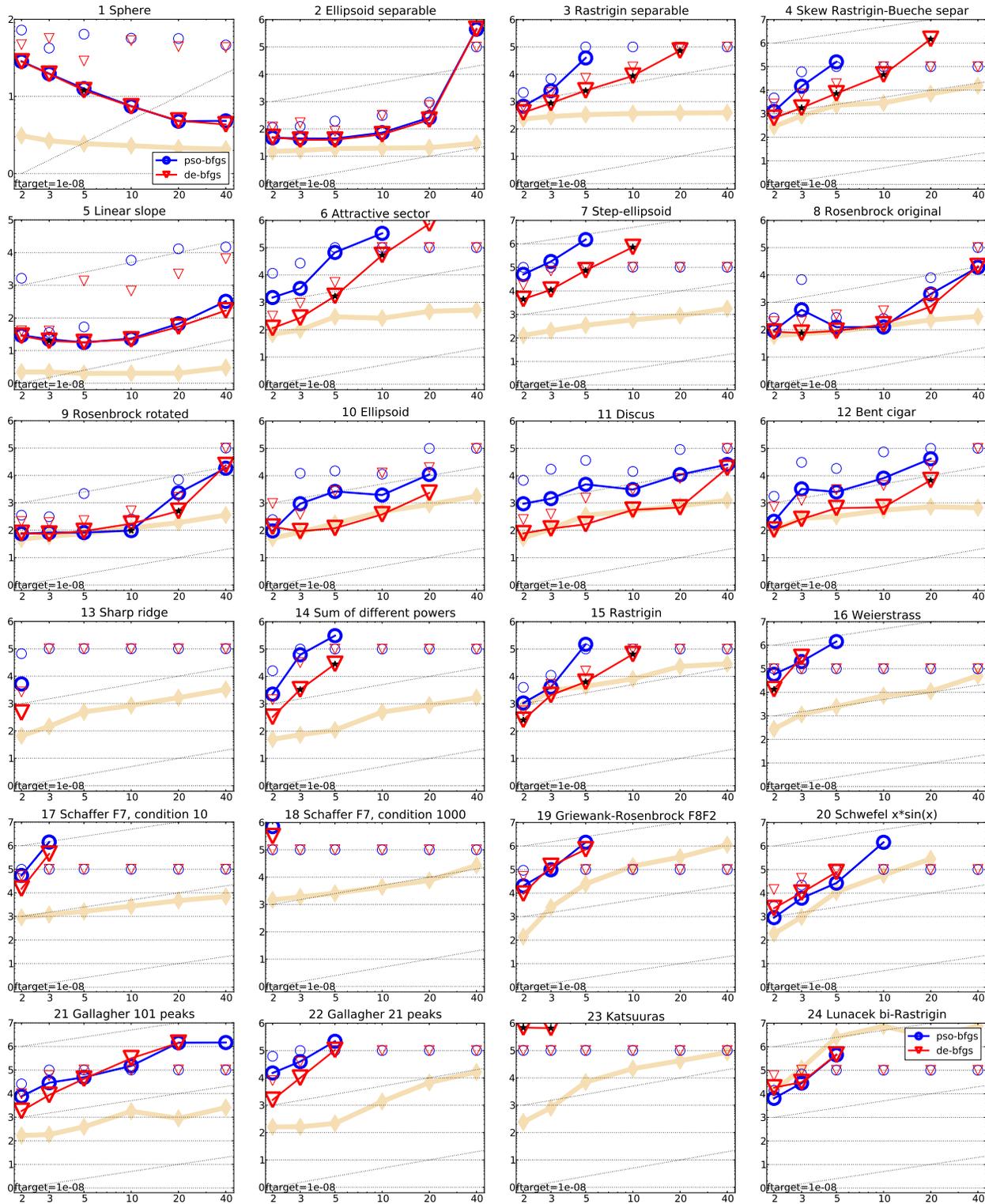


Figure 1: Expected running time (ERT in number of  $f$ -evaluations) divided by dimension for target function value  $10^{-8}$  as  $\log_{10}$  values versus dimension. Different symbols correspond to different algorithms given in the legend of  $f_1$  and  $f_{24}$ . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Horizontal lines give linear scaling, slanted dotted lines give quadratic scaling. Black stars indicate statistically better result compared to all other algorithms with  $p < 0.01$  and Bonferroni correction number of dimensions (six). Legend:  $\circ$ : pso-bfgs,  $\nabla$ : de-bfgs.

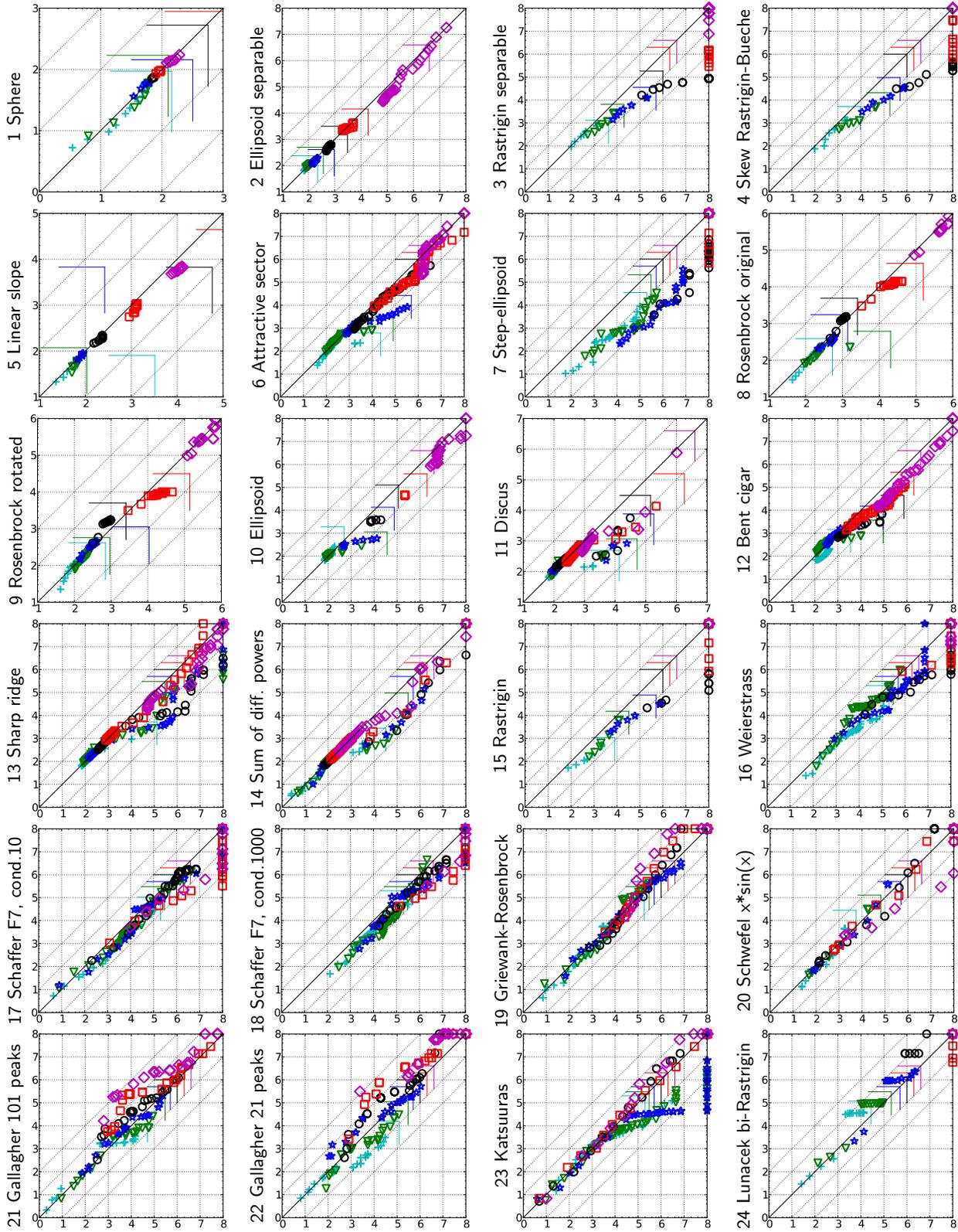


Figure 2: Expected running time (ERT in  $\log_{10}$  of number of function evaluations) of pso-bfgs ( $x$ -axis) versus de-bfgs ( $y$ -axis) for 46 target values  $\Delta f \in [10^{-8}, 10]$  in each dimension on functions  $f_1$ – $f_{24}$ . Markers on the upper or right edge indicate that the target value was never reached. Markers represent dimension: 2: +, 3:  $\nabla$ , 5: \*, 10:  $\circ$ , 20:  $\square$ , 40:  $\diamond$ .

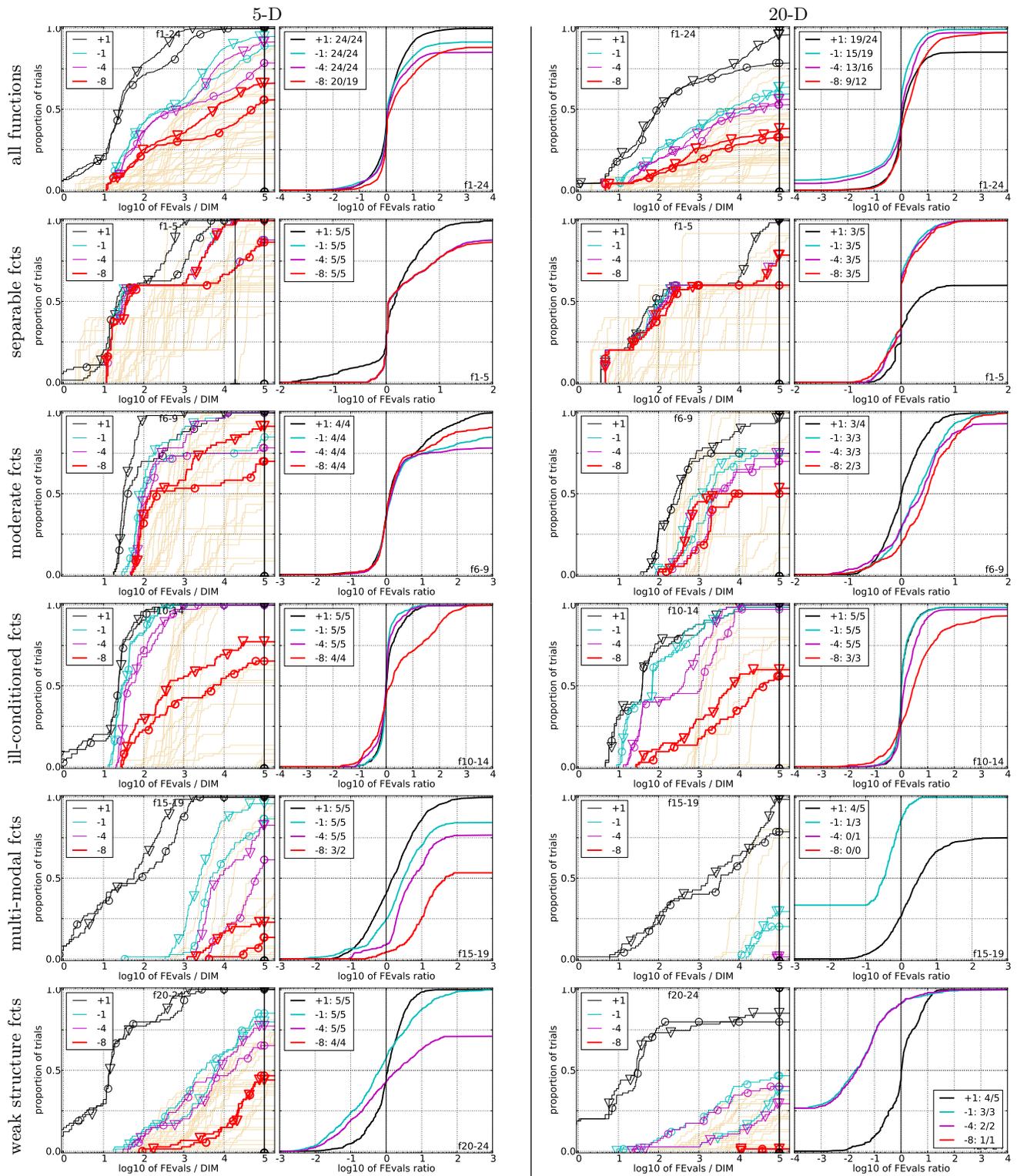


Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension  $D$  (FEvals/ $D$ ) to reach a target value  $f_{\text{opt}} + \Delta f$  with  $\Delta f = 10^k$ , where  $k \in \{1, -1, -4, -8\}$  is given by the first value in the legend, for pso-bfgs ( $\circ$ ) and de-bfgs ( $\nabla$ ). Light beige lines show the ECDF of FEvals for target value  $\Delta f = 10^{-8}$  of all algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of pso-bfgs divided by de-bfgs, all trial pairs for each function. Pairs where one trial failed are visible in the limits being  $> 0$  or  $< 1$ . The legends indicate the number of functions that were solved in at least one trial (pso-bfgs first).

## 5-D

## 20-D

$\Delta f$	1e+1	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f$	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
<b>f<sub>1</sub></b>	11	12	12	12	12	15/15	<b>f<sub>1</sub></b>	43	43	43	43	43	15/15
1: pso	3.1(3)	5.2(0.2)	5.2(0.2)	5.2(0.2)	5.2(0.2)	15/15	1: pso	1.9(0.2)	2.2	2.2	2.2	2.2	15/15
2: de <sub>-</sub>	3.3(2)	4.9(0.2)	4.9(0.2)	4.9(0.2)	4.9(0.2)	15/15	2: de <sub>-</sub>	2.0(0.2)	2.2	2.2	2.2	2.2	15/15
<b>f<sub>2</sub></b>	83	88	90	92	94	15/15	<b>f<sub>2</sub></b>	385	387	390	391	393	15/15
1: pso	1.7(0.7)	1.8(0.7)	1.9(0.7)	2.1(0.7)	2.3(0.8)	15/15	1: pso	5.2(3)	5.6(3)	6.7(5)	7.6(5)	12(12)	15/15
2: de <sub>-</sub>	1.4(0.3)	1.5(0.3)	1.7(0.3)	1.9(0.3)	2.0(0.3)	15/15	2: de <sub>-</sub>	5.7(4)	6.2(4)	6.5(4)	7.1(5)	10(5)	15/15
<b>f<sub>3</sub></b>	716	1637	1646	1650	1654	15/15	<b>f<sub>3</sub></b>	5066	7635	7643	7646	7651	15/15
1: pso	9.4(10)	121(155)	120(159)	120(158)	120(154)	13/15	1: pso	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
2: de <sub>-</sub>	<b>2.0(1)*</b>	<b>7.7(5)*<sup>3</sup></b>	<b>7.6(5)*<sup>3</sup></b>	<b>7.6(5)*<sup>3</sup></b>	<b>7.6(5)*<sup>3</sup></b>	15/15	2: de <sub>-</sub>	<b>56(24)*<sup>3</sup></b>	<b>181(159)*<sup>3</sup></b>	<b>181(150)*<sup>3</sup></b>	<b>181(151)*<sup>3</sup></b>	<b>191(159)*<sup>3</sup></b>	13/15
<b>f<sub>4</sub></b>	809	1688	1817	1886	1903	15/15	<b>f<sub>4</sub></b>	4722	7666	7700	7758	1.4e5	9/15
1: pso	14(12)	410(479)	381(414)	367(399)	364(392)	7/15	1: pso	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
2: de <sub>-</sub>	<b>3.8(2)*<sup>2</sup></b>	<b>20(17)*<sup>3</sup></b>	<b>19(15)*<sup>3</sup></b>	<b>18(15)*<sup>3</sup></b>	<b>18(15)*<sup>3</sup></b>	15/15	2: de <sub>-</sub>	<b>143(83)*<sup>3</sup></b>	<b>3893(4441)*<sup>3</sup></b>	<b>3876(3901)*<sup>3</sup></b>	<b>3847(4259)*<sup>3</sup></b>	<b>212(213)*<sup>3</sup></b>	1/15
<b>f<sub>5</sub></b>	10	10	10	10	10	15/15	<b>f<sub>5</sub></b>	41	41	41	41	41	15/15
1: pso	6.8(0.3)	8.8(0.6)	8.9(0.6)	8.9(0.6)	8.9(0.6)	15/15	1: pso	22(16)	31(32)	33(36)	33(36)	33(36)	15/15
2: de <sub>-</sub>	6.5(2)	9.0(2)	9.0(2)	9.0(2)	9.0(2)	15/15	2: de <sub>-</sub>	14(8)	24(16)	24(16)	26(20)	26(20)	15/15
<b>f<sub>6</sub></b>	114	281	580	1038	1332	15/15	<b>f<sub>6</sub></b>	1296	3413	5220	6728	8409	15/15
1: pso	6.4(9)	3.1(4)	1.9(2)	1.5(2)	54(98)	11/15	1: pso	9.4(8)	40(36)	148(193)	198(267)	352(392)	0/15
2: de <sub>-</sub>	5.1(2)	2.5(0.9)	1.5(0.6)	1.3(2)	2.8(3)	15/15	2: de <sub>-</sub>	6.8(5)	17(16)	32(36)	96(152)	295(334)	2/15
<b>f<sub>7</sub></b>	24	1171	1572	1572	1597	15/15	<b>f<sub>7</sub></b>	1351	9503	16524	16524	16969	15/15
1: pso	589(940)	930(922)	2363(2591)	2363(2733)	2326(2372)	1/15	1: pso	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
2: de <sub>-</sub>	<b>8.9(6)*<sup>3</sup></b>	<b>10(22)*<sup>3</sup></b>	<b>9.2(17)*<sup>3</sup></b>	<b>9.2(17)*<sup>3</sup></b>	<b>41(64)*<sup>3</sup></b>	10/15	2: de <sub>-</sub>	<b>502(747)*<sup>3</sup></b>	$\infty$ <sup>3</sup>	$\infty$ <sup>3</sup>	$\infty$ <sup>3</sup>	$\infty$ <sup>3</sup>	0/15
<b>f<sub>8</sub></b>	73	336	391	410	422	15/15	<b>f<sub>8</sub></b>	2039	4040	4219	4371	4484	15/15
1: pso	3.0(2)	1.6(0.8)	1.5(0.7)	1.5(0.7)	1.4(0.7)	15/15	1: pso	1.6(1)	4.7(5)	6.0(4)	6.3(4)	9.0(12)	15/15
2: de <sub>-</sub>	2.8(2)	1.1(0.5)	1.1(0.4)	1.1(0.4)	1.1(0.4)	15/15	2: de <sub>-</sub>	1.5(1)	2.8(4)	3.2(3)	3.2(3)	3.2(3)	15/15
<b>f<sub>9</sub></b>	35	214	300	335	369	15/15	<b>f<sub>9</sub></b>	1716	3277	3455	3594	3727	15/15
1: pso	4.1(1)	1.6(0.9)	1.3(0.6)	1.2(0.5)	1.1(0.5)	15/15	1: pso	1.7(0.7)	5.7(4)	7.1(5)	6.9(5)	6.9(4)	15/15
2: de <sub>-</sub>	4.2(0.7)	1.9(0.9)	1.5(0.7)	1.3(0.6)	1.2(0.5)	15/15	2: de <sub>-</sub>	1.8(1)	2.8(1)	<b>2.7(1)*</b>	<b>2.7(1)*</b>	<b>2.7(1)*</b>	15/15
<b>f<sub>10</sub></b>	349	574	626	829	880	15/15	<b>f<sub>10</sub></b>	7413	10735	14920	17073	17476	15/15
1: pso	1.3(1)	0.84(0.9)	0.79(0.8)	0.62(0.6)	1.7(0.6)	15/15	1: pso	29(15)	20(10)	14(7)	12(6)	12(6)	14/15
2: de <sub>-</sub>	0.75(0.7) $\downarrow$	0.49(0.4) $\downarrow$ <sup>2</sup>	0.48(0.4) $\downarrow$ <sup>2</sup>	0.39(0.3) $\downarrow$ <sup>2</sup>	0.50(0.9) $\downarrow$	15/15	2: de <sub>-</sub>	6.1(6)	4.3(4)	3.1(3)	2.7(3)	2.6(3)	15/15
<b>f<sub>11</sub></b>	143	763	1177	1467	1673	15/15	<b>f<sub>11</sub></b>	1002	6278	9762	12285	14831	15/15
1: pso	0.60(0.1) $\downarrow$ <sup>3</sup>	0.13(0.0) $\downarrow$ <sup>4</sup>	0.10(0.0) $\downarrow$ <sup>4</sup>	0.09(0.0) $\downarrow$ <sup>4</sup>	0.11(0.0) $\downarrow$ <sup>4</sup>	15/15	1: pso	0.25(0.2) $\downarrow$ <sup>3</sup>	0.05(0.0) $\downarrow$ <sup>4</sup>	0.04(0.0) $\downarrow$ <sup>4</sup>	0.04(0.0) $\downarrow$ <sup>4</sup>	0.05(0.0) $\downarrow$ <sup>4</sup>	15/15
2: de <sub>-</sub>	0.70(0.2) $\downarrow$	0.15(0.0) $\downarrow$ <sup>4</sup>	0.11(0.0) $\downarrow$ <sup>4</sup>	0.11(0.0) $\downarrow$ <sup>4</sup>	0.12(0.1) $\downarrow$ <sup>4</sup>	15/15	2: de <sub>-</sub>	0.20(0.1) $\downarrow$ <sup>4</sup>	0.05(0.0) $\downarrow$ <sup>4</sup>	0.04(0.0) $\downarrow$ <sup>4</sup>	0.04(0.0) $\downarrow$ <sup>4</sup>	0.05(0.0) $\downarrow$ <sup>4</sup>	15/15
<b>f<sub>12</sub></b>	108	371	461	1303	1494	15/15	<b>f<sub>12</sub></b>	1042	2740	4140	12407	13827	15/15
1: pso	2.8(2)	1.5(1)	1.7(1)	0.73(0.5)	0.86(0.9)	15/15	1: pso	2.2(2)	2.3(2)	3.7(3)	5.0(4)	21(24)	13/15
2: de <sub>-</sub>	2.8(1)	1.9(1)	1.9(1)	0.80(0.5)	1.1(0.7)	15/15	2: de <sub>-</sub>	1.4(0.8)	1.6(1)	1.7(1)	<b>1.1(0.6)*<sup>3</sup></b>	<b>4.6(4)*<sup>2</sup></b>	15/15
<b>f<sub>13</sub></b>	132	250	1310	1752	2255	15/15	<b>f<sub>13</sub></b>	652	2751	18749	24455	30201	15/15
1: pso	0.99(0.1)	0.84(0.1) $\downarrow$ <sup>4</sup>	0.22(0.0) $\downarrow$ <sup>4</sup>	181(286)	$\infty$ 5.0e5	0/15	1: pso	1.2(0.1)	0.51(0.0) $\downarrow$ <sup>4</sup>	0.21(0.1) $\downarrow$ <sup>3</sup>	133(166)	$\infty$ 2.0e6	0/15
2: de <sub>-</sub>	1.0(0.2)	0.87(0.1) $\downarrow$ <sup>2</sup>	0.23(0.0) $\downarrow$ <sup>4</sup>	2.6(1.0)	<b>529(597)*<sup>2</sup></b>	0/15	2: de <sub>-</sub>	1.3(0.1)	0.53(0.0) $\downarrow$ <sup>4</sup>	0.18(0.2) $\downarrow$ <sup>4</sup>	92(116)	$\infty$ 2.0e6	0/15
<b>f<sub>14</sub></b>	10	58	139	251	476	15/15	<b>f<sub>14</sub></b>	75	304	932	1648	15661	15/15
1: pso	2.3(3)	1.6(0.2)	0.98(0.1)	0.76(0.1)	134(203)	4/15	1: pso	1.7(0.6)	0.98(0.2)	0.63(0.1) $\downarrow$ <sup>4</sup>	0.57(0.1) $\downarrow$ <sup>4</sup>	892(958)	0/15
2: de <sub>-</sub>	1.0(1)	1.5(0.3)	0.95(0.1)	0.71(0.1)	6.4(10)	13/15	2: de <sub>-</sub>	1.7(0.4)	0.90(0.2)	0.62(0.1) $\downarrow$ <sup>4</sup>	0.56(0.1) $\downarrow$ <sup>4</sup>	125(151)	0/15
<b>f<sub>15</sub></b>	511	19369	20073	20769	21359	14/15	<b>f<sub>15</sub></b>	30378	3.1e5	3.2e5	4.5e5	4.6e5	15/15
1: pso	10(7)	39(36)	37(38)	36(33)	35(32)	8/15	1: pso	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 2.0e6	0/15
2: de <sub>-</sub>	<b>3.8(3)*</b>	<b>1.6(1)*<sup>2</sup></b>	<b>1.6(1)*<sup>2</sup></b>	<b>1.5(1)*<sup>2</sup></b>	<b>1.5(1)*<sup>2</sup></b>	15/15	2: de <sub>-</sub>	<b>28(15)*<sup>3</sup></b>	$\infty$ <sup>3</sup>	$\infty$ <sup>3</sup>	$\infty$ <sup>3</sup>	$\infty$ 2.0e6 <sup>3</sup>	0/15
<b>f<sub>16</sub></b>	120	2662	10449	11644	12095	15/15	<b>f<sub>16</sub></b>	1384	77015	1.9e5	2.0e5	2.2e5	15/15
1: pso	6.9(8)	75(97)	111(130)	298(323)	593(622)	1/15	1: pso	542(550)	$\infty$	$\infty$	$\infty$	$\infty$ 2.0e6	0/15
2: de <sub>-</sub>	8.0(8)	<b>21(9)*</b>	33(48)	30(43)	585(663)	0/15	2: de <sub>-</sub>	595(508)	$\infty$	$\infty$	$\infty$	$\infty$ 2.0e6	0/15
<b>f<sub>17</sub></b>	5.2	899	3669	6351	7934	15/15	<b>f<sub>17</sub></b>	63	4005	30677	56288	80472	15/15
1: pso	1.5(1.0)	21(19)	13(21)	9.2(12)	907(979)	0/15	1: pso	19(15)	$\infty$	$\infty$	$\infty$	$\infty$ 2.0e6	0/15
2: de <sub>-</sub>	3.0(3)	35(110)	<b>10(27)*</b>	<b>6.9(16)*</b>	145(160)	0/15	2: de <sub>-</sub>	17(19)	<b>327(293)*<sup>3</sup></b>	<b>952(1028)*<sup>3</sup></b>	$\infty$ <sup>3</sup>	$\infty$ 2.0e6 <sup>3</sup>	0/15
<b>f<sub>18</sub></b>	103	3968	9280	10905	12469	15/15	<b>f<sub>18</sub></b>	621	19561	67569	1.3e5	1.5e5	15/15
1: pso	24(18)	16(10)	38(38)	53(58)	$\infty$ 5.0e5	0/15	1: pso	210(159)	$\infty$	$\infty$	$\infty$	$\infty$ 2.0e6	0/15
2: de <sub>-</sub>	<b>5.7(7)*</b>	28(64)	19(27)	20(24)	587(645)	0/15	2: de <sub>-</sub>	<b>69(61)*</b>	<b>1503(1689)*<sup>3</sup></b>	$\infty$	$\infty$	$\infty$ 2.0e6	0/15
<b>f<sub>19</sub></b>	1	242	1.2e5	1.2e5	1.2e5	15/15	<b>f<sub>19</sub></b>	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
1: pso	69(94)	60(51)	3.6(3)	3.6(3)	19(19)	1/15	1: pso	3614(6276)	<b>1.4(1)*</b>	$\infty$ <sup>2</sup>	$\infty$ <sup>2</sup>	$\infty$ 2.0e6 <sup>2</sup>	0/15
2: de <sub>-</sub>	40(38)	71(65)	4.3(4)	4.3(4)	11(11)	2/15	2: de <sub>-</sub>	3206(3658)	5.4(6)	$\infty$	$\infty$	$\infty$ 2.0e6	0/15
<b>f<sub>20</sub></b>	16	38111	54470	54861	55313	14/15	<b>f<sub>20</sub></b>	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
1: pso	4.9(0.9)	3.5(3)	2.4(2)	2.4(2)	2.4(2)	14/15	1: pso	7.1(1)	$\infty$	$\infty$	$\infty$	$\infty$ 2.0e6	0/15
2: de <sub>-</sub>	4.1(2)	10(14)	7.1(7)	7.0(9)	7.0(10)	10/15	2: de <sub>-</sub>	6.3(4)	$\infty$	$\infty$	$\infty$	$\infty$ 2.0e6	0/15
<b>f<sub>21</sub></b>	41	1674	1705	1729	1757	14/15	<b>f<sub>21</sub></b>	561	14103	14643	15567	17589	15/15
1: pso	1.8(2)	1.6(2)	1.6(2)	1.6(2)	29(55)	13/15	1: pso	1.7(1)	0.56(0.9)	0.55(0.9)	0.57(1)	96(119)	1/15
2: de <sub>-</sub>	2.2(2)	2.7(3)	2.7(3)	2.7(3)	16(15)	12/15	2: de <sub>-</sub>	10(11)	17(26)	16(25)	15(24)	205(247)	1/15
<b>f<sub>22</sub></b>	71	938	1008	1040	1068	14/15	<b>f<sub>22</sub></b>	467	23491	24948	26847	1.3e5	12/15
1: pso	1.6(2)	3.4(4)	3.2(4)	3.3(4)	170(237)	5/15	1: pso	1.7(2)	<b>10(13)*<sup>3</sup></b>	<b>9.0(13)*<sup>3</sup></b>	<b>8.6(12)*<sup>3</sup></b>	209(238)	0/15
2: de <sub>-</sub>	6.7(15)	23(24)	21(22)	21(22)	131(244)	8/							

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**Algorithm 1:** Pseudocode of the implemented memetic algorithm

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**Input:** Objective function,  $f : S \subset \mathbb{R}^n \rightarrow \mathbb{R}$ ; algorithm PSO/DE:  $algo$ ; swarm size:  $N$ ; memetic strategy: **memetic**, maximum function evaluations: **maxfev**; unification factor: **UF**; use mutation: **mut**; probability for local search:  $\rho$

**Output:** Best detected solution:  $x^*$ ,  $f(x^*)$ .

```

// Initialization
1 for  $i = 1, 2, \dots, N$  do
2   Initialize position  $x_i$  and velocity  $u_i$ 
3   Set  $p_i \leftarrow x_i$  // Initialize best position
4    $f_i \leftarrow f(x_i(0))$  // Evaluate particle
5    $f_i^p \leftarrow f_i$  // Best position
6    $local_i \leftarrow 0$  // Best position is minimum is set to false
7 end
// Update Best Indices
8 Calculate global best index  $g_1$  and local best index  $g_2$ 
// Main Iteration Loop
9 Set  $t \leftarrow 0$ 
10 while termination criterion do
// Update Swarm
11 if  $algo = 'pso'$  then
12   for  $i = 1, 2, \dots, N$  do
13     Calculate local best velocity update for particle  $u_i^l$ 
14     using  $g_1$ 
15     Calculate global best velocity update for particle  $u_i^g$ 
16     using  $g_2$ 
17     if  $mut = 1$  then
18       // Unified PSO with mutation
19        $R \leftarrow N(\mu, \sigma)$ 
20       if  $rand() \leq 0.5$  then
21          $u_i \leftarrow RUFu_i^l + (1 - UF)u_i^g$  // Unified PSO +
22         Mutate local term
23       else
24          $u_i \leftarrow UFu_i^l + R(1 - UF)u_i^g$  // Unified PSO +
25         Mutate global term
26       end
27     else
28        $u_i \leftarrow UFu_i^l + (1 - UF)u_i^g$  // Unified PSO
29     end
30      $x_i = x_i + u_i$  // Update particle's position
31   end
32 else if  $algo = 'de'$  then
33   for  $i = 1, 2, \dots, N$  do
34      $x_i \leftarrow p_i$  // Replicate best positions  $p$  to swarm
35     array  $x$ 
36   end
37   for  $i = 1, 2, \dots, N$  do
38     Calculate  $u_i$  using a strategy from Eqs. (6)–(??)
39     if  $rand() \leq C$  then
40        $x_i \leftarrow u_i$ 
41     else
42        $x_i \leftarrow p_i$ 
43     end
44   end
45 // Evaluate Swarm
46 for  $i = 1, 2, \dots, N$  do
47    $f_i \leftarrow f(x_i)$  // Evaluate particle
48 end
49 // Update Best Positions
50 for  $i = 1, 2, \dots, N$  do
51   if  $f_i < f(p_i)$  then
52      $p_i \leftarrow x_i$ 
53      $f_i^p \leftarrow f_i$ 
54   end
55 end
56 Calculate global best index  $g_1$  and local best index  $g_2$ 
57 Apply one of the schemes using  $\rho$ 
58 Calculate global best index  $g_1$  and local best index  $g_2$ 
59 // If all best positions are local minima, restart
60 if  $\sum_{i=1}^N local_i = N$  then
61   Keep global best particle and reinitialize the swarm
62 end
63 end

```

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