

**FORBIDDEN STATES AND THE THREE-BODY
BOUND STATE COLLAPSE**

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Forbidden States and the Three-Body Bound State Collapse.

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Abstract

The appearance of bound states with large binding energies of several hundred MeV's in the three-body system, known as bound state collapse, is investigated. For this purpose three classes of two body-potentials are employed, namely, local potentials equivalent to nonlocal interactions possessing a continuum bound state, in addition to the usual negative-energy bound state, local potentials with a strong attractive well sustaining a forbidden state, and supersymmetric transformation potentials. It is firstly shown that the local potentials equivalent to the above nonlocal ones have a strong attractive well in the interior region which supports, in addition to the physical deuteron state, a second bound state (usually called pseudo bound state) with large binding energy, which is responsible for the bound state collapse in the three-body (and in general to the N-body) system. Secondly, it is shown that local potentials with a forbidden state also generate a three-body bound state collapse, implying that the role played by the forbidden state is similar to the

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one played by the pseudo bound state. Finally, it is shown that the removal of the forbidden state via supersymmetric transformations results also in the disappearance of the collapse. Thus, one can safely argue that the presence of unphysical bound states with large binding energies in the two-body system is responsible for the bound state collapse in the three-body system.

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I. INTRODUCTION

The three-body bound state collapse (BSC) [1-5], *i.e.* the appearance of bound states with large binding energies in the three-body system, has been the subject of several studies in the past. It was found, long ago, that the nonlocal separable potential of rank-1 of Tabakin [1] generates a large binding energy for the three-body system [2]. This came as a surprise as this potential predicts fairly well the two-body data and, so, there was no apparent reason why such an unphysical bound state with large binding energy should appear in the three-body system.

Subsequent studies with rank-2 separable potentials [3,4] have shown that the BSC could be related to two-body continuum bound states (CBS), *i.e.* to the existence of an S-matrix pole on the real positive energy-axis. This was the case for separable potentials while for purely local potentials or superposition of local and nonlocal potentials the collapse was not observed. Some aspects of the BSC for the Tabakin potential were also studied by Rupp *et al.* [5]. In that work a rank-1 potential, similar to the Tabakin potential, was constructed and used to calculate the three-body binding energy as a function of the nonlocal parameter β , *i.e.* the inverse of the nonlocality range of the potential. Several radial S-wave functions for different values of β were constructed and compared with the deuteron wave function

of the Graz-II potential [6]. The comparison revealed that, in contrast to the deuteron wave function of the Graz-II potential, these bound state wave functions had a node at short distances which moves outwards with increasing β , while the corresponding three-body binding energies increased eventually leading to a collapse of the three-body system. This was an indication that the BSC is related to the range of the nonlocality, at least for this kind of potentials.

Almost a decade later, Delfino *et al.* [7,8] were able to show that for rank-1 separable potentials with one-term form factor of Yamaguchi type, the BSC could essentially be linked to the Thomas effect [9], *i.e.* to the drastic increase of the three-particle binding energy as the range of the two-particle potential tends to zero. Thus, these authors were able to establish an equivalence between the Thomas effect and the phenomenon of the collapse by means of the range of the potential. In contrast, when the form factor is a sum of Yamaguchi terms and the nonlocality parameter could not be taken as a measure of the range of the potential, Delfino *et al.* [8] noticed that if this type of potentials supports a CBS, in addition to the physical deuteron state, the CBS wave-function was identical to the wave function of a negative energy bound state, the so called pseudo-bound state, which was responsible for the collapse. An important suggestion of that work was that one should expect similar results for all potentials which support at least another bound state in addition to the physical deuteron state. This bound state is usually called, in the Resonating Group Model (RGM), Pauli forbidden state (PFS) [10].

In the presence of a PFS the physical deuteron state becomes an excited bound state and its wave-function has a node. The relation between that node and the three-body binding energy, has been investigated by Nakaichi-Maeda [11] who employed the Kukulin nucleon-nucleon (NN) potential [12]. This interaction, in addition to the physical deuteron state, sustains also a PFS which for the triplet channel is ~ 58 MeV, for the singlet is ~ 440 MeV, while for the couple channel system is even deeper, ~ 525 MeV [13]. Thus, for this potential, the physical deuteron state is an excited bound state and the corresponding wave function has a node. By varying the magnitude of the inner amplitude of the wave function and the

position of the node, Nakaichi–Maeda confirmed that the BSC is connected to the nodal behavior of the deuteron wave function.

The purpose of this paper is threefold. Firstly, to take further the findings of Delfino *et al.* [7,8] and show that they can be generalized to any kind of two-particle potential which has a bound state with large binding energy in addition to the physical deuteron state. By looking at the nodal behavior of the wave functions we will also confirm, in a more rigorous way, the results of Nakaichi–Maeda [11]. Secondly, by constructing a local potential sustaining at least one forbidden state we will show that it can cause a BSC in the three-body system. Finally, we will remove the forbidden state via supersymmetric (SUSY) transformations [14,15], and show that this results in the disappearance of the BSC.

The paper is organized as follows: In Sect. II, we present a short description of nonlocal potentials of rank-1 sustaining a CBS, we construct their phase equivalent local interactions, and use them to obtain the trinucleon binding energy. The relevance of forbidden states of purely local interactions to the BSC are discussed in Sect. III. In Sect. IV we briefly present the supersymmetric transformations used to remove the forbidden states. Finally, in Sect. V we discuss our results and draw our conclusions.

II. NONLOCAL INTERACTIONS

A. Short Review

For convenience, let us recall, briefly, the relevant formulas describing a two-particle system in an S-state. In our investigations we shall use the nonlocal potentials of Table I of Ref. [8] which are rank-1 separable interactions,

$$V(p, q) = \lambda g(p)g(q), \quad (1)$$

with a form factor consisting of a sum of Yamaguchi terms,

$$g(k) = \frac{\alpha_1}{k^2 + \beta_1^2} + \frac{\alpha_2}{k^2 + \beta_2^2}. \quad (2)$$

The corresponding two particle t-matrix at a given energy $E = k^2$ ($\hbar^2/2\mu = 1$), is given by

$$t(p, q; k^2) = g(p) \frac{\lambda}{D(k^2)} g(q) \quad (3)$$

with

$$D(k^2) = 1 - \frac{2\lambda}{\pi} \int_0^\infty \frac{g^2(p)p^2 dp}{k^2 - p^2 + i\epsilon}. \quad (4)$$

If the system has a bound state at $E_b = -\gamma^2$ then from Eq. (4) we obtain

$$D(\gamma^2) = 1 + \lambda \sum_{i,j=1}^2 \frac{\alpha_i \alpha_j}{(\gamma + \beta_i)(\gamma + \beta_j)(\beta_i + \beta_j)}. \quad (5)$$

The corresponding bound state wave function is given by

$$\Phi_b(p) = -N \frac{g(p)}{(\gamma^2 + p^2)} \quad (6)$$

where N is the normalization constant.

The potentials of Table I support a CBS at a positive energy $E_c = p_c^2$ if

$$g(p_c) = 0 \quad \text{and} \quad D(p_c^2) = 0. \quad (7)$$

From Eqs. (2) and (4) and the conditions (7) one gets for a CBS wave function,

$$\Phi_c(p) = -N \frac{(\alpha_1 + \alpha_2)}{(p^2 + \beta_1^2)(p^2 + \beta_2^2)}. \quad (8)$$

Two important aspects concerning this wave function were already noted in Ref. [8], namely, that it does not depend directly on the energy p_c^2 and that at large distances it decays exponentially as $\exp(-\beta_i r)$ and not as $\exp(-p_c r)$. It was further noted that it reduces to the bound state (6) if one identifies the binding energy γ^2 with β_i^2 and the form factor with $(p^2 + \beta_j^2)^{-1}$, where $\beta_i(\beta_j)$ is the smaller (greater) of β_1 and β_2 . In other words, the CBS wave function has the behavior of a normal negative energy bound state wave function and this pseudo-bound state is responsible for the appearance of an extra bound state in the three-body system.

It is noted that for this type of nonlocal potentials the parameter β_i cannot be taken as a measure of the range of the potential. However, the range of the nonlocality can be deduced by constructing equivalent local interactions in coordinate space which we shall discuss next.

B. Equivalent Local Interactions

There are many ways to construct local interactions equivalent to nonlocal ones. A particular localization method which is well suited for our investigation is the one based on two linear independent solutions of the Schrödinger equation. The method is outlined in Refs. [16–18] and we refer to these works for more details. This type of equivalent local potential (ELP) (sometimes called quantal or Wronskian ELP) depends on energy. Since we are interested in the shape and range of these potentials as well as in their strength for interpretation purposes, it suffices to construct them at some fixed energy. Another way, is of course, to construct ELP's via inverse scattering techniques in order to generate ℓ -dependent but energy independent interactions [19–21]. However, this would have led to unnecessary complications without gaining more insight into what we are trying to do.

The ELP's to the nonlocal interactions of Table I are shown in Fig. 1 for $k = 2.0 \text{ fm}^{-1}$. It is seen that they are similar in shape and have an attractive strength at short distances which is quite large compared with the strength of a typical NN interaction. Another aspect of these potentials should be noted, namely the existence of a hump, which suggests a repulsion in the interaction region and thus resonances may also appear [21]. Such a hump is characteristic of local potentials equivalent to nonlocal interactions which fit the two-nucleon scattering data at high energies [11,17,22]. The striking similarity of the n - α local potentials equivalent to nonlocal potentials and of the corresponding two-body bound state wave functions presented in Ref. [21] with those of the present work is worth mentioning. Looking now at the three-body binding energies generated by the nonlocal potentials of Table I, we note that the shorter the range of the potential the larger is the three-body binding energy. This is in agreement with the findings of Delfino *et al.* [7] obtained with rank-1 nonlocal potentials with one-term Yamaguchi form factor and their results were shown to be related to the Thomas effect. The present results suggest that this relation is also valid for potentials with a form factor consisting of a sum of Yamaguchi terms and therefore it is a more general statement.

In Fig. 2 one of the ELP's employed, potential 1, is shown for different momenta. It is seen that the main characteristics of the potential do not change significantly and thus the previous results by Rupp *et al.* [5] obtained with an equivalent local potential to that of Tabakin are corroborated.

The wave functions of the excited states, *i.e.* the physical two-body states, are shown in Fig. 3. It is seen that there is a node in the interior region which moves to shorter distances as the energy of the ground state increases. This is not unexpected as the attractive well of the potential is shifted in the interior region and assumes the characteristics of a δ -function (hence the relation to the Thomas effect, see Fig. 2) and therefore the position of the node of the excited state is also shifted closer to zero. These results are in qualitative agreement with those of Nakaichi-Maeda [11]. In other words the appearance of a bound state with a large binding energy in the three-body system (collapse), is related to the nodal behavior of the physical two-body wave function.

III. LOCAL INTERACTIONS

There are many local potential models which determine fairly well the deuteron properties. Since our main concern is the three-body bound state collapse, it is sufficient to choose out of this multitude a simple local one which reproduces the binding energy of the deuteron. The aforementioned NN potential of Kukulín and collaborators [12] is best suited for our investigations. This potential has a deep attractive well at short distances which results from a six-quark model in the interior region and generates a PFS with a large binding energy. This implies that the corresponding physical two-body bound state wave function has an inner node which simulates the repulsive core of the traditional NN potentials. The form of this potential is

$$V(r) = V_0 \exp(-\alpha r^2) + V_1(1 - \exp(-\beta r)) \frac{\exp(-\mu r)}{\mu r}. \quad (9)$$

By varying its parameters one can move the forbidden state above or below the physical deuteron state which we keep fixed at $E_b = 2.225$ MeV. Thus the state corresponding to E_b

can be an excited or a ground state of the two-body system and thus it may or may not have a node.

Several sets of parameters were used which give rise to different shapes, ranges, and depths of the potential. Four characteristic examples are presented in Table II together with the resulting two- and three-body binding energies. These potentials are plotted in Fig. 4. Potential 1 is much more attractive and supports a forbidden state at $E_g = 77.513$ MeV. As compared to the other three potentials, potential 2 has a different shape and a much longer range, the forbidden state being at $E_g = 17.486$ MeV. Potential 3 sustains only the physical deuteron state while potential 4 supports in addition to it an excited bound state at $E_e = 0.074$ MeV. It is noted that potentials 3 and 4 are similar to soft core NN potentials and fit only the binding energy but not the scattering phase shifts. Therefore, with these examples we can pinpoint which property of the potential is most important for the collapse.

In order to calculate the three-body binding energy, we utilize the Faddeev formalism. For this we transform the potentials, Eq. (9), in momentum space and the potential matrix,

$$V(p, q) = \frac{V_0}{4} \sqrt{\pi} \alpha \left[\exp\left(-\frac{(p-q)^2}{4\alpha}\right) - \exp\left(-\frac{(p+q)^2}{4\alpha}\right) \right] + \frac{V_1}{4\mu} \ln \left[\frac{(\mu^2 + (p+q)^2) ((\mu + \beta)^2 + (p-q)^2)}{(\mu^2 + (p+q)^2) ((\mu + \beta)^2 + (p+q)^2)} \right], \quad (10)$$

is then used to obtain the two-body t -matrix needed in the Faddeev equation for the bound states. For three bosons in an S -state one has [23]

$$\psi(p, q) = \frac{8}{\pi q \sqrt{3}} \int_0^\infty q' dq' \int_{|2q-q'|/\sqrt{3}}^{(2q+q')/\sqrt{3}} p' dp' \frac{t(p, (p'^2 + q'^2 - q^2)^{1/2}; s - q^2) \psi(p', q')}{s - p'^2 - q'^2} \quad (11)$$

where

$$t(p, k; z) = V(p, k) - \frac{2}{\pi} \int_0^\infty \frac{V(p, k') t(k', k; z) k'^2 dk'}{k'^2 - z}. \quad (12)$$

Due to the variable limits of p' it is impractical to solve Eq. (11) by converting it to a matrix form and then apply the usual eigenanalysis techniques. Furthermore, the form of our potentials (deep attractive well) requires special attention and care. Thus the method of successive iterations has been employed [24] and the results obtained were reasonably stable.

We point out here that the two-body ground state energies for potentials 1 and 2 are larger than the binding energy of the deuteron. In contrast, the two-body ground state energy for potential 3 is fixed at 2.225 MeV while for potential 4 at 2.282 MeV for reasons which we shall explain later. The corresponding wave functions for the two-body ground states are shown in Fig. 5 while those for the excited states in Fig. 6.

We have searched for three-body binding energies in the region of $(-1000, 0)$ MeV. The results are presented in Table II. For potentials 1, 2, and 4 we located 2 three-body bound states corresponding to the 2 two-body bound states. The three-body binding energy is at its maximum value at $E_t = 163.85$ MeV for potential 1 and decreases to $E_t = 38.35$ MeV for potential 2 while there is no collapse for potential 3 which supports only the physical deuteron state. Comparing these results with the range of the potentials shown in Fig. 4 we see that the three-body ground state energy is larger when the range of the potential is smaller – an indirect manifestation of the relation between the BSC and the Thomas effect. Looking now at the nodal behavior of the wave functions in Fig. 6, we note that the collapse is more enhanced when the node is in the interior region. This is in line with the results found in the previous section.

In order to get an excited bound state above the physical deuteron state, we slightly increased, for potential 4, the two-body ground state energy to $E_b = 2.822$ MeV and obtained $E_e = 0.074$ MeV. The corresponding three-body binding energies are 0.308 MeV and 5.96 MeV *i.e.*, no collapse was detected in this case. It seems that one has a contradiction here with the results of the nonlocal potentials which show a collapse also for potentials fulfilling the condition (7). As was already noticed by Delfino *et al.* [7,8], however, for this type of potentials the two-body pseudo-bound state has the same behavior as the physical bound state. Therefore, we can conclude that only forbidden states with large binding energies cause the three-body bound state collapse. This argument will be also supported by removing the forbidden states via supersymmetric transformations.

IV. SUPERSYMMETRIC POTENTIALS

Quite often, in nuclear physics problems, the constructed two-body potential has an attractive well that sustains unphysical bound states which must be removed or projected out from the spectrum before the potential is used in calculations. One way to achieve this is via SUSY transformations in which one can add or remove a bound state from the spectrum. The method has been extensively discussed in Refs. [14,15] and therefore we shall recall here briefly its main features.

We consider the radial Schrödinger equation

$$H_0\psi_0(r) \equiv \left[-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} + V_0(r)\right]\psi_0(r) = E\psi_0(r) \quad (13)$$

for the Hamiltonian H_0 which has in its spectrum an unphysical two-body bound state $E \equiv E_b^{(0)}$. This state can be removed by factorizing H_0 and its supersymmetric partner Hamiltonian H_1

$$H_0 = A_0^+ A_0^- + \epsilon_0, \quad H_1 = A_0^- A_0^+ + \epsilon_0, \quad (14)$$

where

$$A^- = (A_0^+)^{\dagger} = -\frac{d}{dr} + \frac{d}{dr} \ln[\psi_0(r, \epsilon_0)]. \quad (15)$$

Here $\psi_0(r, \epsilon_0)$ is the solution of the Schrödinger equation at the factorization energy ϵ_0 . The SUSY-1 potential V_1 that corresponds to the partner Hamiltonian H_1 is given by

$$V_1 = V_0 - 2\frac{d^2}{dr^2} \ln[\psi_0(E_0^{(0)})]. \quad (16)$$

where $\epsilon_0 \leq E_0^{(0)}$, $E_0^{(0)}$ being the ground state energy of H_0 . Whereas V_1 and V_0 have the same spectra (except that V_1 does not sustain a two-body ground state at $E_0^{(0)}$), they are not phase equivalent. To achieve phase equivalence a second supersymmetric transformation is needed to obtain the SUSY-2 potential [15]

$$V_2 = V_0 - 2\frac{d^2}{dr^2} \ln[\psi_0(E_0^{(0)})\psi_1(E_0^{(0)})]. \quad (17)$$

The latter potential is fully phase equivalent to V_0 except that the ground state of V_0 has been removed from the spectrum. The fundamental difference between V_0 and V_2 is that the latter has a repulsive singular core at short distances

$$V_2 \sim V_0 + \frac{2(2\ell + 3)}{r^2} \sim \frac{(2 + \ell)(3 + \ell)}{r^2} \quad (18)$$

instead of an attractive well. It is interesting to note that the difference $\delta_\ell(0) - \delta_\ell(\infty)$ may be a multiple of π despite the fact that the ground state is removed, *i.e.*, the Levinson theorem is not applicable to this type of singular potentials, as it has been shown long ago by Swan [25].

One may argue here that, as the resulting potential is shallow and singular, it should be expected that no deep bound state is generated in the three-body system. However, as the three-body bound state collapse was found to be related to the resonance behavior of the Jost function we endeavored to go through the three-body calculations once more as the SUSY transformations might generate a new resonance spectrum [21] that could be of relevance.

We have applied the above method to potential 1 and potential 2 of the preceding section and checked again for three-body binding energies in the region $(-1000, 0)$ MeV. Only one three-body bound state was found for each potential namely at 4.72 MeV for potential 1 and at 7.14 MeV for potential 2 *i.e.*, no collapse was detected. Thus the removal of the forbidden state of the two-body system resulted in the disappearance of the second three-body bound state as well.

V. DISCUSSION AND CONCLUSIONS

The role played by unphysical two-body bound states in the appearance of a collapsed state in the three-body system has been investigated. For this purpose three classes of potentials were employed, namely, local potentials equivalent to nonlocal interactions which produce the BSC, local interactions which in addition to the physical deuteron state sustain

a second bound state with large binding energy and SUSY transformation potentials. We shall discuss them in turn.

Rank-1 nonlocal separable potentials with a Yamaguchi-type form factor may possess bound states in the continuum, *i.e.* the S-matrix has a pole on the real positive energy-axis. These bound states at positive energies are called pseudo-bound states, because in all respects they behave similarly to negative energy states. Since the nonlocal potentials cannot be used directly for interpretation purposes, we resorted to the construction of ELP's which can provide information of how the underlying nonlocality is manifested in configuration space. In the present work, we constructed quantal ELP's to the above rank-1 nonlocal interactions and showed that they have a strong attractive well that tends to have a δ -function behavior and sustain a deep bound state of nature similar to the PFS. This implies that the bound states in the continuum are mapped, via the localization procedure, onto the positive imaginary energy-axis. The three-body ground state energy becomes then extremely large. In other words, when one has at a two-body level a strong, short range attractive potential that generates the Thomas effect, then there is a BSC in the three- and, in general, in the N-body system – a finding which is in agreement with that of Delfino and co-workers [7,8].

We have extended our investigations to include purely local interactions having unphysical two-body bound states, *i.e.* PFS with large binding energies, and calculated the corresponding three-body binding energies. We have demonstrated that an increase(decrease) of the two-body binding energy of the unphysical state resulted in an increase/decrease of the three-body binding energy. Thus, the BSC of the three-body system is directly connected to the presence of an unphysical two-body state. We wish to mention further that variations of the binding energy of the unphysical two-body state resulted also in variations of the inner amplitude and the position of the node of the physical deuteron state. This is an indirect prove of the results of Nakaichi-Maeda [11]. Of course, one must be careful about the role of the node. The existence of a node in the physical two-body state wave function generated by a local or nonlocal interaction, is also a manifestation of a strong attraction in

the interior region.

The PFS is usually present in two-cluster systems where antisymmetrization is used to construct the underlying intercluster interaction which is, in general, nonlocal. However, local potentials can also be constructed to have one or more PFS which generate in few-cluster systems the appearance of a set of unphysical bound states. One such potential is the α - α potential of Buck *et al.* [26] employed in Ref. [27] to study the spectrum of the 3α and 4α system. It was found there [27] that the appearance of a set of unphysical bound states disappears once the PFS are removed from the spectrum via SUSY transformations. In the present work we found a similar result, namely that the removal of the unphysical bound state from the two-body spectrum results also in the disappearance of the BSC in the three-body system.

In conclusion, from the above discussion one can safely argue that the presence of unphysical bound states with large binding energies in the two-body system is responsible for the BSC in the three-body and, in general, in the N-body system.

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TABLES

TABLE I. The parameters for the nonlocal potentials, Eqs. (1,2), together with the collapse momentum p_c and the three-body binding energy B_3 for $\alpha_2 = -1.0$ and $\beta_1 = 1.4$ (fm^{-1}).

Pot.	α_1	β_2 (fm^{-1})	λ (fm^{-3})	p_c (fm^{-1})	B_3 (MeV)
1	0.05	8.47	2491.4	1.307	902.3
2	0.1	5.73	1096.1	1.213	644.8
3	0.13	4.92	831.3	1.167	578.5
4	0.15	4.53	729.3	1.149	539.4
5	0.2	3.82	573.9	1.097	486.5

TABLE II. The set of parameters used in the two-body local potential of Eq. (9) together with the corresponding two- and three-body binding energies B_2 and B_3 respectively.

Pot.	V_0 (MeV)	α (fm^{-2})	V_1 (MeV)	β (fm^{-1})	μ (fm^{-1})	B_2 (MeV)	B_3 (MeV)
1	178.24	0.2	5.0	1.0	0.7	2.225	4.60
						77.513	163.85
2	36.5	0.03	10.0	3.0	0.7	2.225	7.26
						17.486	38.35
3	45.88	0.5	30.0	1.4	0.7	2.225	4.74
4	4.7	0.007	3.0	1.4	0.2	0.074	0.308
						2.822	5.96

FIGURES

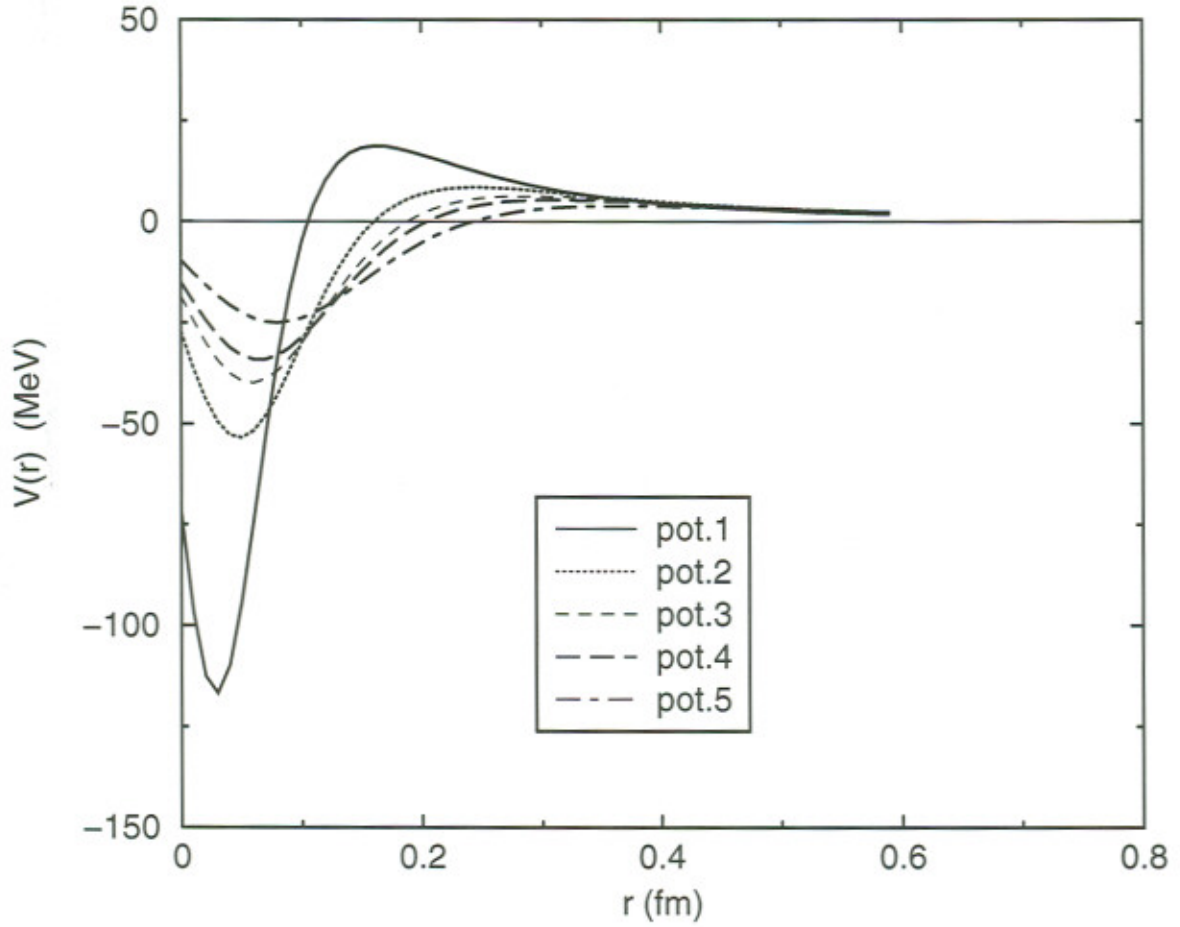


FIG. 1. Local potentials equivalent to the two-body nonlocal interactions of Table I for $k = 2.0 \text{ fm}^{-1}$. The strong, short-ranged, attractive well of potential 1 generates a BSC in the three-body system.

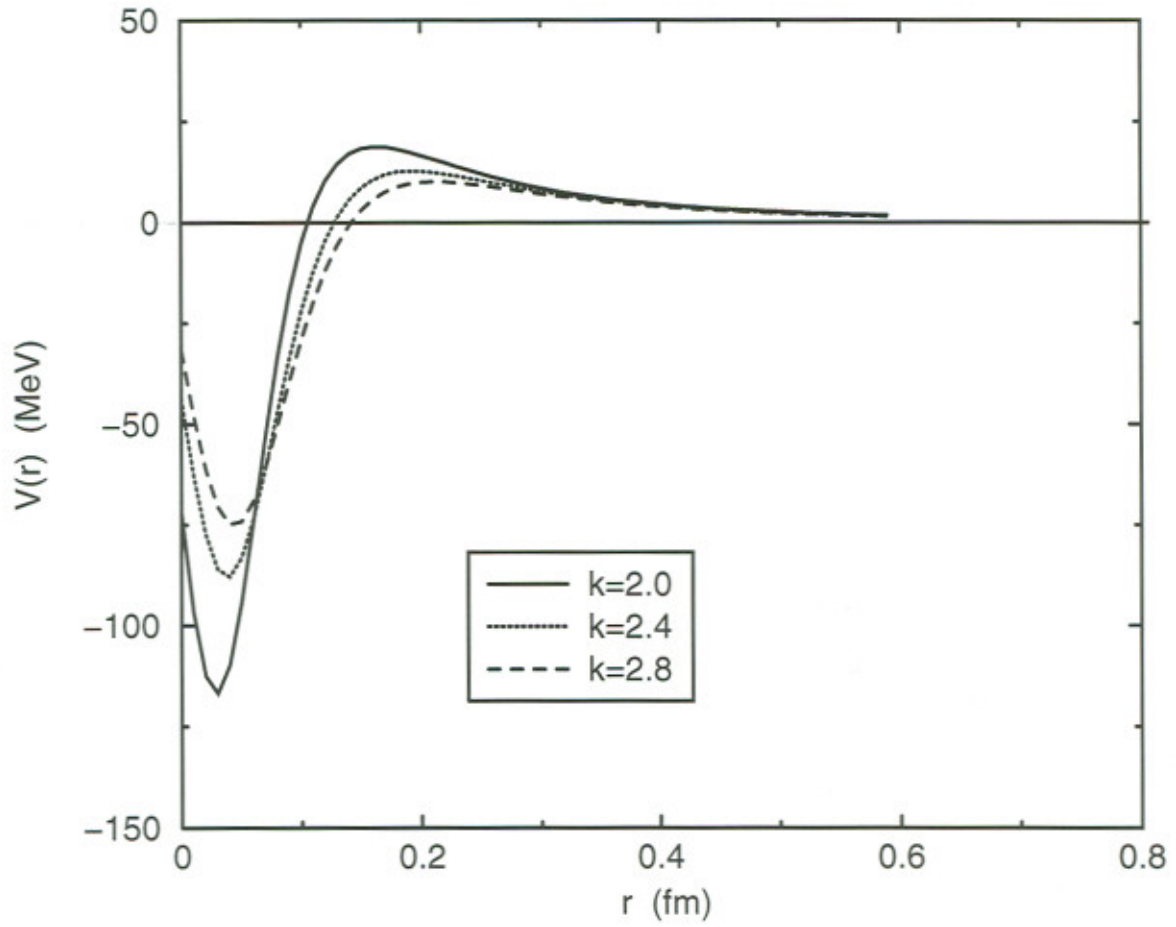


FIG. 2. Local potentials equivalent to potential 1 of Table I for $k = 2.0 \text{ fm}^{-1}$, $k = 2.4 \text{ fm}^{-1}$, and $k = 2.8 \text{ fm}^{-1}$.

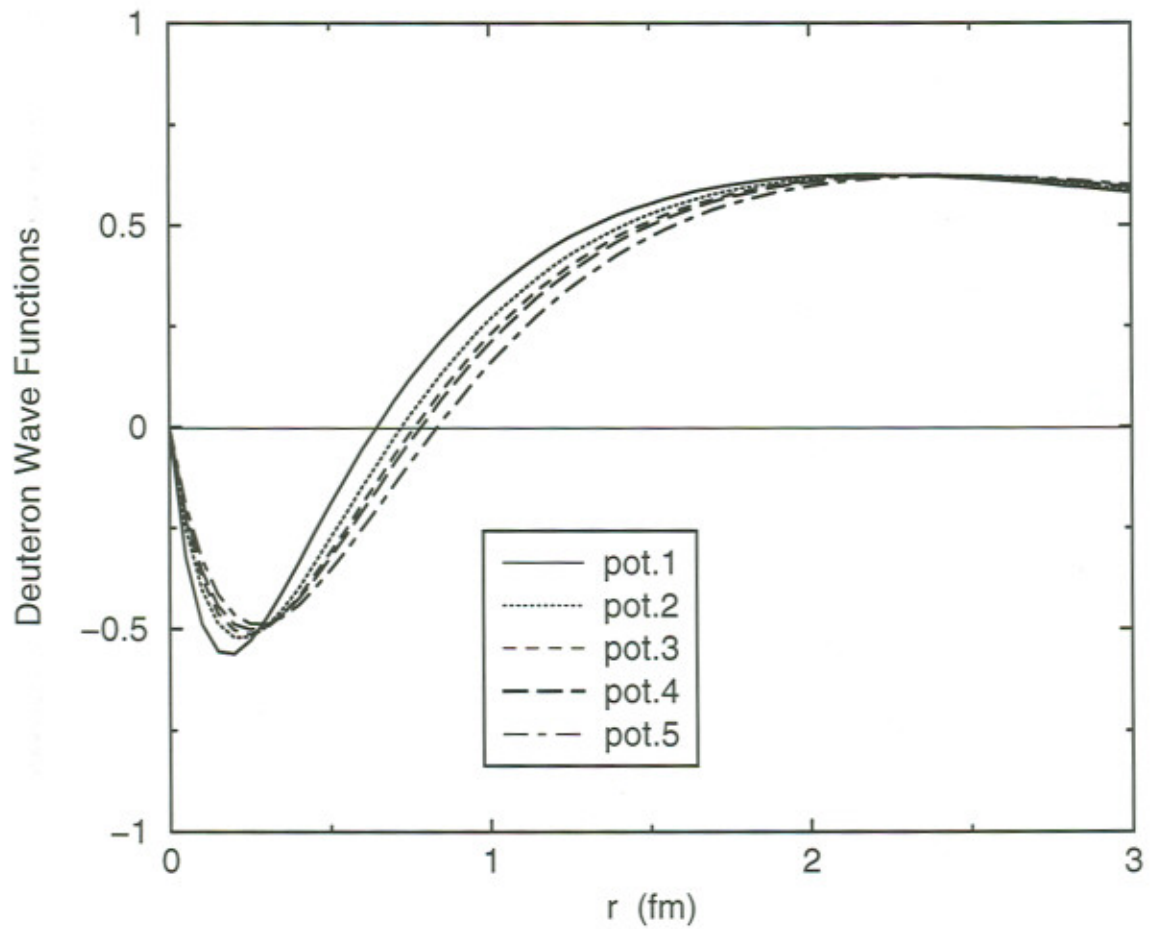


FIG. 3. The short range behavior of the physical deuteron wave function generated by the potentials of Table I. The position of the node depends on the range of the attractive well.

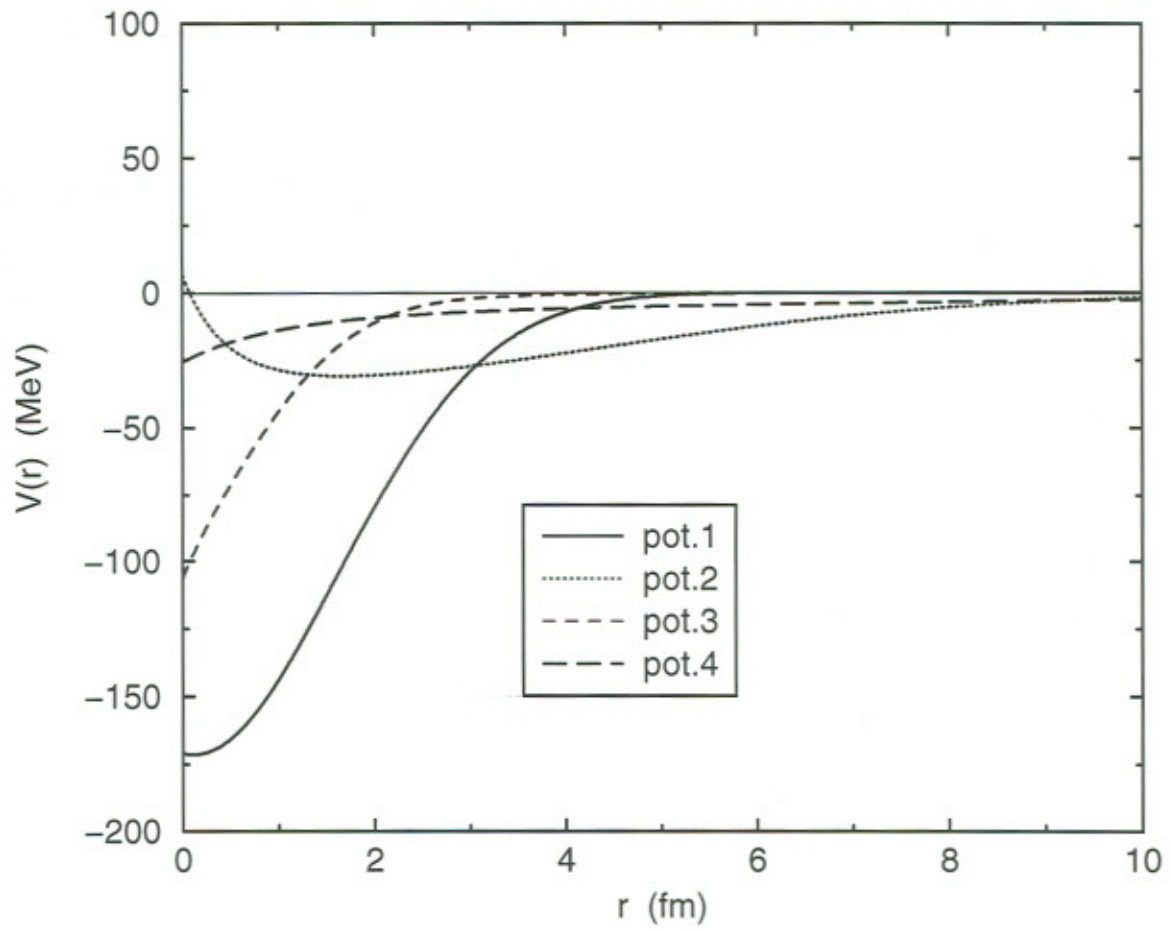


FIG. 4. The local potentials corresponding to Eq. (9). The parameters are those given in Table II.

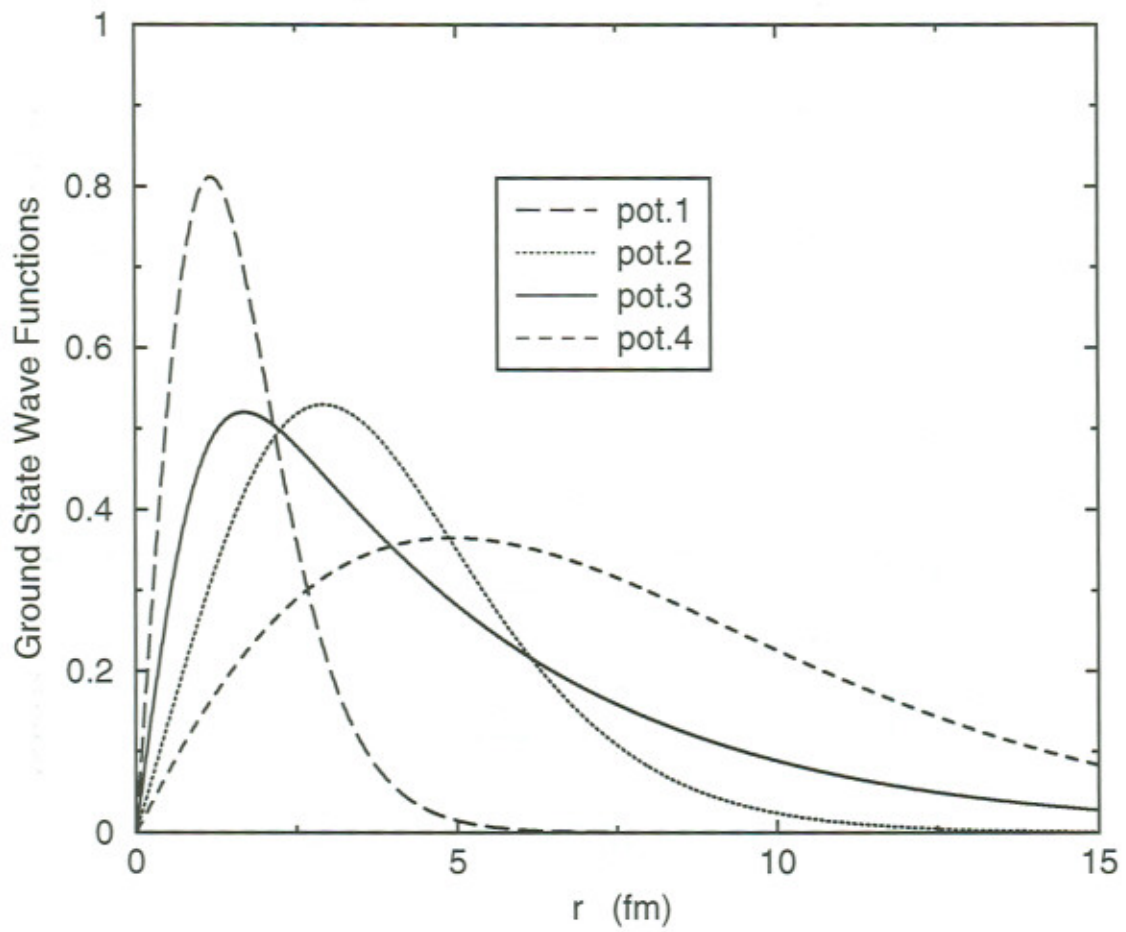


FIG. 5. Ground state wave functions of the potentials of Table II. Their peak and spread is directly linked to the shape and strength of the attractive well.

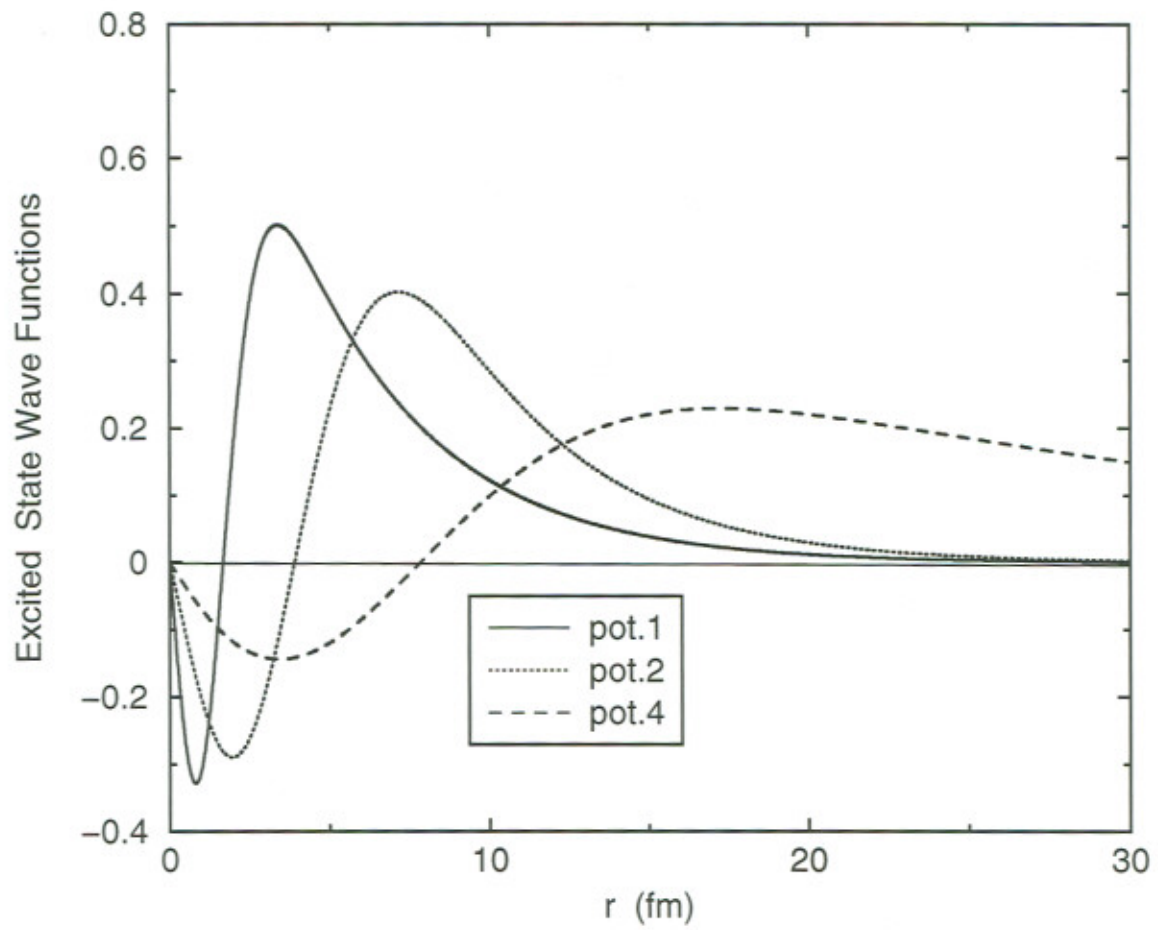


FIG. 6. Excited state wave functions of the potentials of Table II. Short-range attractive wells shift the node in the interior region.