ELECTROMAGNETIC DETECTION OF BURIED SPHEROIDAL OBJECTS

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1. SUMMARY

In this work, we examine the direct electromagnetic scattering problem of spherical waves by a buried spheroidal perfect conductor. The proposed analysis is based on the integral equation formalism of the problem and the purpose is the establishment of a multiparametric model describing analytically the scattering process under consideration. The outcome of the analysis is the determination of the scattered field in the observation environment along with its multivariable dependence on the several physical and geometric parameters of the investigated system.

2. INTRODUCTION

The present work concerns the investigation of the three-dimensional direct scattering problem of electromagnetic spherical waves by a prolate spheroidal perfect conductor, which is embedded in a semi-infinite dielectric medium.

The investigation of the aforementioned scattering problem is a very difficult task with several complication factors. It is well known that the embedding environment interface complicates drastically the analysis of the scattering process, while the geometric and physical characteristics of the scatterer constitute the most important and intricate parameters of the problem.

Within the framework of arbitrary scatterer shape, the suggested techniques for handling the direct scattering problem belong to the numerical regime [1-4]. Several fine techniques belonging to the regime of Boundary Elements Methods (BEM's) are evoked for the numerical treatment of the problem based on the establishment of the integral equation formalism [5,6]. Nevertheless, in case that we have a priori information concerning the geometric features of the scatterer, it is possible to develop analytical methods facing the scattering problem and leading to the establishment of multiparametric models describing the studied process. The work at hand examines the case of the prolate spheroidal scatterer, which simulates perfectly a convex body that lacks symmetry in only one direction.

Section 3 provides the mathematical formulation of the scattering problem under consideration. The main outcome of this section is the establishment of appropriate integral representations for the electric fields in the two half-space media. In Section 4 we expand the electric field in scatterer's region in terms of the spheroidal vector wave functions and force this expansion to satisfy the boundary condition on the scatterer's surface. Exploiting orthogonality arguments of the underlying spheroidal functions, we obtain fully algebraic equations with unknowns the electric field expansion coefficients.

The handling of the integral representation for the electric field in the host medium via the aforementioned spectral decomposition in spheroidal coordinates is presented briefly in Section 5. A lot of the analysis is dedicated to encounter the coexistence of the cylindrical with the spheroidal geometry in order to obtain fully analytical expressions for the above integral representations. Based on these expressions, we arrive at algebraic equations resulting from the investigation of the electric field in the asymptotic realm. These equations are combined with those originated from the boundary condition satisfaction and met in Section 4 in order to form the final non-homogeneous algebraic system whose solution provides with the spectral decomposition coefficients. Finally, in Section 6, it is explained that the knowledge of these coefficients is actually equivalent to the determination of the scattered electric field in the observation environment.

3. FORMULATION OF THE PROBLEM

We consider two separate subregions characterized by different electric permittivities $\varepsilon_i, i=(1,2)$, separated by a flat infinite interface S_δ, on which suitable impedence conditions are satisfied (Fig.1). The media occupying the two half-spaces $V_i, i=(1,2)$ are isotropic, homogeneous and non-magnetic, having magnetic permeability μ_0. The upper half-space is the region where we can stimulate or measure electromagnetic fields. In contrast, the lower half-space corresponds to the propagation environment, where we usually do not have access or we cannot make any measurements.

A prolate spheroidal scatterer with surface S_{sph} is embedded in subregion (2). The scatterer is assumed to have semiaxes a_0, b_0 $(a_0 > b_0)$, focal distance a, to be orientated vertically with respect to the interface S_δ and its center is located at a distance δ from the boundary S_δ. The origin of the coordinate system coincides with the center of the spheroidal object.

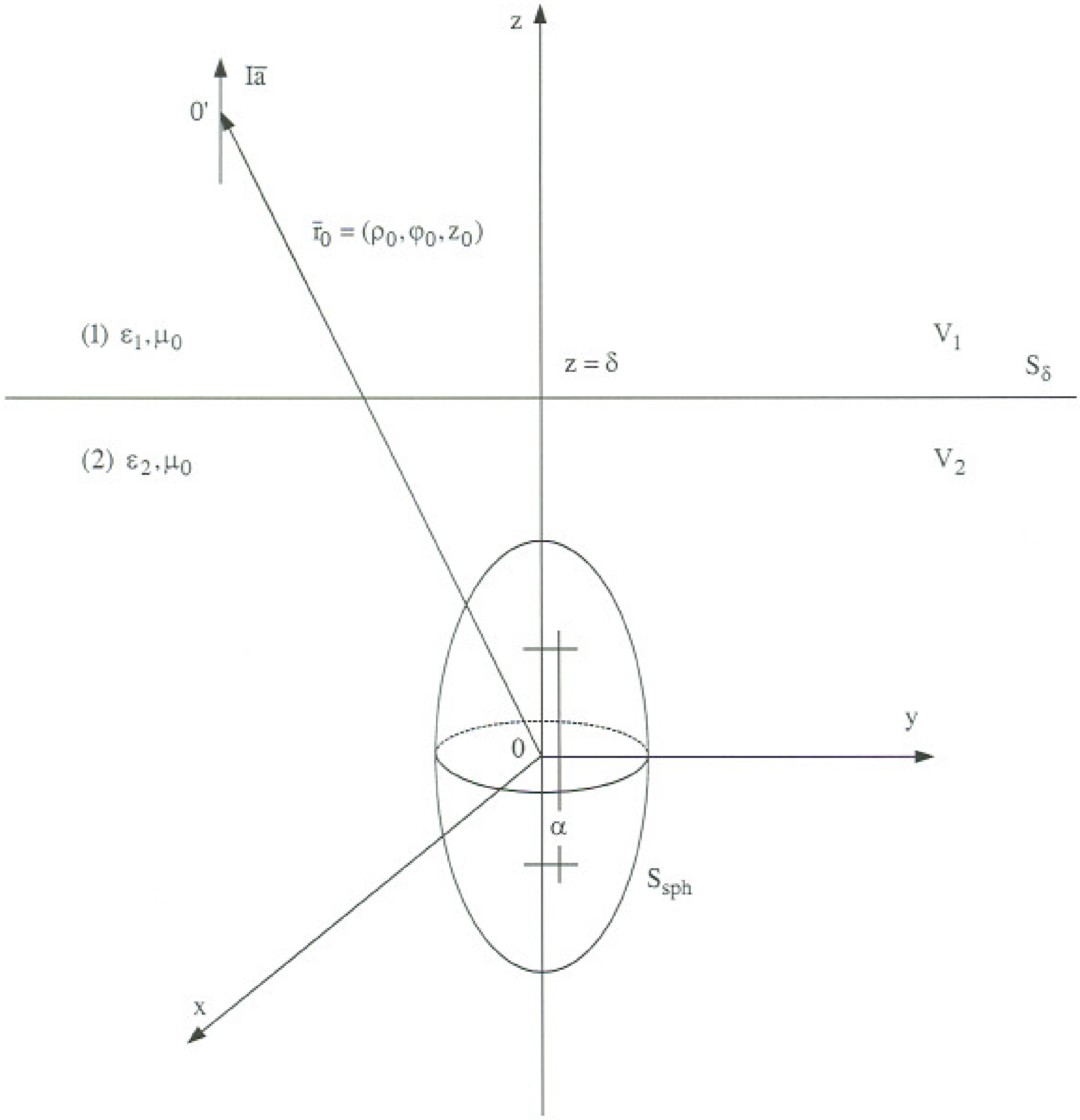

Figure 1: The System Geometry

A point source emanating time-harmonic electromagnetic spherical waves is located in region (1), at a position O' with cylindrical coordinates (ρ_0, ϕ_0, z_0). The electromagnetic field generated by the point source constitutes the incident field, the interference of which with the interface S_δ along with the spheroidal surface S_{sph}, leads to the creation of the scattered wave. This interference is strongly dependent on the boundary conditions on the scatterer's surface. For the problem under discussion, we assume that the spheroidal body constitutes a perfect conductor. The scattered wave encodes all the information concerning the physical characteristics of the scatterer along with the relevant geometrical features of the system under consideration. The total electromagnetic field is formed by the superposition of the incident field with the scattered one and is denoted by the vector pair $(\overline{E}_i, \overline{H}_i)$, $i = (1,2)$ in each region. These fields satisfy the non-homogeneous time-reduced Maxwell's equations concerning harmonic time dependence of the form

$\exp\{-i\omega t\}$. The non-homogeneous term is the current density $\overline{J}_i(\overline{r})$, which is given by

$$\overline{J}_i(\overline{r}) = \begin{Bmatrix} \mathrm{Ia}\hat{\mathrm{a}}\delta(\overline{r} - \overline{r}_0) & i = 1 \\ \overline{0} & i = 2 \end{Bmatrix}, \tag{1}$$

where $\hat{\mathrm{a}}$ is the unit vector indicating the source orientation, Ia is the magnitude of the current moment of the source expressed in terms of its length a and its constant amplitude I [7] and $\overline{r}_0$ denotes the source position vector.

On the interface S_δ, the electric field satisfies the following impedence conditions

$$\hat{z} \times \left[\overline{E}_1(\overline{r}) - \overline{E}_2(\overline{r})\right] = \overline{0}, \tag{2.a}$$

$$\hat{z} \times \left[\nabla \times \overline{E}_1(\overline{r}) - \nabla \times \overline{E}_2(\overline{r})\right] = \overline{0}, \quad \overline{r} \in S_\delta. \tag{2.b}$$

Furthermore, the tangential component of the total electric field on the spheroidal surface S_{sph} vanishes, i.e.

$$\hat{n} \times \overline{E}_2(\overline{r}) = \overline{0}, \quad \overline{r} \in S_{sph}, \tag{3}$$

where $\hat{n}$ is the outward unit normal vector to the scatterer's surface.

In addition, the electric fields satisfy the radiation conditions

$$\lim_{r \to \infty} r\left[\nabla \times \overline{E}_i(\overline{r}) - ik_i \hat{r} \times \overline{E}_i(\overline{r})\right] = \overline{0}, \quad \overline{r} \in V_i, i = (1,2), \tag{4}$$

where $k_i = \dfrac{\omega}{c_i} = \omega\sqrt{\mu_0 \varepsilon_i}$, $i = (1,2)$ are the wave numbers in the two media.

Finally, the electric fields are susceptible of the following integral representations via the well-known dyadic Green's functions [7]

$$\begin{aligned} \overline{E}_1(\overline{r}) = i\omega\mu_0 &\int_{V_1} \overline{\overline{G}}_e^{(11)}(\overline{r}, \overline{r'}) \cdot \overline{J}_1(\overline{r'})\, dV' \\ &+ \int_{S_{sph}} \overline{\overline{G}}_e^{(12)}(\overline{r}, \overline{r'}) \cdot \left[\hat{n}' \times \nabla' \times \overline{E}_2(\overline{r'})\right] dS'_{sph}, \quad \overline{r} \in V_1, \end{aligned} \tag{5}$$

$$\begin{aligned} \overline{E}_2(\overline{r}) = i\omega\mu_0 &\int_{V_1} \overline{\overline{G}}_e^{(21)}(\overline{r}, \overline{r'}) \cdot \overline{J}_1(\overline{r'})\, dV' \\ &+ \int_{S_{sph}} \overline{\overline{G}}_e^{(22)}(\overline{r}, \overline{r'}) \cdot \left[\hat{n}' \times \nabla' \times \overline{E}_2(\overline{r'})\right] dS'_{sph}, \quad \overline{r} \in V_2. \end{aligned} \tag{6}$$

In the sequel, our aim is the exploitation and adaptation of the above integral representations in order to determine the electric fields from the knowledge of the physical and geometric characteristics of the scatterer.

4. SPECTRAL DECOMPOSITION IN SPHEROIDAL GEOMETRY AND BOUNDARY CONDITIONS INVESTIGATION

The electric field in region (2) is preferable to be expressed in terms of the spheroidal solenoidal vector wave functions [8, 9], which constitute a basis in the space of Maxwell's equations solutions. This spectral representation is expected to fit suitably to the boundary conditions imposed on the spheroid S_{sph}. Consequently, it holds that

$$\overline{E}_2(\overline{r})=\sum_{j=3}^{4}\sum_{m'=0}^{\infty}\sum_{n'=m'}^{\infty}\left[A^{(j)}_{\substack{e\\o}m'n'}\,\overline{M}^{(j)}_{\substack{e\\o}m'n'}(\overline{r})+B^{(j)}_{\substack{e\\o}m'n'}\,\overline{N}^{(j)}_{\substack{e\\o}m'n'}(\overline{r})\right],\ \overline{r}\in V_2. \qquad (7)$$

We force representation (7) to obey the perfect conductor boundary condition (3) and we project the resulting vector equation on the tangential unit vectors $\hat{\theta},\hat{\phi}$ of the spheroidal system. Afterwards we project the resulted equations functionally on the complete set of functions $P_n^m(\cos\theta)\begin{Bmatrix}\cos(m\phi)\\ \sin(m\phi)\end{Bmatrix}$, $m=0,1,2,\ldots;n\geq m$ in the θ,ϕ-space with weight function $w=(\cosh^2\mu_0-\cos^2\theta)\sin\theta$. In this analysis, specific "inner" products arise, which contain the products $\langle\hat{a}\cdot\overline{\mathrm{A}},P_n^m(\cos\theta)\sin\theta\begin{Bmatrix}\cos(m\phi)\\ \sin(m\phi)\end{Bmatrix}\rangle$ with $\hat{a}=(\hat{\theta},\hat{\phi});\overline{\mathrm{A}}=\left(\overline{M}^{(j)}_{\substack{e\\o}m'n'},\overline{N}^{(j)}_{\substack{e\\o}m'n'}\right)$, which also give birth to "simple" ϕ-inner products along with 18 "complicated" θ-inner products. The complete derivation of these θ-brackets is of extended complexity, although analytical, and is omitted for simplicity. Finally, we obtain for every pair of parameters (m,n) with $m=0,1,2,\ldots;n\geq m$ the following set of algebraic equations, which is equivalent to the boundary condition on S_{sph}

$$\sum_{j=3}^{4}\sum_{n'=m}^{\infty}\{A^{(j)}_{e,mn'}\left(\pi\hat{\delta}_{0m}\right)$$

$$\cdot[\sinh\mu_0\cosh\mu_0 R^{(j)}_{mn'}(\cosh\mu_0;c_2)\Re^{4,m}_{n,n'}-\sinh\mu_0 R^{(j)'}_{mn'}(\cosh\mu_0;c_2)\Re^{6,m}_{n,n'}]$$

$$+B^{(j)}_{o,mn'}\frac{2}{k_2 a}(m\pi)$$

$$\cdot\{[\frac{(\sinh^2\mu_0-1)}{\sinh\mu_0}R^{(j)}_{mn'}(\cosh\mu_0;c_2)+\sinh\mu_0\cosh\mu_0 R^{(j)'}_{mn'}(\cosh\mu_0;c_2)]\Re^{1,m}_{n,n'}$$

$$+\frac{1}{\sinh\mu_0}R^{(j)}_{mn'}(\cosh\mu_0;c_2)[2\Re^{3,m}_{n,n'}+\Re^{5,m}_{n,n'}+\Re^{2,m}_{n,n'}]\}\}=0, \qquad (8)$$

$$\sum_{j=3}^{4}\sum_{n'=m}^{\infty}\{A_{o,mn'}^{(j)}(\pi(1-\delta_0))$$

$$\cdot[\sinh\mu_0\cosh\mu_0 R_{mn'}^{(j)}(\cosh\mu_0;c_2)\Re_{n,n'}^{4,m}-\sinh\mu_0 R_{mn'}^{(j)'}(\cosh\mu_0;c_2)\Re_{n,n'}^{6,m}]$$

$$+B_{e,mn'}^{(j)}\frac{2}{k_2a}(-m\pi)$$

$$\cdot\{[\frac{(\sinh^2\mu_0-1)}{\sinh\mu_0}R_{mn'}^{(j)}(\cosh\mu_0;c_2)+\sinh\mu_0\cosh\mu_0 R_{mn'}^{(j)'}(\cosh\mu_0;c_2)]\Re_{n,n'}^{1,m}$$

$$+\frac{1}{\sinh\mu_0}R_{mn'}^{(j)}(\cosh\mu_0;c_2)[2\Re_{n,n'}^{3,m}+\Re_{n,n'}^{5,m}+\Re_{n,n'}^{2,m}]\}\}=0, \tag{9}$$

$$\sum_{j=3}^{4}\sum_{n'=m}^{\infty}\{A_{o,mn'}^{(j)}(m\pi)$$

$$\cdot\{\cosh\mu_0 R_{mn'}^{(j)}(\cosh\mu_0;c_2)[\sinh^4\mu_0\Re_{n,n'}^{1,m}+2\sinh^2\mu_0\Re_{n,n'}^{3,m}+\Re_{n,n'}^{7,m}]\}$$

$$-B_{e,mn'}^{(j)}\frac{2(\pi\hat{\delta}_{0m})}{k_2a}\{[\sinh^2\mu_0(\sinh^2\mu_0-1)R_{mn'}^{(j)}(\cosh\mu_0;c_2)$$

$$+\sinh^4\mu_0\cosh\mu_0 R_{mn'}^{(j)'}(\cosh\mu_0;c_2)]\Re_{n,n'}^{4,m}$$

$$+\sinh^2\mu_0[4R_{mn'}^{(j)}(\cosh\mu_0;c_2)+\cosh\mu_0 R_{mn'}^{(j)'}(\cosh\mu_0;c_2)]\Re_{n,n'}^{8,m}$$

$$+\sinh^2\mu_0[\left(\frac{k_2a}{2}\right)^2\sinh^2\mu_0 R_{mn'}^{(j)}(\cosh\mu_0;c_2)+2\cosh\mu_0 R_{mn'}^{(j)'}(\cosh\mu_0;c_2)]\Re_{n,n'}^{6,m}$$

$$+\left(\frac{k_2a}{2}\right)^2 R_{mn'}^{(j)}(\cosh\mu_0;c_2)[2\sinh^2\mu_0\Re_{n,n'}^{13,m}+\Re_{n,n'}^{15,m}]$$

$$+R_{mn'}^{(j)}(\cosh\mu_0;c_2)[\sinh^2\mu_0\Re_{n,n'}^{9,m}+\Re_{n,n'}^{16,m}+2\Re_{n,n'}^{14,m}-\sinh^2\mu_0\Re_{n,n'}^{10,m}]\}\}=0, \tag{10}$$

$$\sum_{j=3}^{4}\sum_{n'=m}^{\infty}\{A_{e,mn'}^{(j)}(-m\pi)$$

$$\cdot\{\cosh\mu_0 R_{mn'}^{(j)}(\cosh\mu_0;c_2)[\sinh^4\mu_0\Re_{n,n'}^{1,m}+2\sinh^2\mu_0\Re_{n,n'}^{3,m}+\Re_{n,n'}^{7,m}]\}$$

$$-B_{o,mn'}^{(j)}\frac{2\pi(1-\delta_0)}{k_2 a}\{[\sinh^2\mu_0(\sinh^2\mu_0-1)R_{mn'}^{(j)}(\cosh\mu_0;c_2)$$

$$+\sinh^4\mu_0\cosh\mu_0 R_{mn'}^{(j)'}(\cosh\mu_0;c_2)]\Re_{n,n'}^{4,m}$$

$$+\sinh^2\mu_0[4R_{mn'}^{(j)}(\cosh\mu_0;c_2)+\cosh\mu_0 R_{mn'}^{(j)'}(\cosh\mu_0;c_2)]\Re_{n,n'}^{8,m}$$

$$+\sinh^2\mu_0[\left(\frac{k_2 a}{2}\right)^2\sinh^2\mu_0 R_{mn'}^{(j)}(\cosh\mu_0;c_2)+2\cosh\mu_0 R_{mn'}^{(j)'}(\cosh\mu_0;c_2)]\Re_{n,n'}^{6,m}$$

$$+\left(\frac{k_2 a}{2}\right)^2 R_{mn'}^{(j)}(\cosh\mu_0;c_2)[2\sinh^2\mu_0\Re_{n,n'}^{13,m}+\Re_{n,n'}^{15,m}]$$

$$+R_{mn'}^{(j)}(\cosh\mu_0;c_2)[\sinh^2\mu_0\Re_{n,n'}^{9,m}+\Re_{n,n'}^{16,m}+2\Re_{n,n'}^{14,m}-\sinh^2\mu_0\Re_{n,n'}^{10,m}]\}\}=0, \tag{11}$$

where the terms multiplying the unknown expansion coefficients $A_{\substack{e\\o}mn'}^{(j)}, B_{\substack{e\\o}mn'}^{(j)}$ are specific known functions of the concrete μ_0 characterizing the spheroidal surface. We mention here that the combination of these equations with the corresponding non-homogeneous ones resulting from the integral representation (6), after being amenable to asymptotic analysis in the far-field region, will lead to the determination of the expansion coefficient.

5. DETERMINATION OF THE ELECTRIC FIELD IN THE HOST ENVIRONMENT

The integral representation (6) is characterized by the fact that its surface integral involves functions expressed in different geometrical coordinate systems. As a matter of fact, the electric field "lives" in spheroidal geometry, since it is expressed via the spectral decomposition (7), while the kernel dyadic $\overline{\overline{G}}_e^{(22)}(\overline{r},\overline{r'})$ is expressed in terms of the cylindrical solenoidal vector wave functions [7]. The substitution of the aforementioned eigenfunction expansions in the surface integral of representation (6) along with extended use of several vector analysis arguments lead to the creation of a plethora (38 terms) of surface integrals on the spheroidal surface. These integrals are of "mixed-type" in the sense that their integrands constitute products of scalar functions expressed in the two different coordinate systems. The analytical treatment of these integrals constitutes the most demanding part of the analytical burden of this work, since their determination is based on the investigation and handling of suitable addition theorems connecting the two geometries and the exploitation of special function's properties. Their exact values have been fully determined and they are analytical expressions of cumbersome form so they are omitted for the sake of brevity.

Coming back to the integral representation (6) and remarking that the volume integral is faced easily since it includes the Dirac current density (1), we proceed with the application of stationary phase techniques adapted to this kind of problems [7] to obtain the following asymptotic equation

$$
\begin{aligned}
&\sum_{j=3}^{4} \frac{1}{k_2 r} e^{ik_2 r\,\mathrm{sgn}(j+1)} \sum_{m'=0}^{\infty}\sum_{n'=m'}^{\infty} e^{-i\frac{\pi}{2}(n'+1)\mathrm{sgn}(j+1)} \\
&\cdot\{A^{(j)}_{\substack{e\\o}m'n'}[\frac{S_{m'n'}(\cos\theta;c_2)}{\sin\theta}\begin{Bmatrix}-m'\sin(m'\phi)\\ m'\cos(m'\phi)\end{Bmatrix}\hat{\theta}+\sin\theta S'_{m'n'}(\cos\theta;c_2)\begin{Bmatrix}\cos(m'\phi)\\ \sin(m'\phi)\end{Bmatrix}\hat{\phi}] \\
&\qquad + i\,\mathrm{sgn}(j+1)B^{(j)}_{\substack{e\\o}m'n'} \\
&\cdot[-\sin\theta S'_{m'n'}(\cos\theta;c_2)\begin{Bmatrix}\cos(m'\phi)\\ \sin(m'\phi)\end{Bmatrix}\hat{\theta}+\frac{S_{m'n'}(\cos\theta;c_2)}{\sin\theta}\begin{Bmatrix}-m'\sin(m'\phi)\\ m'\cos(m'\phi)\end{Bmatrix}\hat{\phi}]\} \\
&+\frac{e^{ik_2 r}}{4\pi k_2 r\sin\theta}\sum_{m=0}^{\infty}(2-\delta_0)(-i)^{m+1}\begin{Bmatrix}\cos(m\phi)\\ \sin(m\phi)\end{Bmatrix} \\
&\cdot\left\{\begin{aligned}
&i\sum_{j=3}^{4}\sum_{n'=m}^{\infty}\{A^{(j)}_{\substack{e\\o}mn'}I^{A}_{1,\substack{e\\o}jmn'}(h_2,\lambda_0)+B^{(j)}_{\substack{e\\o}mn'}I^{B}_{1,\substack{e\\o}jmn'}(h_2,\lambda_0)\}\hat{\phi}\\
&+\sum_{j=3}^{4}\sum_{n'=m}^{\infty}\{A^{(j)}_{\substack{e\\o}mn'}I^{A}_{2,\substack{e\\o}jmn'}(h_2,\lambda_0)+B^{(j)}_{\substack{e\\o}mn'}I^{B}_{2,\substack{e\\o}jmn'}(h_2,\lambda_0)\}\hat{\theta}
\end{aligned}\right\}_{\substack{\lambda_0=k_2\sin\theta\\ h_2=-k_2\cos\theta}} \\
&= i\omega\mu_0(\mathrm{Ia})\frac{e^{ik_2 r}}{4\pi r\sin\theta\sqrt{k_1^2-k_2^2\sin^2\theta}}\cos\theta\sum_{m=0}^{\infty}(2-\delta_0)(-i)^{m+1}\begin{Bmatrix}\cos(m\phi)\\ \sin(m\phi)\end{Bmatrix} \\
&\cdot\left\{i\,c(\theta)\left[\overline{M}^{(0)}(h_1)\cdot\hat{\mathrm{a}}\right]\hat{\phi}+d(\theta)\left[\overline{N}^{(0)}(h_1)\cdot\hat{\mathrm{a}}\right]\hat{\theta}\right\}_{h_1=\sqrt{k_1^2-k_2^2\sin^2\theta}}, \\
&\qquad z<\min\{z_{sph}\}, \qquad \theta\in(\pi/2,\pi].
\end{aligned}
\tag{12}
$$

The asymptotic expression (12) is first projected on the unit vectors $\hat{\theta}$ and $\hat{\phi}$ of the spherical coordinate system, which asymptotic analysis has revealed as the primitive one. Projecting then functionally the resulted equations on azimuthal functions $\begin{Bmatrix}\cos(m\phi)\\ \sin(m\phi)\end{Bmatrix}$, $m=0,1,2,\ldots$ and subsequently multiplying with basis θ-functions and averaging over a dense partition of the interval $(\pi/2,\pi]$ we obtain, for every pair of indices (m,n) with $m=0,1,2,\ldots;n\geq m$, a set of four non-homogeneous algebraic equations. These equations along with those originated from the boundary condition satisfaction, i.e. Eqs. (8)-(11), constitute a block system of eight algebraic equations for every pair of parameters (m,n). This system is of infinite dimension since it includes infinite summations over the infinite unknowns and therefore a truncation procedure needs to be imposed for its numerical treatment. Indeed, we followed a truncation procedure in a systematic way, adapted to the inner structure of the aforementioned blocks to obtain the following matrix formulation of the truncated system

$$\mathbf{A}_k^{(m)}\,\mathbf{c}_k^{(m)}=\mathbf{d}_k^{(m)},\ k=1,2,3,\ldots, \tag{13}$$

$$\mathbf{A}_k^{(m)}=\begin{bmatrix} \mathbf{E}_{m,m,m} & \mathbf{E}_{m,m+1,m} & \mathbf{E}_{m,m+2,m} & \cdots & \mathbf{E}_{m,m+2k-1,m} \\ \mathbf{E}_{m,m,m+1} & \mathbf{E}_{m,m+1,m+1} & \mathbf{E}_{m,m+2,m+1} & \cdots & \mathbf{E}_{m,m+2k-1,m+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{E}_{m,m,m+k-1} & \mathbf{E}_{m,m+1,m+k-1} & \mathbf{E}_{m,m+2,m+k-1} & \cdots & \mathbf{E}_{m,m+2k-1,m+k-1} \end{bmatrix}, \tag{14}$$

$$\mathbf{c}_k^{(m)}=\left[\underline{c}_{m,m}^T \quad \underline{c}_{m,m+1}^T \quad \underline{c}_{m,m+2}^T \quad \cdots \quad \underline{c}_{m,m+2k-1}^T\right]^T, \tag{15}$$

$$\mathbf{d}_k^{(m)}=\left[\underline{d}_{m,m}^T \quad \underline{d}_{m,m+1}^T \quad \underline{d}_{m,m+2}^T \quad \cdots \quad \underline{d}_{m,m+k-1}^T\right]^T, \tag{16}$$

$$\mathbf{E}_{m,n',n}=\left[\underline{E}_{mn',n}^{(1)\ T},\ \underline{E}_{mn',n}^{(4)\ T},\ \underline{E}_{mn',n}^{(2)\ T},\ \underline{E}_{mn',n}^{(3)\ T},\ \underline{E}_{mn',n}^{(8)\ T},\ \underline{E}_{mn',n}^{(7)\ T},\ \underline{E}_{mn',n}^{(6)\ T},\ \underline{E}_{mn',n}^{(5)\ T}\right]^T, \tag{17}$$

$$\underline{E}_{mn',n}^{(i)}=\left[D_{mn',n}^{(i),A_e} \quad D_{mn',n}^{(i),A_o} \quad D_{mn',n}^{(i),B_e} \quad D_{mn',n}^{(i),B_o}\right], \quad i=1,2,\ldots 8, \tag{18}$$

$$D_{mn',n}^{(i),s}=\begin{Bmatrix} 0, & (i,s)=(1,A_o),(1,B_e),(4,A_o),(4,B_e) \\ 0, & (i,s)=(2,A_e),(2,B_o),(3,A_e),(3,B_o) \\ 0, & (i,s)=(5,B_o),(6,B_e),(7,A_o),(8,A_e) \end{Bmatrix}, \tag{19}$$

$$\underline{c}_{mn'}=\left[A_{e,mn'}^{(3)} \quad A_{o,mn'}^{(3)} \quad B_{e,mn'}^{(3)} \quad B_{o,mn'}^{(3)}\right]^T, \tag{20}$$

$$\underline{d}_{m,n}=\left[0 \quad 0 \quad 0 \quad 0 \quad d_{m,n}^{(8)} \quad d_{m,n}^{(7)} \quad d_{m,n}^{(6)} \quad d_{m,n}^{(5)}\right]^T. \tag{21}$$

In the above expressions, the notation $D_{mn',n}^{(i),s}; i=1,2,\ldots 8;\ s=(A_e,A_o,B_e,B_o)$ is used for the coefficients of the unknowns $A_{e,mn'}^{(3)}, A_{o,mn'}^{(3)}, B_{e,mn'}^{(3)}, B_{o,mn'}^{(3)}$ in each one of the eight equations consisting a specific block for concrete parameters m,n, while $d_{m,n}^{(i)}, i=1,2,\ldots 8$ denotes the right-hand sides of these equations.

6. DETERMINATION OF THE ELECTRIC FIELD IN SCATTERING REGION

Once the electric field $\overline{E}_2(\overline{r})$ is determined (through the numerical calculation of the spheroidal expansion coefficients) the determination of the electric field $\overline{E}_1(\overline{r})$ in the scattering region is accomplished through the integral representation (5). It is noticed first that the volume integral (in this representation constitutes the incident wave) is independent of the specific scatterer and usually is not the subject of the measurement process. In contrast, the surface integral stands for the scattered wave $\overline{E}_1^{sc}(\overline{r})$ and encodes all the information concerning the spheroidal scatterer. This surface integral is first fed with the known spectral spheroidal expansion of the electric field in the complementary region and the appropriate expression for the dyadic kernel involved and after that is being amenable to asymptotic analysis applying stationary phase arguments. This yields the following representation of the scattered field in spherical coordinates

$$\overline{E}_1^{sc}(\overline{r}) = -\frac{e^{ik_1 r}}{4\pi r \sin\theta\sqrt{k_2^2 - k_1^2 \sin^2\theta}} \cos\theta \sum_{m=0}^{\infty} (2-\delta_0)(-i)^{m+1} \begin{Bmatrix} \cos(m\phi) \\ \sin(m\phi) \end{Bmatrix}$$

$$\cdot \left\{ \begin{array}{l} i\, c'(\theta) \displaystyle\sum_{n'=m}^{\infty} \{ A_{\substack{e\\o}mn'}^{(3)} I'^{A}_{1,\substack{e\\o}(3)mn'}(-h_2,\lambda_1) + B_{\substack{e\\o}mn'}^{(3)} I'^{B}_{1,\substack{e\\o}(3)mn'}(-h_2,\lambda_1) \}\, \hat{\phi} \\ + d'(\theta) \displaystyle\sum_{n'=m}^{\infty} \{ A_{\substack{e\\o}mn'}^{(3)} I'^{A}_{2,\substack{e\\o}(3)mn'}(-h_2,\lambda_1) + B_{\substack{e\\o}mn'}^{(3)} I'^{B}_{2,\substack{e\\o}(3)mn'}(-h_2,\lambda_1) \}\, \hat{\theta} \end{array} \right\}_{\substack{\lambda_1 = k_1 \sin\theta \\ h_2 = \sqrt{k_2^2 - k_1^2 \sin^2\theta}}} \tag{22}$$

which is valid for $k_1 r >> 1$ and $\theta \in (0, \pi/2]$ and constitutes the final expression that renders feasible the determination of the electric field in the observation environment.

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