AN IDENTIFICATION METHOD FOR VECTOR PREISACH-TYPE MODELS OF HYSTERESIS

A. Ktena, D.I. Fotiadis, P.D. Spanos
And C.V. Massalas

24 – 2001

Preprint, no 24 – 01 / 2001

Department of Computer Science
University of Ioannina
45110 Ioannina, Greece

An identification method for vector Preisach-type models of hysteresis

A. Ktena[(1)], D. I. Fotiadis[(1)], P. D. Spanos[(2)] and C. V. Massalas[(3)]

[(1)] Dept. of Computer Science, University of Ioannina, GR 45110 Ioannina, Greece
[(2)] Dept. of Mechanical Engineering and Materials Science, George R. Brown School of Engineering, Rice University, Houston, USA
[(3)] Dept. of Materials Science, University of Ioannina, GR 45110 Ioannina, Greece

1. SUMMARY

In a hysteresis model following the Preisach formalism, the output sequence, $\mathbf{f}(t)$, is obtained by integrating the characteristic probability density function, $\rho(\alpha,\beta)$, of the elementary hysteresis operators, γ_{ab}, operating on the input sequence $\mathbf{u}(t)$ over the Preisach plane. The model can be one - or two - dimensional depending on the dimensionality of the hysteresis operator chosen. The vector version has been designed for perfectly oriented systems with uniaxial anisotropy under vector inputs. Angular dispersion of orientation axes is accounted for by superimposing the responses of angularly distributed perfectly oriented models. The identification method accompanying it is using data from a major hysteresis curve and a least-squares fitting procedure for the parameters of the characteristic density. Results using two different operators, the "sw-astroid" and the "diamond" are obtained and compare well will experimental hysteresis data on two different magnetic samples.

2. INTRODUCTION

The abstract formulation and speed of the resulting calculations have placed the Preisach formalism among the favorites in the modeling of the particularly complex phenomenon of hysteresis. Originally designed for ferromagnetic hysteresis, it has been extended to applications in shape memory alloys (sma), rocks, economics and other systems with hysteresis. Preisach models have successfully been used in magnetic recording simulations, electrical steel lamination loss calculations, finite element calculations and control of sma actuators – all problems involving lengthy calculations where the speed of the Preisach model based algorithms is much needed.

The inherently scalar nature of the classical model has drawn a considerable amount of criticism on the grounds that the one-dimensional treatment of a hysteresis process is not always valid and a lot of information is lost when modeling the one-dimensional projection of a vector process. However, the classical model whose properties have been extensively and exhaustively studied remains a very popular and efficient model [1]. To address the issue of the modeling of vector hysteresis, vector extensions of the original formalism have been developed [2]. The vector formulations divert from the original one and so do their properties. The higher complexity of the vector models complicates the identification process as well. In the scalar case, the identification of the model can be carried out through detailed measurements of the characteristic density of the system being modeled as outlined by Everett [3]. It is not obvious how this method can be extended to the vector case, so alternative methods are being considered. One such approach is based on a major loop measurement and consists in determining the parameters of the probability density function chosen to model the system in consideration [4-6].

In the following section, we discuss the building blocks of the model: two vector hysteresis operators and their features and the vector extension of the original formalism. An identification procedure using major loop data and a least-squares algorithm that optimizes the density parameters for the given data [6] is described in section 3. In order to test the identification method in conjunction with the vector model, results using the two vector operators are presented in section 4. The results are compared with experimental data from two ferromagnetic samples.

2. MODELING OF HYSTERESIS

The definition of hysteresis adopted in this work and used to describe the systems to which Preisach-type modeling applies is that of hysteresis as *rate independent memory effect* [7]. According to this definition, in a system with hysteresis the current output is a function of the current input as well as previous inputs and/or the initial state. The system can store information, it has *memory*. For every input there may be more than one equilibrium states. The resulting state depends on the history of the system, on the previous equilibrium states. When a system with hysteresis is *bistable* is characterized by a hysteresis loop like the one shown in Fig. 1. The curve is traced along the path ABC (descending branch) or CDA (ascending branch). For $u \geq u_{s+}$ (or $u \leq u_{s-}$), the output is increasing (or decreasing) monotonically with the input, hysteresis vanishes, the processes are reversible and the resulting states are uniquely defined and *stable*. For $u_{s-} < u(t) < u_{s+}$, $f(t)$ is a *metastable* state and a nonlinear function of previous states [13]. If the loop ABCDA delimits the space of all possible states for any given input, it is called a *major loop*. A point inside the major loop can be attained through several trajectories called *minor loops*. Because hysteresis is modeled as a rate-independenet phenomenon, the rate of change of the input in the systems under consideration must be slow enough to allow for any transients to die out so that the quasistatic treatment applies.

2. 1 The Preisach formalism

According to the Preisach formalism, hysteresis is the result of superposition of scalar local hysteresis operators γ_{ab} (Fig. 2b). The system being modeled is viewed as a collection of subcomponents each of which has a hysteresis characteristic γ_{ab} with different switching points (a,b). The displacement of the loop from the origin, $u_d = \frac{a+b}{2}$, corresponds to the effective interactions experienced by a given component. If the subcomponents are isolated or the sum of interactions one of them experiences is zero the corresponding loop is centered at the origin and $a = -b$. The loop halfwidth, or half distance between the two critical values is $u_c = \frac{a-b}{2}$.

The system is modeled as a distribution of upper and lower switching points (a,b) obtained from the characteristic density of the system $\rho(a,b)$ defined over the Preisach plane (Fig. 2a), bounded by $u_c = 0$, $u = u_{s+}$, and $u = u_{s-}$, where u_{s+} and u_{s-} are the input values leading to positive and negative saturation respectively: $\forall a,b \quad a \le u_{s+}, b \ge u_{s-}$. The response of the system, $f(t)$, to an input, $u(t)$, is the integral of the output states of each elementary loop weighed by the probability density function $\rho(a,b)$:

$$f(t) = \iint_{a \ge b} \rho(a,b)\gamma_{ab}u(t)\,da\,db\,. \tag{1}$$

Since the model is quasistatic, time is discretized and an input sequence $u_0, u_1, ..., u_n$ is assumed instead of a continuous input time function. When an input $u_0 > u_{s+}$ is applied, the system "saturates" in the positive state where all the operators are in the +1-state. Decreasing the input to $u_{s-} < u_1 < u_{s+}$ all operators with $u_{s+} > b > u_1$ will switch to -1. A horizontal boundary separating the regions of +1- and -1- states is established at $b = u_1$ and the change in output, $\Delta f = f_1 - f_0$, is obtained by integrating the density over the triangle ABC. Decreasing the input to $u_{s-} < u_2 < u_1$, all operators with $b > u_2$ will revert to -1 and a perpendicular boundary segment appears (Fig. 2a). The change in output is then given by the integral over the triangle CDE. This way, at the end of an input sequence a *staircase boundary* is established between areas of positive and negative state. The horizontal and vertical segments, a direct consequence of the discontinuity of the operator at the switching points, clearly indicate the past input extrema. Therefore, the boundary serves as *memory* keeping track of the history of the system. The integrals of the density over the triangular area of change are called *Everett functions* [3] and can be used for the identification and the *inverse* model.

The classical model, due to the operator $\gamma_{\alpha\beta}$ is able to model irreversible processes (switching) only and yields congruent loops [1]. There are systems, like the ferromagnets, where reversible processes (rotations) take place and non-congruent lops are observed. A vector extension of the Preisach model is therefore needed.

2.2 The two-dimensional model

To model vector hysteresis, the model must be able to respond to vector inputs and allow for reversible as well as irreversible rotations of the output vector. This can be achieved by substituting the scalar switch $\gamma_{\alpha\beta}$ by a vector operator:

$$f(t) = \iint\limits_{a \geq b} \rho(a,b)\gamma_{ab}\boldsymbol{u}(t)\,da\,db . \tag{2}$$

2.2.1 Vector hysteresis operators

The vector operators used in this work are shown in Fig. 3. The sw-astroid [8] (Fig. 3a) is the locus of the equation $u_e^{2/3} + u_h^{2/3} = 1$, where u_e and u_h are the components of the input u(t) along the easy and hard axis respectively. The solution is the tangent to the astroid passing from the tip of the input vector. Switching occurs only when the output vector crosses the astroid from the inside out. Otherwise the output vector rotates reversibly. The astroid equation results from the minimization of the free (Gibbs) energy equation for an ellipsoidal magnetic particle with *uniaxial anisotropy* under the influence of an applied field. $u_x \tan\phi - u_y + \sin\phi = 0$. The solution φ is the angle of the output vector with respect to the easy axis of the astroid. Because the solution of a transcendental equation can be quite time consuming, the transformation $k = \tan\frac{\phi}{2}$ is used and the roots of the equation: $u_y k^4 + 2(u_x - 1)k^3 + 2(u_x + 1)u - u_y = 0$ are used.

The second vector operator, the diamond (Fig. 3b), is the first order approximation of the sw-astroid: $u_e + u_h = 1$. It is computationally more efficient but without physical attributes.

In order to demonstrate the differences between the two operators the following experiment is performed. Inputs of constant magnitude are rotated 360° and the output is calculated in both cases (Fig. 4). For very small inputs, the response is practically identical. As the input increases but not enough to cause switching, the sw-astroid rotates harder than the diamond. For large inputs, the sw-astroid allows for more switching.

2.2.2 The model

Because the vector operators assume a system with uniaxial anisotropy, the vector model described by Eq. (1) is a model for *perfectly oriented* systems with uniaxial anisotropy. Where needed, *dispersion* of orientations can be added by superimposing the responses of angularly distributed perfectly oriented models:

$$f(t) = \int_{-\pi/2}^{\pi/2} \rho(\phi)\,d\phi \iint\limits_{a \geq b} \rho(a,b)\gamma_{ab}\boldsymbol{u}(t)\,da\,db , \tag{3}$$

where $\rho(\varphi)$ is a probability density function of angles.

The vector model does not possess the congruency property (Fig. 5), as expected, since it allows for rotations and, according to Mayergoyz' theorem [1], it is not a Preisach model. Therefore, we shall call it a Preisach-type model. The vector properties of this model are very good and in agreement with experiments [2,4]. A theoretical experiment testing the vector properties of the model is shown in Fig. 6. The transverse component of the output is plotted against the longitudinal component for a major loop input sequence applied at 0°, 45° and 90° to the main easy axis of the model described by Eq. (2). The model using the sw-astroid allows for more switching as expected. Fig. 7 shows major loops obtained by the classical model in Eq. (1), by the perfectly oriented model in Eq. (2), and by the model with dispersion in (3) using both operators. The scalar and perfectly oriented models yield identical results for fields along the main easy axis, as expected. Adding dispersion makes switching easier, which is in agreement with experimental evidence, and accounts for the reversible behaviour and the slope of the curve near saturation.

3 IDENTIFICATION

Because in the Preisach-type models the mathematical tools developed for the Preisach model [1,3] no longer apply, the characteristic density $\rho(a,b)$ cannot be directly measured with the help of Everett functions. The alternative approach is to fit the parameters of a known probability density functions (pdf) to some points on a major hysteresis curve. This method is more appropriate for use with a general application model because it is not restricted by the type of material or system, the 1D or 2D treatment of the problem and the ability to measure the Everett functions.

The bivariate probability density function of upper and lower switching points can be equivalently expressed in terms of the transformation $u'_c = \frac{a-b}{\sqrt{2}}$ and $u'_d = \frac{a+b}{\sqrt{2}}$: $\rho(a,b) = \rho(u'_c, u'_d)$ (Fig. 2).

There are systems, like ferromagnets, supporting the assumption that the variables u'_c and u'_d are independent and therefore $\rho(a,b) = \rho(u'_c, u'_d) = \rho(u'_c)\rho(u'_d)$. Then, if σ_a^2, σ_b^2 are the variances of a and b, it can be shown that, $\sigma_a^2 = \sigma_b^2$ and symmetrical loops are obtained. It can also be shown that for loops centered at the origin, $\mu_a = \mu_b$ must hold, where μ_a, μ_b are the mean values of a and b. This implies that the mean interaction $\mu_d = 0$ which is a valid assumption in ferromagnets.

In [5], the density was constructed as a product of the two independent variables, u_c, u_d: $\rho(a,b) = \rho(u'_c, u'_d) = \rho(u'_c)\rho(u'_d)$. The four parameters μ_c, σ_c, μ_d, σ_d μ_c were determined according to some empirical rules outlined in [5]. Finding appropriate values for σ_c and σ_d was the most difficult part in this process being more of an art rather than a science.

The need to come up with a more systematic identification method applicable to as many classes of systems as possible pointed to the direction of using a bivariate probabilty density function as a basis for the Preisach distribution and apply a least-squares curve - fitting procedure. The method used in [5] will be hereafter called "old" and the one presented here "new". In either method, a pdf of angles is involved to account for dispersion. It is a gaussian centered at 0° with its standard deviation being the sole parameter controlling the squareness $S = \frac{f\,|_{u=0}}{f_{max}}$ of the loop.

The obvious bivariate pdf to use with the "new" method is the normal one with five parameters to be determined: $\mu_a, \mu_b, \sigma_a, \sigma_b$ and the correlation parameter r between a and b. An array of i points of the experimental loop is fed to the least squares algorithm along with an array of initial estimates of the parameters and the algorithm iterates on the parameter values until $\sum_i (f_{mod} - f_{exp})^2 < \varepsilon$, where ε is a small positive number.

4 RESULTS AND DISCUSSION

The model in Eq. (3) and the identification method described is tested using experimental data from a homogeneous SmFeN and an inhomogeneous α-Fe/SmFeN sample obtained from the literature [8]. In ferromagnets, hysteresis occurs during the switch from positive to negative magnetization. For an applied magnetic field (input) $H(t)$, the resulting magnetization (output) is a function of the applied field as well as an internal interaction field which is in turn a function of the magnetization $M(t)$. Hence the resulting magnetization state contains a positive feedback mechanism leading to hysteresis: $M(t) = M(H(t), M(t))$. Ferromagnets, well-known for their energy and information storage capabilities, are a typical example of the type of hysteresis described by (4). Furthermore, hysteresis in a ferromagnet is a vector process and a vector model is appropriate.

In the results which follow the output is normalized with respect to the maximum value attained experimentally, or to saturation magnetization, and ranges from -1 to +1. The Preisach plane is coded as a $K \times K$ array. K depends on the desired degree of discretization and the maximum experimental input values observed. Each element of the array holds a hysteresis operator and the height (weight) of the density at the given point. The input is operated on by γ_{ab} at each array element, its output state is decided and then multiplied by the weight. Summing over the weighed outputs of each element yields the aggregate output for a given input.

In the case of ferromagnets, because of the symmetry of the loops, $\mu_a = \mu_b, \sigma_a = \sigma_b$. Also, it turns out that the correlation parameter r has a slight effect on the shape of the loop; high positive values yield slightly higher loop squareness. So r is, generally, taken to be 0. The characteristic density parameters obtained through the optimization are summarized in Table I. The resulting calculated major loops for the two samples are shown in Figs. 8-9. Four curves are displayed in each figure: the experimental curve (exp), the curve obtained using the sw-astroid operator and the old identification method (old/vector), the curve obtained using the scalar operator and the new identification method (new/scalar), and the curve obtained using the diamond operator and the new identification method (new/vector). The importance of using a vector model is obvious in both cases, especially for the points near saturation. The improvement due to the new identification method is important in both cases, while in the case of the inhomogeneous sample (Fig. 9) the fitting is excellent. For the generation of the curves labeled 'vector/new' the vector model with the diamond operator was used. The results obtained with the sw-astroid operator were either not as good (the homogeneous sample) or equally good (inhomogeneous sample) in which case the computationally faster "diamond" was preferred.

Table 1: The density parameters used in the identification of the two samples

	SmFeN		α-Fe/SmFeN	
op	diamond	sw-astroid	diamond	sw-atroid
K	80	80	50	70
μ	58.2282	61.1324	30.2099	40.8309
σ	2.2383	4.0074	2.8029	2.8716

5 CONCLUDING REMARKS

A Preisach-type 2D vector model for anisotropic media has been used to reproduce the hysteresis major loops in two ferromagnetic samples. Two vector operators, the sw-astroid and the diamond, have been used and their performance has been tested and compared. The identification method used to determine the characteristic density parameters is based on a least-squares fitting procedure using data from an experimental major loop curve. The results demonstrate significant improvement over results obtained with an older identification procedure and the necessity of vector modeling in the case of materials like ferromagnets where hysteresis is a vector process.

Work in progress involves the testing and development of more operators, scalar and vector, and the refinement of the identification procedure. Other bivariate densities as well as products of single-variable densities are being tested and the possibility of using vector data for the identification is being considered.

References

1. I. D. Mayergoyz, Mathematical models of hysteresis, Physical Review Letters, 56(15), 1518-1521 (1986).
2. Stanley H. Charap and Aphrodite Ktena, Vector Preisach modeling, J. Appl. Phys., 73, 5818-5823 (1993).
3. D. H. Everett, A general approach to hysteresis, Trans. Faraday. Soc., 51, 1551-1557 (1955).
4. Aphrodite Ktena and Stanley H. Charap, Vector Preisach Modeling and Recording Applications, IEEE Trans. Magn., 29 (6), 3661-3663 (1993).
5. A. Ktena, D.I. Fotiadis and C.V. Massalas, IEEE Trans. Mag., New 2-D model for inhomogeneous permanent magnets, 36(6) (2000).
6. A. Ktena, D.I. Fotiadis, P.D. Spanos, C.V. Massalas, A Preisach model identification procedure and simulation of hysteresis in ferromagnets and shape-memory alloys, Phys. B, *in press*.
7. E. C. Stoner and E. P. Wohlfarth, *A mechanism of magnetic hysteresis in heterogeneous alloys*, Phil. Trans. Roy. Soc., A240 (1948), pp. 599-642.
8. E. H. Feutrill, P. G. McCormick and R. Street, Magnetization behavior in exchange-coupled $Sm_2Fe_{14}Ga_3C_2/\alpha$-Fe, J.Phys.D: Appl. Phys. 29 (1996), pp. 2320-2326.

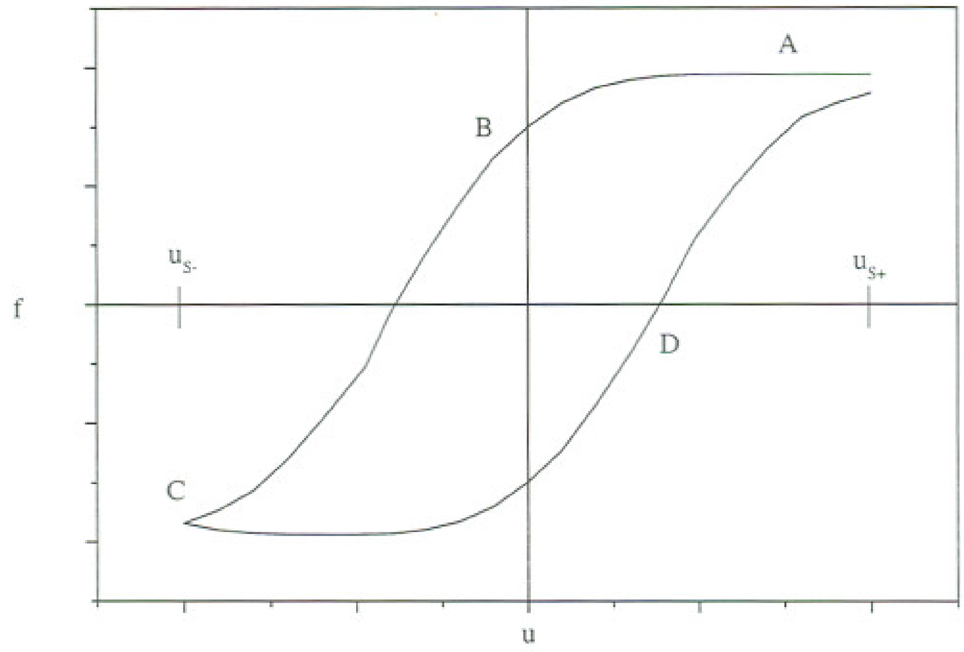

Figure 1: A typical hysteresis loop traced along the path ABCDA.

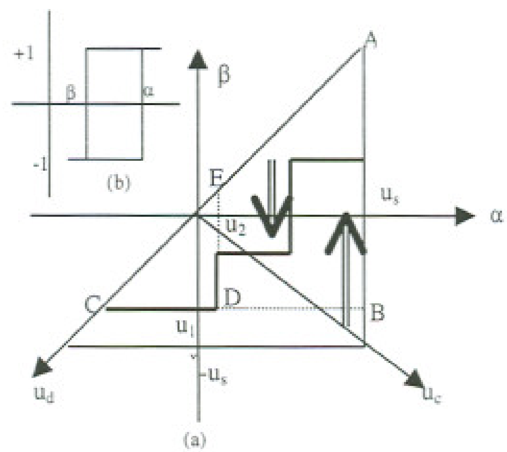

Figure 2: (a) The Preisach plane with the staircase boundary and (b) the scalar operator.

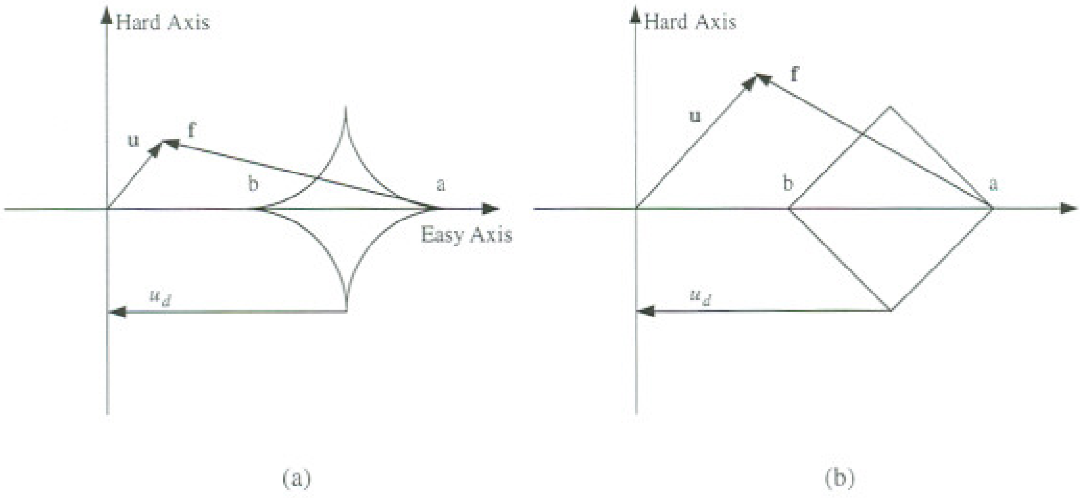

Figure 3: Vector operators: (a) the S-W astroid and (b) the diamond.

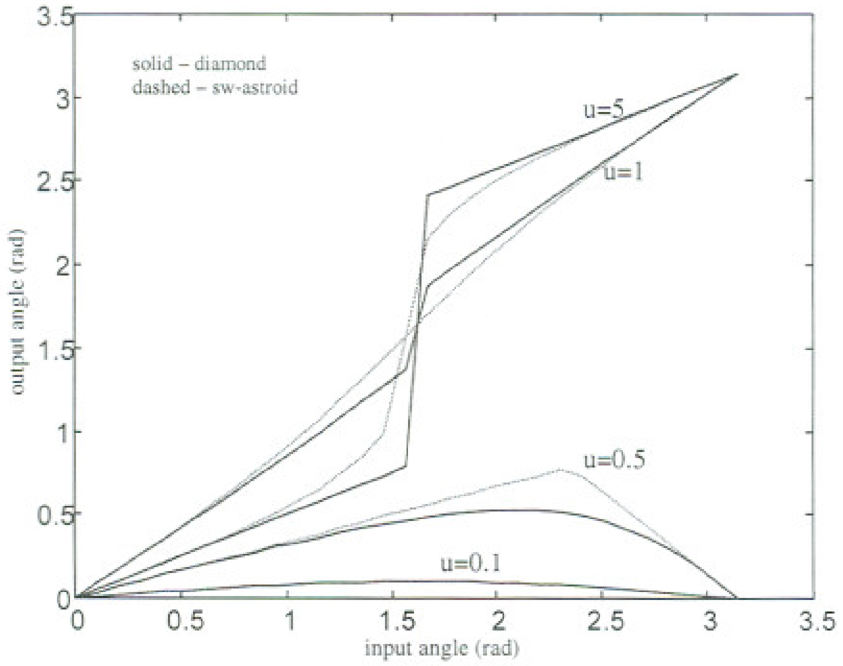

Figure 4: Comparison between the diamond and the sw-astroid hysteresis operators.

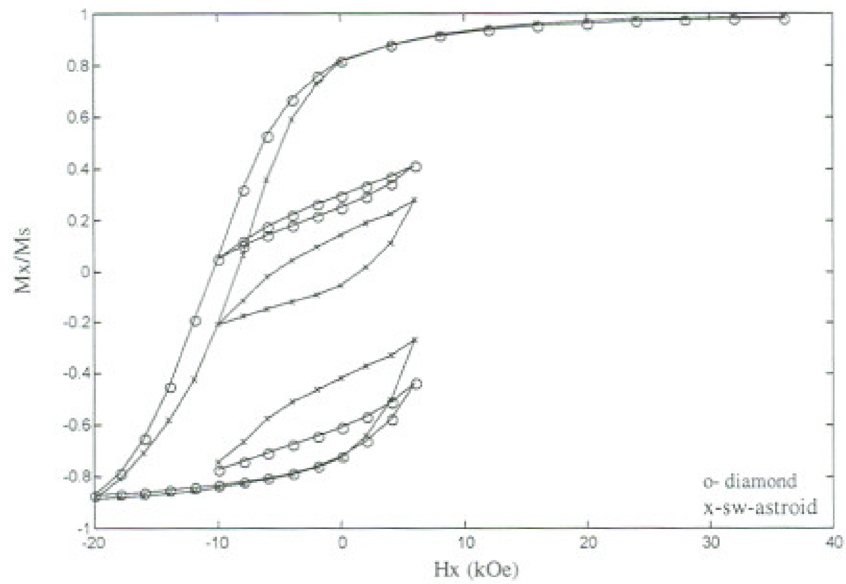

Figure 5: Calcualted non-congruent loop

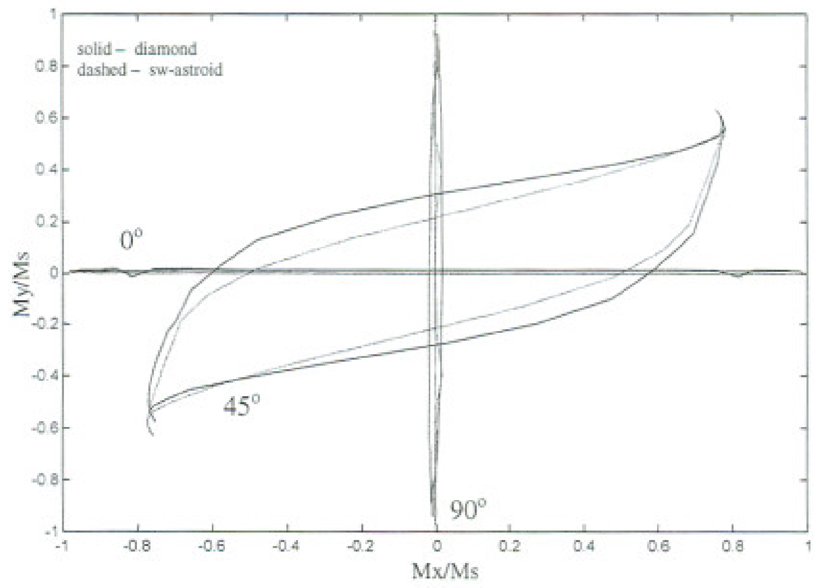

Figure 6: The transverse vs the longitudinal component of the magnetization for a major loop field sequence applied at 0, 45 and 90 degrees to the main easy axis

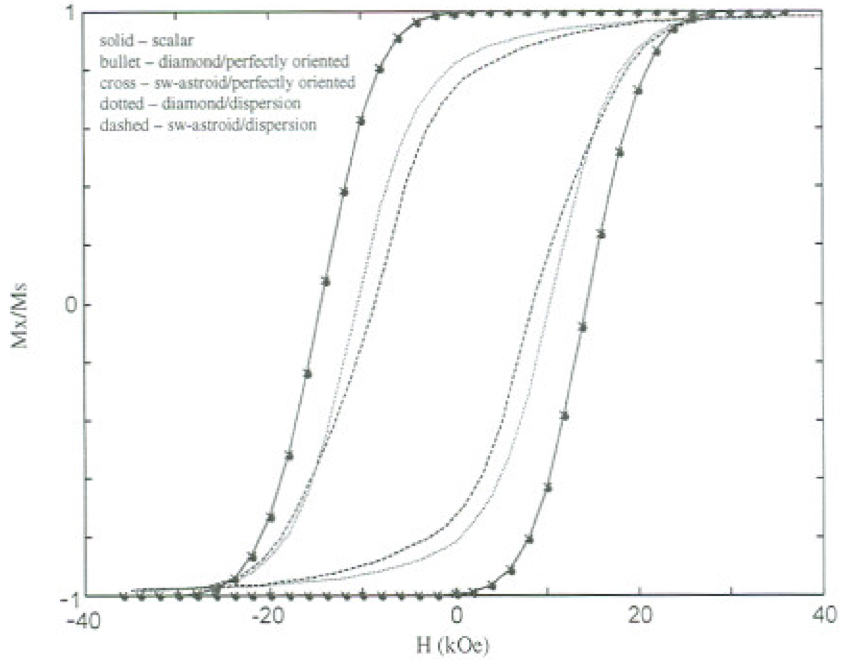

Figure 7 - Calculated major loops using three different operators – The effect of dispersion

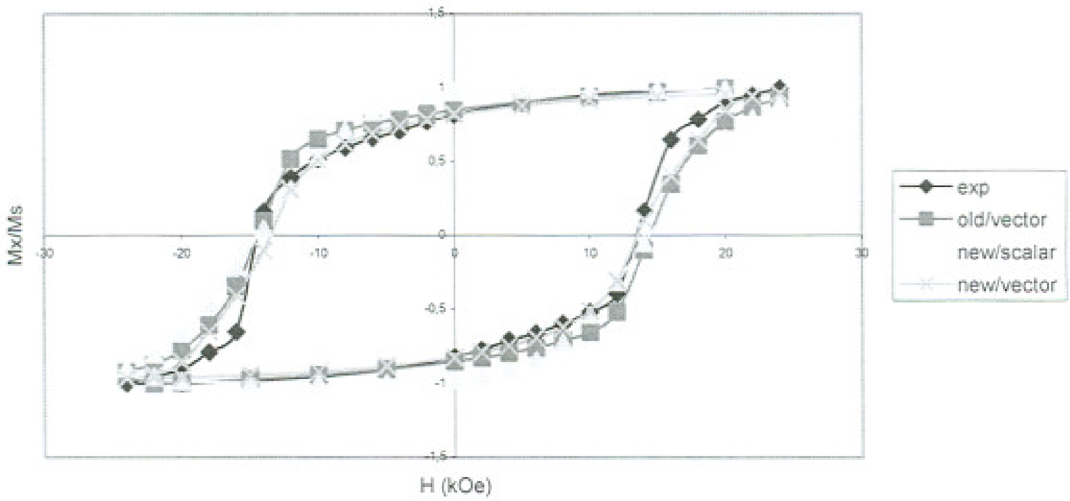

Figure 8: SmFeN experimental and calculated curves

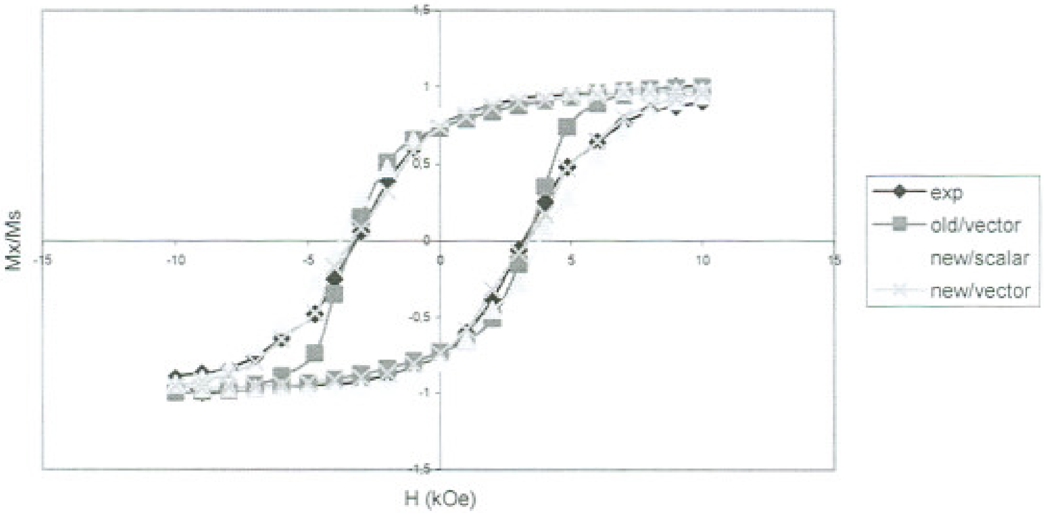

Figure 9: SmFeN/α-Fe experimental and calculated curves