HYSTERESIS MODELING IN MATERIALS AND SYSTEMS

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ABSTRACT

The class of models presented here, targeting the modeling of hysteresis processes in materials and systems, are based on the Preisach formalism. The 1D and 2D formulations are equipped with a set of five different local hysteresis operators, to address different types of systems. The resulting algorithms are efficient enough to be used as core models in simulations or real-time control. Results are shown for the $M(H)$ response in ferromagnets, the $x(T)$ response in shape memory alloys and the $\lambda(H)$ relationship in magnetostrictive materials. The choice and construction of the probability density function of the local hysteresis operators is discussed.

1 INTRODUCTION

Hysteresis is a property of several, both linear and non-linear, systems in which the output lags the input. It may be related to a linear phase lag, to energy dissipation, or to memory properties and metastability. In this work, hysteresis is considered as a rate-independent memory effect where the present output is a complicated non-linear function of the present as well as of past input values. This type of hysteresis presents several interesting properties. From a stability point of view, there are many possible equilibrium states for a given input value. The resulting state depends on the history of the system, on the previous equilibrium states, hence the memory property. This response is due to an underlying complex network of interactions between the system's several components and the external stimulus. For example, in a ferromagnet, hysteresis may arise because of the domain-domain interactions, the stray fields at the grain boundaries, the existence of inhomogeneities and their role as pinning sites, the built-in stresses and anisotropy, the exchange coupling between phases. Hysteresis may be a desired effect, when the stability of information or energy storage is of interest as in the case of data storage media (tapes, disks) or an undesired effect in sensing and actuating applications where the nonlinearity of the response adds to the uncertainty of the sensor or complicates the control of the actuator. In either case, the modeling of hysteresis is important.

Fig. 1 shows a characteristic I/O hysteresis curve, usually referred to as the *major loop* of the system. Such a curve is encountered in the magnetization vs field response, $M(H)$, in ferromagnets, in the strain vs. temperature reponse, $x(T)$, in shape memory alloys (SMAs), in the stress-strain relationship in elastoplasticity etc. The bistability property is obvious: for a given input there are two possible output states. The major loop curve is characteristic of a system, holding a lot of useful information and delimiting the input/output space but it is just a trajectory obtained for a specific input sequence. Other input sequences yield different sets of ascending or descending curves, all of them inside the major loop, called *minor loops*. A point on or inside the curve may be reached in several ways, through various trajectories of this type. Note that the only uniquely defined states are the positive and negative saturation states, $+S$ and $-S$. The treatment of hysteresis in this work is quasistatic: the time elapsing between two consecutive inputs is assumed long enough for the transients to reduce to zero.

2 THE MODEL

Models of hysteresis involve sets of differential equations, minimization of energy equations, finite-element routines or, as in the case presented here, phenomenological models based on local hysteresis operators statistically distributed. Even though hysteresis is due to different mechanisms in each system, the response illustrated in Fig. 1 is typical in most of them. Therefore a phenomenological approach, like the Preisach formalism [1], ignoring the details of the underlying physics may be a useful tool in the modeling of hysteresis in several types of systems or materials.

The Preisach formalism postulates that hysteresis is the aggregate response of a distribution of elementary *hysteresis operators*. The resulting model is computationally efficient and for systems that fulfill certain necessary and sufficient conditions it is quite reliable [1]. The hysteresis operator of the classical Preisach model (CPM) is a relay (Figs. 2a, 2b) that switches between two states, (+1, -1) or (0,1), at two critical input values a, b, with $a > b$. Hence, it is a scalar operator describing only *irreversible switching*. An extensive discussion on the mathematical properties, the hysteresis operator, the identification, and the invert of the CPM can be found in Refs. [1-3]. The inherently scalar nature of the CPM and its inability to model *reversible processes* has led to several modifications and 2D- or 3D-adaptations [1, 4]. In the approach presented here, the original formalism is extended to two dimensions replacing the operator of Fig. 2a by a 2D operator (Figs. 2d, 2e) that allows for irreversible switching, as well as, reversible rotation [4,6].

2.1 The operators

Scalar operators

In the classical model the operator γ_{ab}, hereafter referred to as "cpm1", is a simple relay with output ±1 and upper and lower switching points a and b, respectively (Fig. 2a). For an input u_t, the output f_t is given by

$$f_t = \gamma_{ab} \circ u_t = \min\{1, f_{t-1}\}, \text{ where } \gamma_{ab} = \begin{cases} +1, & u_t > a \\ -1, & u_t < b \end{cases}. \tag{1}$$

It can be used to model hysteresis in ferromagnets, where the output (magnetization) varies between positive and negative saturation, and the major loop is traced in the counter clockwise direction.

The operator in Fig. 2b, "cpm2", is a modification of the classical operator and suitable for hysteresis modeling in SMAs and elastoplasticity where the output (strain) varies between zero and a maximum value:

$$\gamma_{ab} = \begin{cases} 0, & u_t > a \\ +1, & u_t < b \end{cases} \text{ and } f_t = \min\{1, f_{t-1}\}. \tag{2}$$

The operator in Fig. 2c, also known as the "kp" operator [5], allows for a linear transition between the minimum and maximum values, and bi-directional horizontal movement at any point of the ascending or descending curve. It is appropriate for hysteresis modeling in SMAs and elastoplasticity.

For the descending branch,

$$\gamma_{ab} = \begin{cases} 1 & u_t \leq a-\delta \\ \gamma_a & a-\delta < u_t < a+\delta \\ 0 & u_t \geq a+\delta \end{cases}, \qquad \text{where } \gamma_a = 1-\frac{1}{2\delta}(u_t - a + \delta), \tag{3}$$

and

$$f_t = \min\{1, \max\{f_{t-1}, \gamma_a\}\}. \tag{4}$$

For the ascending branch,

$$\gamma_{ab} = \begin{cases} 0 & u_t \geq b+\delta \\ \gamma_b & b+\delta > u_t > b-\delta \\ 1 & u_t \leq b-\delta \end{cases}, \qquad \text{where } \gamma_b = 1-\frac{1}{2\delta}(u_t - b + \delta), \tag{5}$$

and

$$f_t = \max\{0, \min\{f_{t-1}, \gamma_b\}\}, \tag{6}$$

where 2δ is the difference between the two input values at the beginning and end of the switching process.

Vector operators

The Preisach formulation can be extended to two dimensions using vector 2D operators [4]. The operator of Fig. 2d is known as the Stoner-Wohlfarth (sw) astroid, and is borrowed from the theory of ferromagnetism. It results from the minimization of the free energy equation of an ellipsoidal magnetic particle with uniaxial anisotropy under an applied field as the locus of the equation

$$u_x^{2/3} + u_y^{2/3} = 1, \tag{7}$$

where u_x and u_y are the components of the input u along the easy and the hard axes of the particle, respectively. The solution ϕ is the angle of the output vector with respect to the easy axis of the astroid:

$$u_x \tan\phi - u_y + \sin\phi = 0\,; \tag{8}$$

it is the tangent to the astroid passing from the tip of the input vector. Switching occurs only if, during the transition from $f(t-1)$ to $f(t)$, the output vector crosses the astroid from the inside out. Otherwise, the output vector rotates reversibly.

The second vector operator (Fig. 1e), the diamond ("dm"), is the first order approximation of the sw-astroid:

$$u_x + u_y = 1\ , \tag{9}$$

which uses a similar hysteresis mechanism. It is computationally more efficient but it does not have any physical attributes. Both vector operators are used for hysteresis modeling in ferromagnets. For inputs along the x-direction, the vector operators respond identically to the classical scalar operator of Fig. 2a [6].

Modeling the hysteresis process

The parameters a and b of the hysteresis operators are distributed according to a probability density function $\rho(a,b)$ over a half-plane, called the Preisach plane, that is defined by $a \geq b$ [1, 4]. The hysteresis process is then modeled as the aggregate response of the distributed operators to a sequence of inputs. The output $f(t)$ at time t is given by the equation

$$f(t) = \iint_{a\geq b} \rho(a,b)\gamma_{ab} \circ u(t)\,da\,db\,. \tag{10}$$

Using a 2D-operator γ_{ab} instead, Eq. (10) describes a 2D-model for a perfectly oriented system. That is, the easy axes of all operators lie along the same orientation direction. However, actual systems, as a rule, are not perfectly oriented and the question of orientation dispersion needs to be addressed. Systems that are not perfectly oriented can be modeled by

$$f(t) = \int_{-\pi/2}^{\pi/2} \rho(\theta)d\theta \iint_{a \geq b} \rho(a,b)\gamma_{ab} \circ \boldsymbol{u}(t)dadb, \tag{11}$$

where $\rho(\theta)$ is the probability density function of the angles that the easy axes of the 2D operators γ_{ab} form with the model's axis of orientation [4].

When a material contains more than one phase, it may be preferable to use:

$$\mathbf{f}(t) = \int_{-\pi/2}^{\pi/2} \rho(\theta)d\theta \iint_{a \geq b} [w \times \rho_1(a,b) + (1-w) \times \rho_2(a,b)]\gamma_{ab} \circ u(t)dadb \tag{12}$$

where w denotes the percentage content of phase 1 [8].

The identification routine

In order to identify a Preisach-type model for a given system, its characteristic density $\rho(a,b)$ must be determined. In the case of the classical model of Eq. (10), the density $\rho(a,b)$ can be determined experimentally using the Everett functions [1, 4]. However, in the 2D model of Eq. (11), it is not obvious how the effect of the transverse and longitudinal components on the distribution can be decoupled and determined experimentally. In this case, the density is modeled as a bivariate pdf, *e.g.* a bivariate normal, or as the product of two uncorrelated single-variable pdfs or as the weighted sum of densities, not necessarily of the same type. This is useful in the modeling of materials consisting of more than one phase whose interaction with the applied input is distinctly different and reflects in the major loop. Fig. 3 shows the effect the choice of the pdf has on the shape of the $M(H)$ loop. The $\rho(a,b)$ used for these curves is generated as a product of two single variable pdfs of any of the following types:

Normal, $N(x)=\frac{1}{2\pi\sigma}\exp\left[-\left(\frac{x-\mu}{\sigma}\right)^2\right]$, (13)

Lorentzian, $L(x)=\frac{1}{\pi^2\sigma}\left(\frac{1}{1+\left(\frac{x-\mu}{\sigma}\right)^2}\right)$, (14)

or the first-order derivative of sigma, $S(x)=\left(\frac{\exp\left(-\frac{x-\mu}{\sigma}\right)}{1+\exp\left(-\frac{x-\mu}{\sigma}\right)}\right)$. (15)

The major loops have been generated using the vector model of Eq. (2) and the diamond operator of Fig. 1e. The effect of the weighed sum of two gaussians with same mean but different standard deviations is also shown.

In any case, the parameters of the densities are determined using major loop data along with a least-squares fitting procedure [6-8].

3 RESULTS

The model described in Section 2 is applied to data taken on ferromagnetic samples and an SMA sample. The origins and mechanism of hysteresis are quite different in each type of material.

Modeling of hysteresis in ferromagnets

In ferromagnets, hysteresis occurs during the switching from positive to negative magnetization and the opposite. For an applied magnetic field (input), $H(t)$, the resulting magnetization (output), $M(t)$, is a function of the applied field as well as of an internal interaction field, which is in turn a function of the magnetization. Hence, the resulting magnetization state contains a positive feedback mechanism leading to hysteresis: $M(t) = M(H(t), M(t))$.

Fig. 5 shows major ascending curves measured on Gd-film samples, used for recording, annealed at 610 °C and 560 °C prior to the hysteresis measurement. Annealing improves the crystallographic order and sharpens the anisotropy distribution of the sample [9], which is reflected in the phenomenology of the curves. At lower annealing temperatures, the anisotropy distribution width is larger than the threshold value at which the correlation of the magnetic reversal in the sample breaks down. The squareness of the loop, $S = M(0)/M(H_{sat})$, and the coercivity squareness, S^*, a measure of the sterepness of the loop around the coercivity, are lower and a 2D model (Eq. 11) using the diamond operator (Fig. 2e) must be chosen. On the contrary, the 610 °C curve is accurately reproduced by the 1D-model (Eq. 10) using the classical operator (Fig. 2a).

Modeling of hysteresis in shape memory alloys

In shape memory alloys, hysteresis can be observed as the material undergoes the transformation from the martensitic to the austenitic phase and vice versa. The input variable is temperature, $T(t)$, and the output is strain $x(t)$ [5]. The SMA data are modeled using the 1D model of Eq. (10) and the hysteresis operators "cpm2" (Fig. 2b) or "kp" (Fig. 2c). The SMA loops are traced in the opposite direction compared to the magnetic loops. The ascending branch is the one to the left and is traced as the temperature decreases. The output is normalized to the maximum % strain observed and ranges from 0 to 1. Unlike the case of ferromagnets, the loop is not symmetric but skewed and shifted to the right with respect to the origin [6]. Fig. 6 shows major and minor loops obtained using the two operators “cpm2” and “kp” against the corresponding experimental curves obtained on a Nitinol sample. Note that the minor loops are adequately reproduced by the model, even though the data used for the identification were taken from the major loop curve only.

Modeling of mangetostriction

Another example of hysteresis appears in the case of magnetostrictive materials. Under an externally applied field, the change in the Zeeman energy density is counterbalanced by the change in the elastic energy of the bonds. This may result in an increase (positive magnetostriction) or decrease (negative magnetostriction) of the sample length along the direction of the applied field. Because of the microstructure and the ensuing interactions, the $\lambda(H)$ response may exhibit hysteresis yielding the butterfly shape of Fig. 7. As the field decreases from saturation, $\lambda(H_S)=\lambda_S$, the strain of the sample decreases reaching a minimum around the coercivity and then increases back again as the field is further decreasing towards negative saturation. This loop does not look anything like the loop of Fig. 1 but it does look like its derivative: $\lambda(H)=dM/dH$ [10].

It turns out that the $\lambda(H)$ curve can also be modeled by the Preisach model. Assuming scalar deformation, the model of Eq. (10) in conjunction with the modified scalar operator of Fig. 2b is used and the result is shown in Fig. 7. The underlying distribution is obtained as a sum of two gaussians $\rho_1(a,b)$ and $\rho_2(a,b)$ of equal but opposite signs. Their means are such that, $\mu_{a_1}=\mu_{b_2}$ and $\mu_{b_1}=\mu_{a_2}$ (Fig.7). Fig. 6

shows calculated major and minor $\lambda(H)$ loops obtained using the density shown in Fig. 7.

4 CONCLUSIONS

The modeling of hysteresis processes is not a trivial task given the nonlinearity and complexity of the phenomenon. It is shown that Preisach based models are a useful tool in hysteresis modeling, offering flexible and efficient algorithms with satisfactory results regardless of the underlying microstrucutre or physics. Because they are tuned into the system being modeled through the fitting of the parameters of a bivariate probability density, they can adapt to a variety of systems or materials. The results presented indicate that these models can reproduce hysteresis curves of ferromagnets with different characteristics and microstructure, of shape memory alloys undergoing phase transformation, and of magnetostrictive materials.

5 References

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Figure captions

Fig.1: Hysteresis curve: memory and bistability.

Fig.2: Hysteresis operators: (a) cpm1, (b) cpm2, (c) kp, (d) sw, and (e) dm.

Fig.3: Descending curves for various types of densities.

Fig.4: Gd-thin film: experimental and calculated ascending curves.

Fig.5: Nitinol: Experimental and calculated major and minor loops.

Fig.6: Calculated major and minor $\lambda(H)$ curves.

Fig.7: The density used for the $\lambda(H)$ curve.

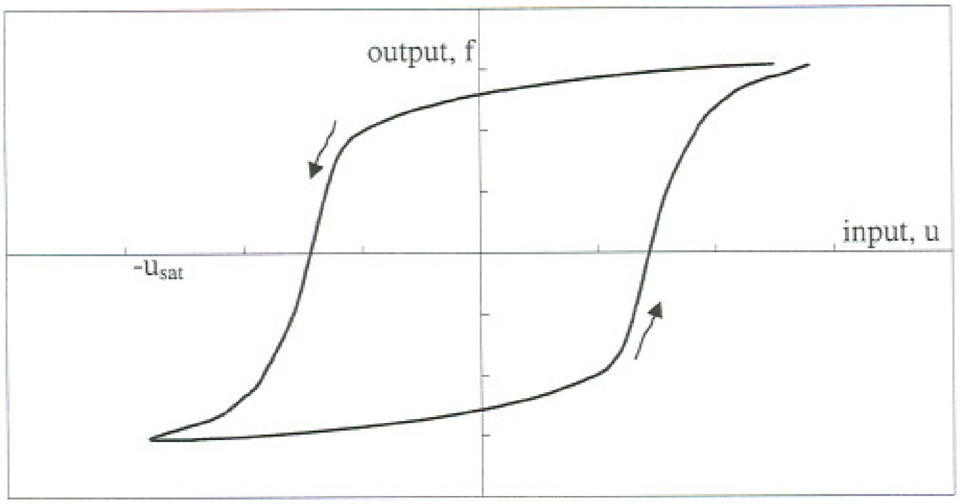


Fig.1

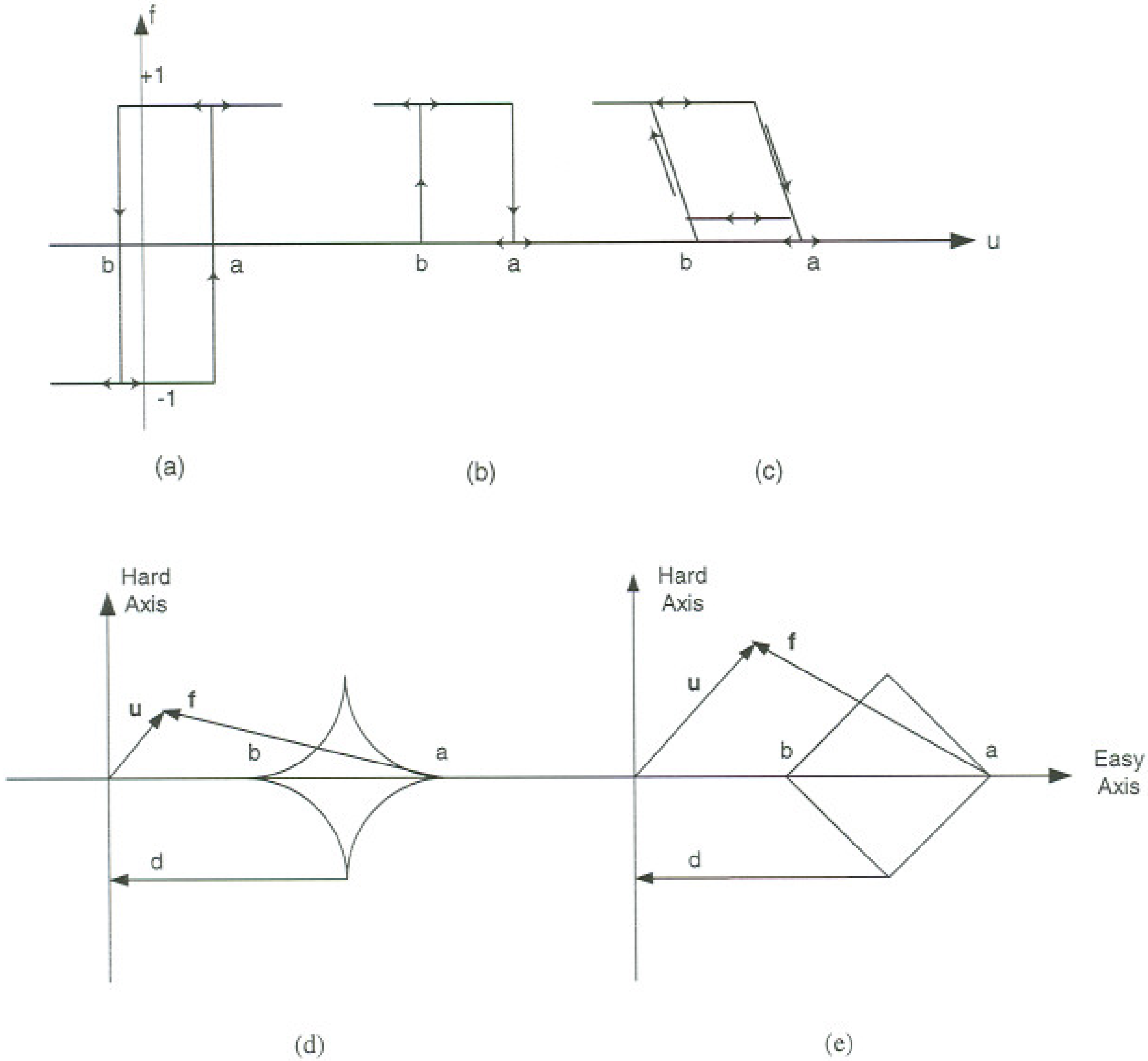


Fig. 2

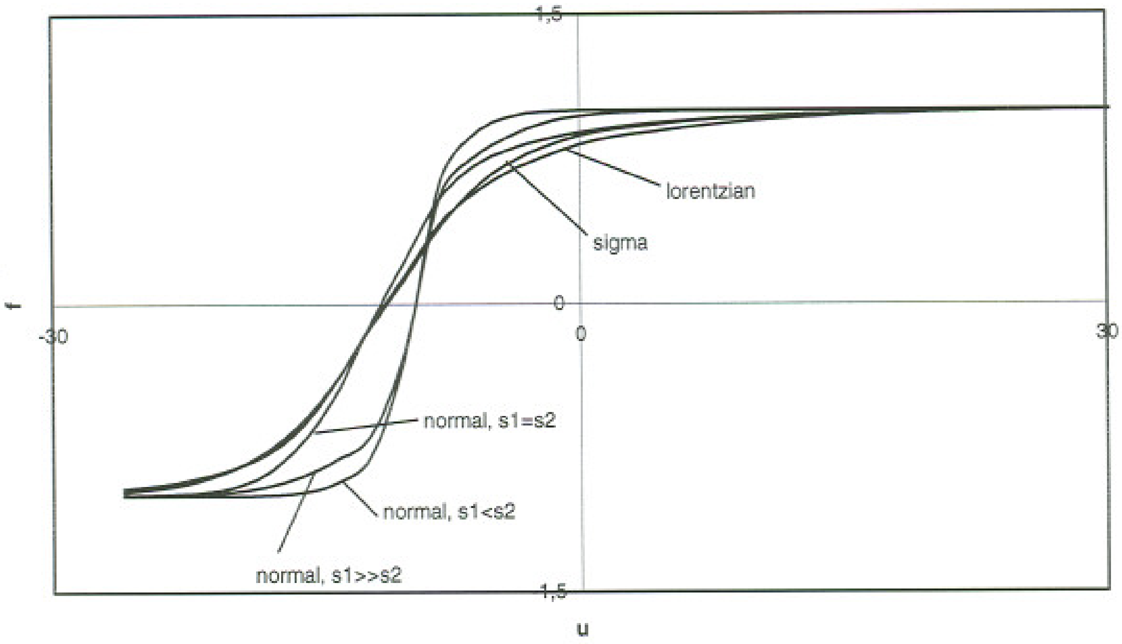
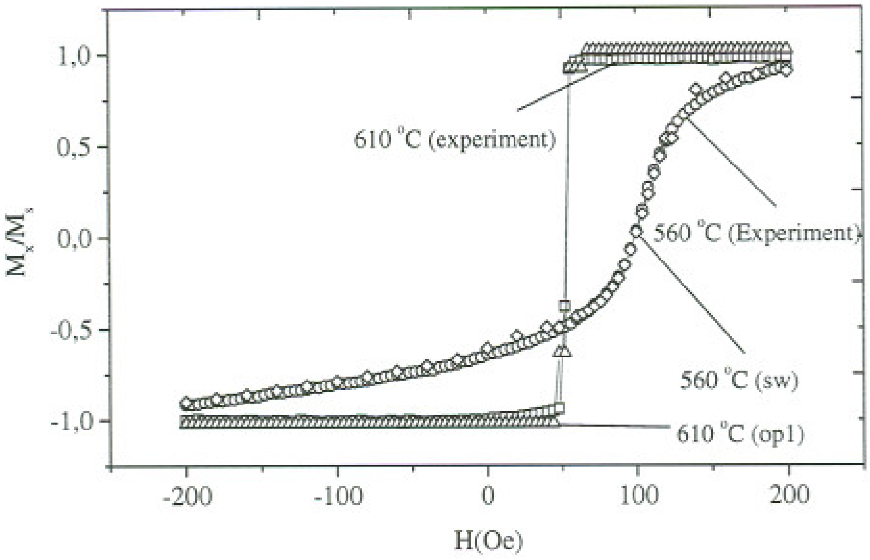

1,5
lorentzian
sigma
f
0
-30
0
30
normal, s1=s2
normal, s1<s2
normal, s1>>s2
1,5
u

Fig.3

Fig. 4



1,0
0,5
0,0
-0,5
-1,0
M_x/M_s
610 °C (experiment)
560 °C (Experiment)
560 °C (sw)
610 °C (op1)
-200
-100
0
100
200
H(Oe)

Fig.5

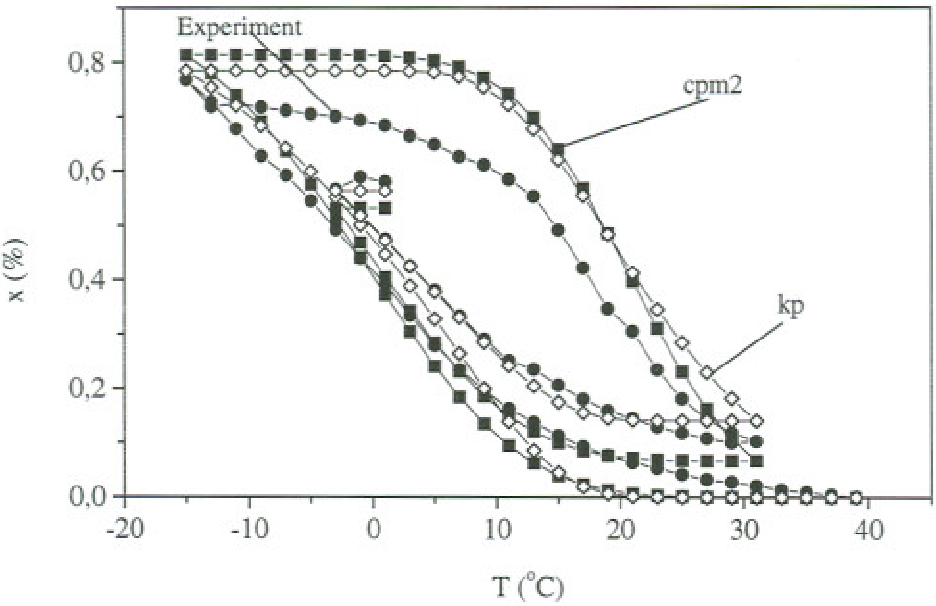


Fig.6

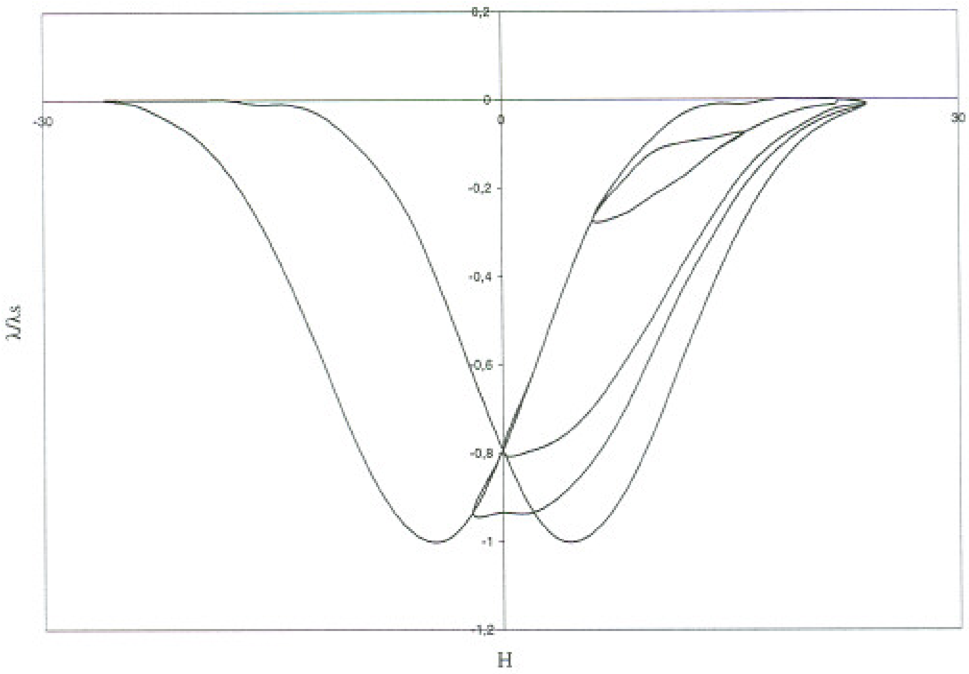


Fig. 7