ELASTIC STABILITY OF SILICONE FERROFLUID INTERNAL TAMPONADE (SFIT) IN RETINAL DETACHMENT SURGERY

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Elastic Stability of Silicone Ferrofluid Internal Tamponade (SFIT) in Retinal Detachment Surgery ⋆

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Abstract

It has been argued that SFIT can provide full (360^{o}) coverage of the retina in retina detachment surgery (RDS). Provided that the produced SFIT is biocompatible, exact knowledge is needed of its elastic stability in the magnetic field produced by the solid magnetic silicon band used as a scleral buckle (MSB). We propose a quantitative, phenomenological model to estimate the critical magnetic field produced by the MSB that "closes" retina tears (RT) and results in the reattachment of the retina. The magnetic "deformation" of SFIT is modeled in accordance with the deformation of a ferrofluid droplet in an external magnetic field.

Key words: Retinal Detachment; Retina Elasticity; Eye Surgery; Ferrofluids.

1 Introduction

Magnetism in living organisms either endogenous (biomagnetism, magnetoferritin, etc.) or artificially implanted (magnetic drug targeting, magnetoliposomes, etc.) is a virgin and versatile field of scientific research and applica-

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tions. Magnetite nanoparticles have been found in bacteria, birds, fish, bees and turtles and help them to navigate in the earth magnetic field [1]. The exact mechanism of magnetic orientation called magnetoreception is not known. Attempts to quantify the physical phenomenon have been reported recently, regarding the magnetized region of the animal as a biological magnetic fluid [2]. The applications of magnetism in medicine and particularly in opthalmology, are as old as the science of magnetism [3]. Specially designed magnets have been used to remove iron and steel foreigner bodies from the patient's eye. A magnetic cell sorter has been used for the isolation of retinal ganglion cells for culture [4]. Magnetic microparticles, called magnetic beads, have been used recently to measure the viscoelastic properties of living cells [5], and among them of the vitreous body of the eye [6].

One of the most common diseases of the posterior segment of the eye is retinal detachment (RD). It occurs when the retina, a transparent membrane that covers the inner surface of the wall of the eye globe, is pulled away of its normal position. This separation, depending on its size results in partial or total loss of vision and gradually blindness. At early stages the problem is cured with laser therapy, while on later ones surgery is required. RDS involves the use of various biomaterials as external buckles and belts or internal tamponades and halogenated gases that aim to keep retina in position with the inner surface of the wall of the eye globe [7]. But these methods have been criticized as ineffectual for repairing large holes in the retina extended up to 360^{o} [8]. The use of an external MSB in combination with a SFIT to cure large RT and holes is proposed in Ref. [8].

If the produced SFIT is non-toxic [9], the conditions under which the magnetic field produced by the MSB results in retina reattachment must be studied in detail. The aim of the present work is to develop, for that purpose, a quantitative phenomenological model with adjustable material and physical parameters. Thus knowledge of the elastic properties of the retina must be combined with magnetostatic properties of the SFIT. The limits of the applicability of the model are determined and clarified.

2 Anatomy of the eye and RD

The eye is a slightly assymetric sphere with an approximate sagittal diameter or length of 2.4 to 2.5cm and a transverse diameter of 2.4cm [10]. A cross-section of the eye in the $yz-$plane is shown in Fig. 1. The eyeball has a volume of about 6.5cm^3. The iris root originates from the anterior face of the ciliary body and extends as a diaphragm centrally. The ciliary body extends from the iris root posteriorly to ora serrata. The lens, a transparent structure is in contact anteriorly with the iris and posteriorly with the vitreous body. Its diameter is approximately $9-10mm$. Anterior and posterior chamber are filled with a transparent fluid called the aqueous humor. Between the lens and the retina is the vitreous cavity which is filled with gel-like transparent material known as the vitreous body. In an adult eye the vitreous body has an average volume of $4cm^3$ [10]. Mechanical properties of the vitreous include static and dynamic features that are related to the viscoelastic characteristics of the vitreous as a whole [11]. The retina, a transparent tissue, covers the inner aspects of the posterior two-thirds of the wall of the globe. Its internal aspect is in contact with the vitreous body and its external aspect is adjacent to the retinal pigment epithelium. The retina is 0.1mm thick at the ora serrata, 0.2mm at the equator and 0.56mm adjacent to the optic nerve head. Retina contains photosensitive cells (photoreceptors) and several layers of neutral cells. This combination generates action pulses relative to the visual image which passes out of the eye to the brain on the optic nerve.

The most sensitive and complex outer portion of the retina has no blood vessels to supply it with oxygen or nutrition directly. For these, the retina has to depend on the choroid which is highly vascular (with plenty of blood vessels). For the retina to be able to use the nutrients, it must remain in "position" (in close proximity with the retinal pigmented epithelium and the choroid). Under certain circumstances, the retina gets detached from the pigmented epithelium. The major cause for RD is the development of hole or break in the retina (RT). Vitreous traction, either intermittent or constant, accounts for the vast majority of RT. Predisposing factors are: degenerate areas of peripheral retina, aging, trauma, etc. Due to these factors the vitreous gel undergoes liquefaction (degenerates into a fluid). Then the fluid vitreous flow through and underneath the retina resulting in complete RD. All the process is depicted in Fig. 2. Immediately following the RD the retina fails to receive or transmit an image with any amount of clarity and the corresponding visual field is lost. If the retina can be attached quickly, most of the vision can be recovered. However, if the retina remains detached, its outer layers containing the important visual elements slowly die and degenerate. The retina could be died completely but for the fact that its inner layers have their own vascular supply which can delay death but cannot save it. The result is total and permanent blindness. Therefore, the longer the retina remains detached, the greater is the degeneration and less the chance of visual recovery.

The present materials that are used in RDS as vitreous substitutes, well known as internal tamponades, have drawbacks. Some, like silicon oil or halogenated gas float up, while others like flurosilicone sink down, thus resulting in partial and not total support to the retina. Moreover they fill the vitreous cavity, decreasing vision, and contact anterior chamber structure, causing cataract and glaucoma [7]. These disadvantages of the currently available treatment modalities, depending on the type and orientation of RD, are minimized or even removed with the use of a SFIT and of a MSB [8].

3 Theory

The placement and shape of the MSB and of the accompanying SFIT is determined by the size and position of the RT. For simplicity we adopt the configuration illustrated in Fig. 3a. The present model addresses one of the many problems encountered in the development of a proper SFIT, that of the estimation of the critical value of the magnetic field produced from the MSB that is capable to stabilize the SFIT and close the holes in the retina. In the presence of the magnetic field produced from the MSB the SFIT will take the shape of the tube represented in Fig. 3a. We assume for simplicity that the cross-section of the tube (yz-plane) is circular of radius R_0, when the MSB is not present, and elliptical with semi-major and semi-minor axis a and b, respectively, when the magnetic field H of the MSB is applied along the $z-$axis (see Fig. 3b). This simple view of the problem is adopted from the model of Bacri et al [12], for the deformation of a ferrofluid droplet in a uniform magnetic field (the long axis of the ellipse indicates the axial direction of the magnetic field but not its polarity). We further assume that a RT, of length ℓ, that establishes an angle θ_0, with the complete retina reattachment position, is firmly attached only to one point with the SFIT and follows its deformations as a "rigid" rod (Fig. 3b). In our model we are not interested for the true deformations of the retina due to the deformations of the SFIT, but rather for the response of the retina to the magnetic field, by conceiving the underling physical mechanism and the elastic properties of the retina. For further simplicity we assume that the RT length ℓ is such that $\ell\theta \approx a - R_0$. In general the rod ℓ that constitutes the RT may not exist at all, we may have a retina hole or break (RB), but our model is still applicable, since in such a case we intent to bring the SFIT in contact with the RB, in order to block the vitreous flow through the retina and obtain retina reattachment. Finally, R_e is the eye diameter, L is the distance of the center of the SFIT from the center of the eyeball and δ_0 the width of the sclera.

Bacri's model proved successful on predicting the shape of ferrofluid droplets with diameters of the order of $1 - 10\mu m$. The cross-section of the SFIT is definitely not circular nor elliptical but that approximation is not far from reality. Also the mean diameter of the cross-section of the SFIT is of the order of $0.1 - 1mm$. In this size scale other effects are also important: (i) gravitational effects which results in non-spherical ferrofluid droplets (the droplet surface tilts responding to gravity), and (ii) the sensitivity of the droplet to the earth's or local magnetic fields [13]. Also the magnetic field that experiences the SFIT is non-uniform. Despite that, for our purposes the original Bacri's model proves useful. We assume that the magnetic field produced from the MSB is rather high to wipe out gravitational and local magnetic fields effects and moreover we ignore magnetic field gradients (the length of the MBS along the $y-$ axis is larger compared to SFIT length). Then the shape of the SFIT of permeability μ_2 in a vitreous fluid of permeability μ_1 is due to the competition between its interfacial energy [12],

$$E_{\rm S} = \sigma 2\pi a^2 K \left(K + \frac{\sin^{-1}\epsilon}{\epsilon} \right) \tag{1}$$

and its magnetic energy [12],

$$E_{\mathrm{M}} = -\frac{V\mu_1 H^2}{2}\frac{1}{\alpha + n}, \tag{2}$$

where σ is the interfacial tension, $K = b/a$ is the aspect ratio, $\epsilon = \sqrt{1-K^2}$ is the eccentricity, H is the applied magnetic field, $V = \frac{4}{3}\pi a b^2$ is the volume of the ellipsoid,

$$\alpha = \frac{\mu_1}{\mu_2 - \mu_1} \tag{3}$$

and

$$n = \frac{K^2}{2\epsilon^3}\left(\log\left(\frac{1+\epsilon}{1-\epsilon}\right) - 2\epsilon\right) \tag{4}$$

is the shape dependent demagnetization factor with $n = \frac{1}{3}$ for the sphere. Assuming that the permeabilities μ_1 and μ_2 are independent of the constant magnetic field H, and the SFIT is incompressible $(ab^2 = R_0^3)$, minimization of the total energy $E_{\mathrm{T}} = E_{\mathrm{S}} + E_{\mathrm{M}}$ with respect to the aspect ratio K, determines the shape of the drop

$$B_m \equiv \frac{H^2 R_0}{\sigma} = \frac{2}{\mu_1}\frac{f_S(K)}{f_M(K)}, \tag{5}$$

where

$$f_{\mathrm{S}}(K) = -\frac{1}{2K^{1/3}\epsilon^2}\left(1 + 2K^2 + (1-4K^2)\frac{\sin^{-1}\epsilon}{K\epsilon}\right), \tag{6}$$

$$f_{\mathrm{M}}(K) = -\frac{1}{(\alpha+n)^2}\frac{K}{2\epsilon^4}\left(\frac{2+K^2}{3}\log\left(\frac{1+\epsilon}{1-\epsilon}\right) - 6\right), \tag{7}$$

and B_m is the dimensionless magnetic Bond number of the SFIT which is shape dependent. Using as fitting parameters α and σ, Eq. (5) can explain quantitatively experimental data. For typical ferrolfuid droplets of the order of 1-10 μm $10^{-7} \leq \sigma \leq 10^{-6} N/m$ and $0.01 \leq \alpha \leq 0.1$ [12,14]. For larger droplets, with diameters of the order of $0.1 - 1mm$, it is expected that the fitting parameters attain higher values, with σ mainly affected.

The deformation of the cross-section of SFIT, like a ferrofluid droplet, in the above described model is responsible for the pressure that experiences the retina, as well as the RT. This pressure has to be estimated in order to know whether or not it results in retina reattachment. For that reason the elastic properties of the retina must be measured and quantified. In order to give an expression for the elastic energy of the retina E_{E}, its physical and mechanical character should be taken into account. In a rather elementary consideration we may assume that it behaves like a linear elastic material:

$$E_{\mathrm{E}} = \frac{1}{2} E e^2 \tag{8}$$

where E is the Young's modulus and e is the strain. Then the stress τ is defined from the Hooke's constitutive law:

$$\tau = \frac{\partial E_{\mathrm{E}}}{\partial e} = Ee. \tag{9}$$

But this is not actually true, since the retina as a biological membrane undergoes large deformations in small applied loads. A common method to estimate the elastic properties of biological soft tissues and membrane, is to apply them usually under biaxial loads [16]. This is similar to the studies of rubber or hyperelastic materials. Then it is typical to express the strain energy function E_{E} is terms of the principal stretches (extension ratios λ_i) of the strain ellipsoid

[17]:

$$E_{\mathrm{E}} = \sum_{j=1}^{N} \frac{\gamma_j}{\beta_j} \left(\lambda_1^{\beta_j} + \lambda_2^{\beta_j} + \lambda_3^{\beta_j} - 3 \right), \tag{10}$$

where γ_j and β_j are parameters that are determined from experiment. Then the Kirchhoff stresses are defined through the constitutive equations:

$$\tau_i = \lambda_i \frac{\partial E_{\mathrm{E}}}{\partial \lambda_i} = \sum_{j=1}^{N} \gamma_j \lambda_i^{\beta_j}. \tag{11}$$

For an incompressible material ($\lambda_1\lambda_2\lambda_3 = 1$) under simple tension ($\lambda_1 = \lambda$, $\lambda_2 = \lambda_3 = 1/\sqrt{\lambda}$) the Kirchhoff stresses reduce to:

$$\tau = \lambda \frac{\partial E_{\mathrm{E}}}{\partial \lambda} = \sum_{j=1}^{N} \gamma_j \left(\lambda^{\beta_j} - \frac{1}{\lambda^{\beta_j/2}} \right). \tag{12}$$

The stresses τ of Eqs. (9), (11) and (12) must be coupled with the pressure that is experiences due to the deformation of the cross-section of the SFIT. Responsible for that pressure is not the interfacial energy E_{S} but the magnetic energy E_{M}. Thus assuming that the distance between the MSB and SFIT is δ the mean pressure p depends on the gradient of E_{M} with respect to δ :

$$p = \frac{1}{\pi b^2} \frac{\partial E_{\mathrm{M}}}{\partial \delta}. \tag{13}$$

But from Fig.3b $\delta = \delta_0 + R_e - L - a$ and due to Eq. (5) we obtain

$$p = -\frac{2\sigma}{R_0} K f_{\mathrm{S}}(K). \tag{14}$$

Thus the coupling between the deformation of the SFIT and the pressure that it experience the retina and the RT is attained by the condition

$$p = \tau. \tag{15}$$

Then due to the model assumptions the strain $e_{zz} = e$ of Fig. 3b can be expressed as

$$e = \frac{a - R_0}{R_0 + L}, \tag{16}$$

and the critical strain that is responsible for retina reattachment is derived from the condition $\delta_c = 0$, or equivalently:

$$e_c = \frac{R_e}{R_0 + L} - 1. \tag{17}$$

Due to the incompressibility condition the critical aspect ratio is

$$K_c = \left(\frac{R_0}{R_e - L}\right)^{3/2} \tag{18}$$

and thus the critical applied magnetic field is derived by

$$H_c = \sqrt{\frac{2\sigma}{\mu_1 R_0}\frac{f_S(K_c)}{f_M(K_c)}}. \tag{19}$$

Provided that there will be performed experiments that will determine σ and α for SFIT and γ_i, β_i for the retina the critical magnetic field for retina reattachment is obtained from Eq. (19). Some rough estimates can be made without resorting to experiments. Due to the incompressibility condition we can rewrite Eq. (16) in terms of K

$$e(K) = \frac{R_0}{R_0 + L}\left(\frac{1}{K^{2/3}} - 1\right). \tag{20}$$

Since in the limit of infinitesimally small applied magnetic fields ($K \to 1$), the linear stress-strain constitutive law (9) with τ determined from (15) should be compatible with (20), we can express σ in terms of the Young modulus E

and the geometric properties as

$$\frac{\sigma}{R_0} \approx \frac{5}{16} \frac{E R_0}{(R_0 + L)}. \tag{21}$$

Thus, for an eyeball with $R_e \approx 1.2cm$, $L \approx 4R_0$, a SFIT with $R_0 \approx 1.2mm$, $\alpha \approx 0.025$ and $E = 2 \times 10^4 Pa$ [19], and for a non-magnetic vitreous body with $\mu_1 \approx \mu_0$ (μ_0 the permeability of vacuum), we obtain $e_c \approx 1$, $\sigma/R_0 \approx 1.25 \times 10^3 Pa$ and $H_c \approx 17.67kA/m$. The above calculated critical magnetic field is almost four orders of magnitude larger than the earth's magnetic field ($\approx 2.39A/m$) in agreement with our assumptions, and the calculated surface tension $\sigma \approx 1.5N/m$, is more than one order of magnitude larger than the reported value for silicon oil/water interface $\sigma_{so} \approx 5 \times 10^{-2} N/m$ [18]. The stress-strain law for a retina with the above material and geometric properties, is plotted in Fig. 4 and compared to the linear elastic material. The fitting with Eq. (12) to the nonlinear stress strain data obtained from Eqs. (14), (15) and (20) corresponded to $\gamma_1 = 5.2539 \times 10^3 Pa$, $\gamma_2 = -5.3694 \times 10^3 Pa$ and $\beta_1 = 1.91743$, $\beta_2 = -4.11081$, and is shown as dashed line in Fig. 4. For retina reattachment $e_c = 1$, the maximum stresses on the retina are $\tau_c = 4.5 \times 10^3 Pa$ or the maximum applied force is $\pi R_0^2 \tau_c = 20.36mN$, about two times larger from related experiments [19–21]. In Fig. 5 we plotted the retina strains e as a function of the magnetic Bond number $B_m \propto H^2$ for varying α. Notice the hysteresis observed for $\alpha = 0.03$ since the retina follows the irreversible deformations of the SFIT. The full curves correspond, as expected, to the nonlinear stress-strain law of Fig. 4, while the dashed to the Hooke's stress-strain law of Eq. (9).

4 Conclusions

The conditions of retinal reattachment with the use of a MSB and a SFIT, that provide 360^{o} coverage of the retina, have been modeled and discussed. A number of problems must be solved before the model becomes functional. SFITs as long term vitreous substitutes they must be biocompatible and non-biodegradable. If they are toxic and carcinogenic, reliable coating must be applied in order to prevent contact with biological tissues. Magnetite microspheres provide a suitable ingredient for SFIT, since they are present in living organisms, compared to the more toxic nickel and cobalt oxides. Experiments have been proposed to determine the material parameters that enter into the model: surface tension σ and dimensionless permeability factor α of the SFIT and elastic modulus γ_i and critical exponents β_i for the retina. The model admits of further improvement after taking into account gravitational as well as local magnetic fields (earth's magnetic field, device fields, etc.). There are also problems that have not been addresses in the present work, and should be taken into account in a more detailed investigation, like: the optimization of the MSB and SFIT properties in order to avoid ischemic effects due to overcritical pressures in the area of interest, or the vibrational stability of the structure due to dynamic effects (walking, head movements, etc.).

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Fig. 1. Anatomy of the eye.

Fig. 2. Stages of Retinal Detachment formation.

Fig. 3. (a) SFIT Geometry. (b) Model Geometry.

Fig. 4. Stress strain law for the retina.

Fig. 5. The strain e vs. the magnetic Bond number $B_m \propto H^2$. for varying α.

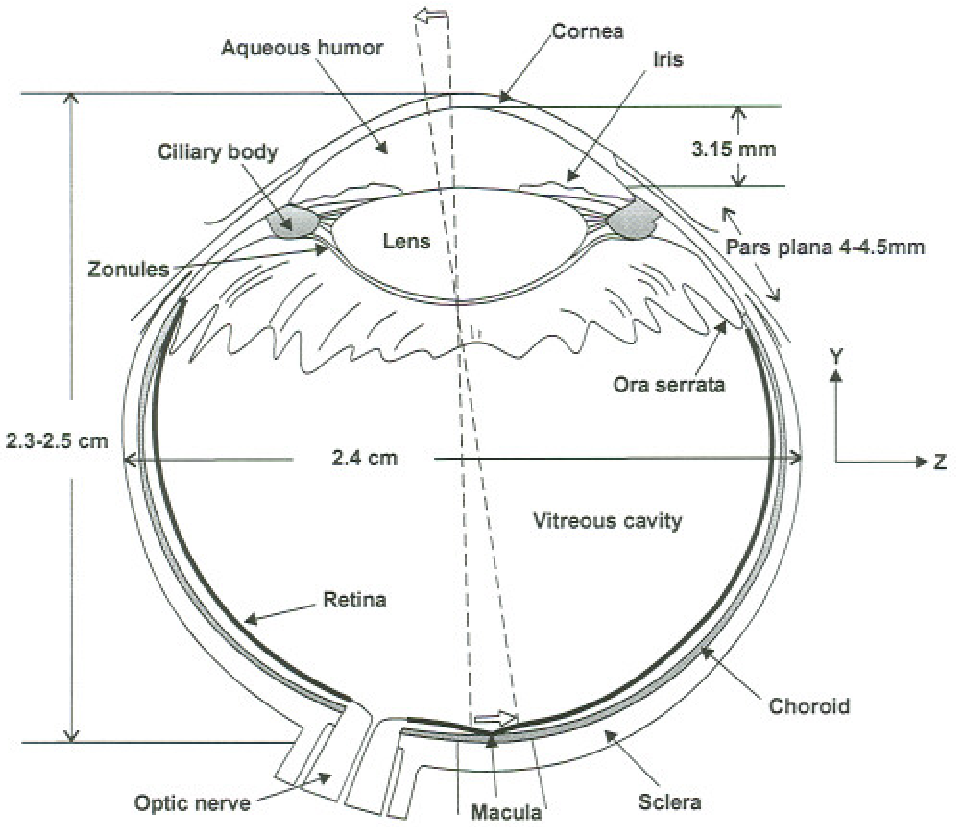

Cornea
Aqueous humor
Iris
3.15 mm
Ciliary body
Lens
Pars plana 4-4.5mm
Zonules
Ora serrata
Y
2.3-2.5 cm
2.4 cm
Z
Vitreous cavity
Retina
Choroid
Optic nerve
Macula
Sclera

Figure 1

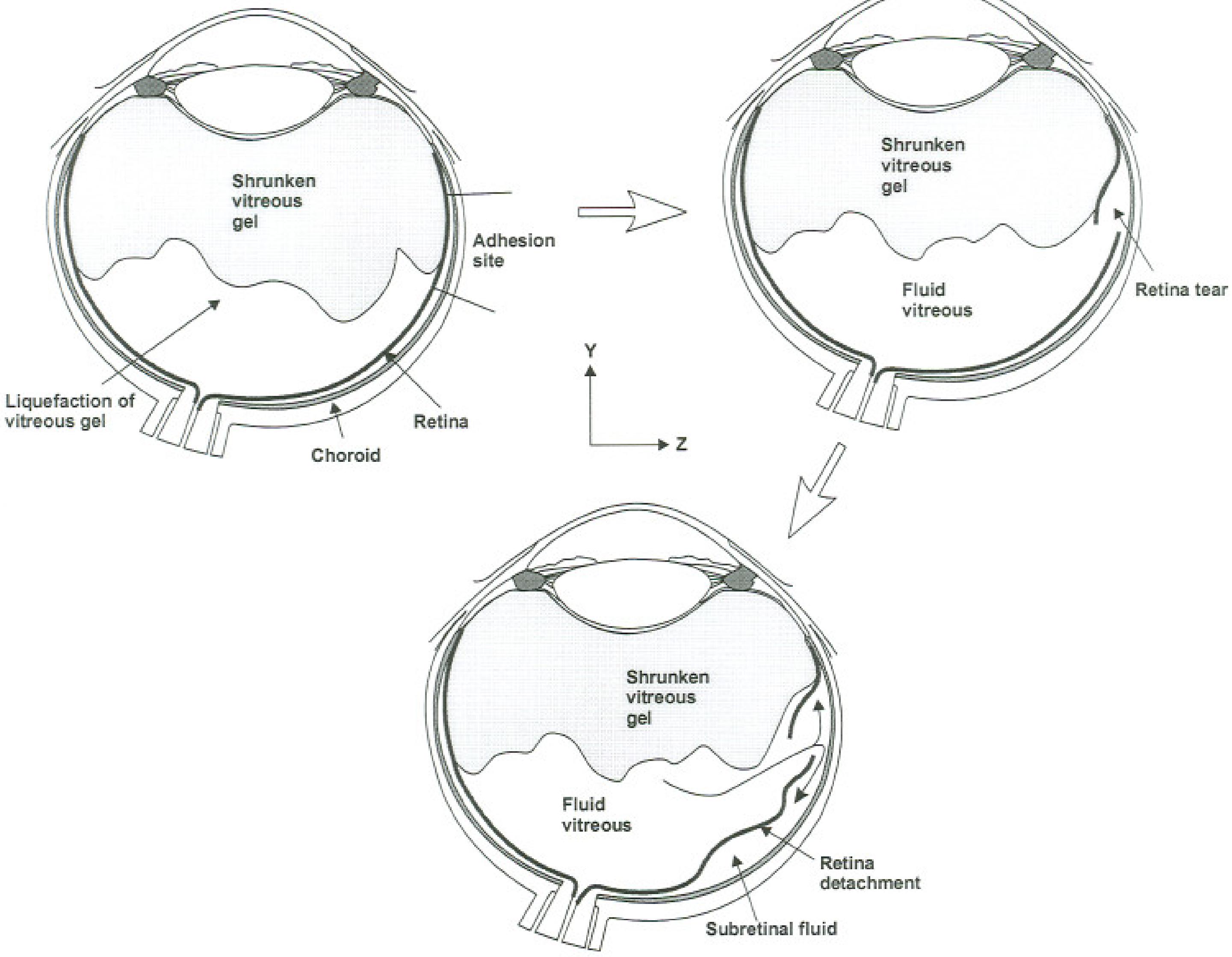
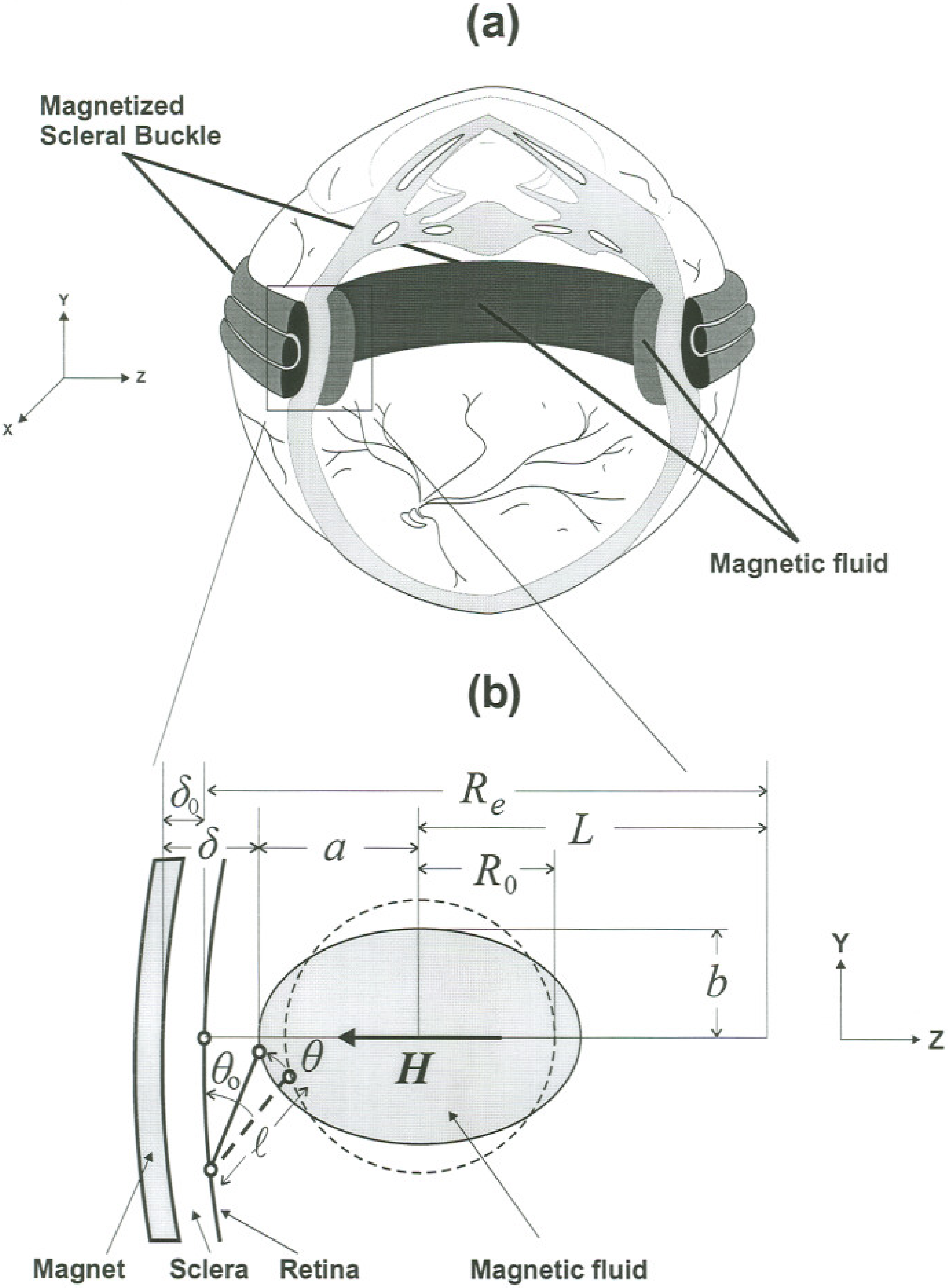

Shrunken vitreous gel
Adhesion site
Liquefaction of vitreous gel
Retina
Choroid
Y
Z
Shrunken vitreous gel
Fluid vitreous
Retina tear
Shrunken vitreous gel
Fluid vitreous
Retina detachment
Subretinal fluid

Figure 2



(a)
Magnetized Scleral Buckle
Magnetic fluid
Y
Z
X
(b)
δ_0
R_e
δ
a
L
R_0
b
Y
Z
θ
θ_0
H
ℓ
Magnet
Sclera
Retina
Magnetic fluid

Figure 3

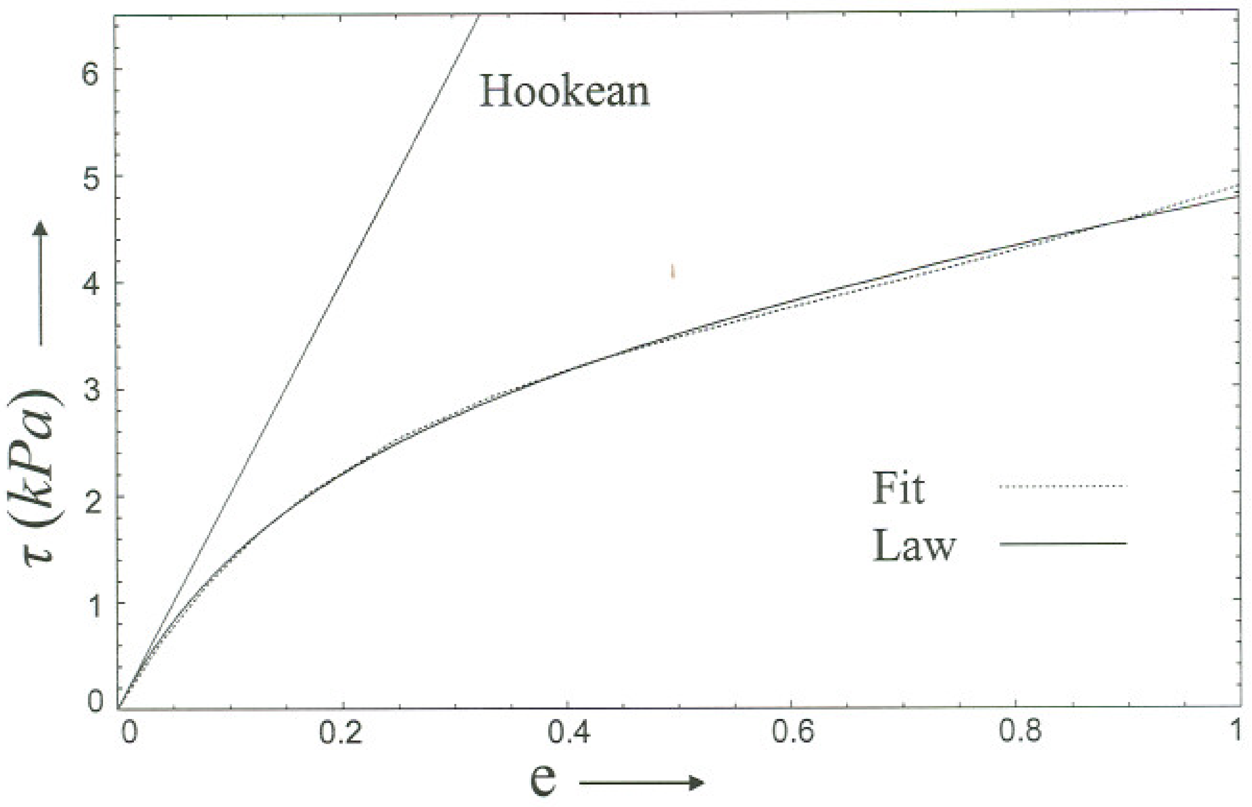


Figure 4

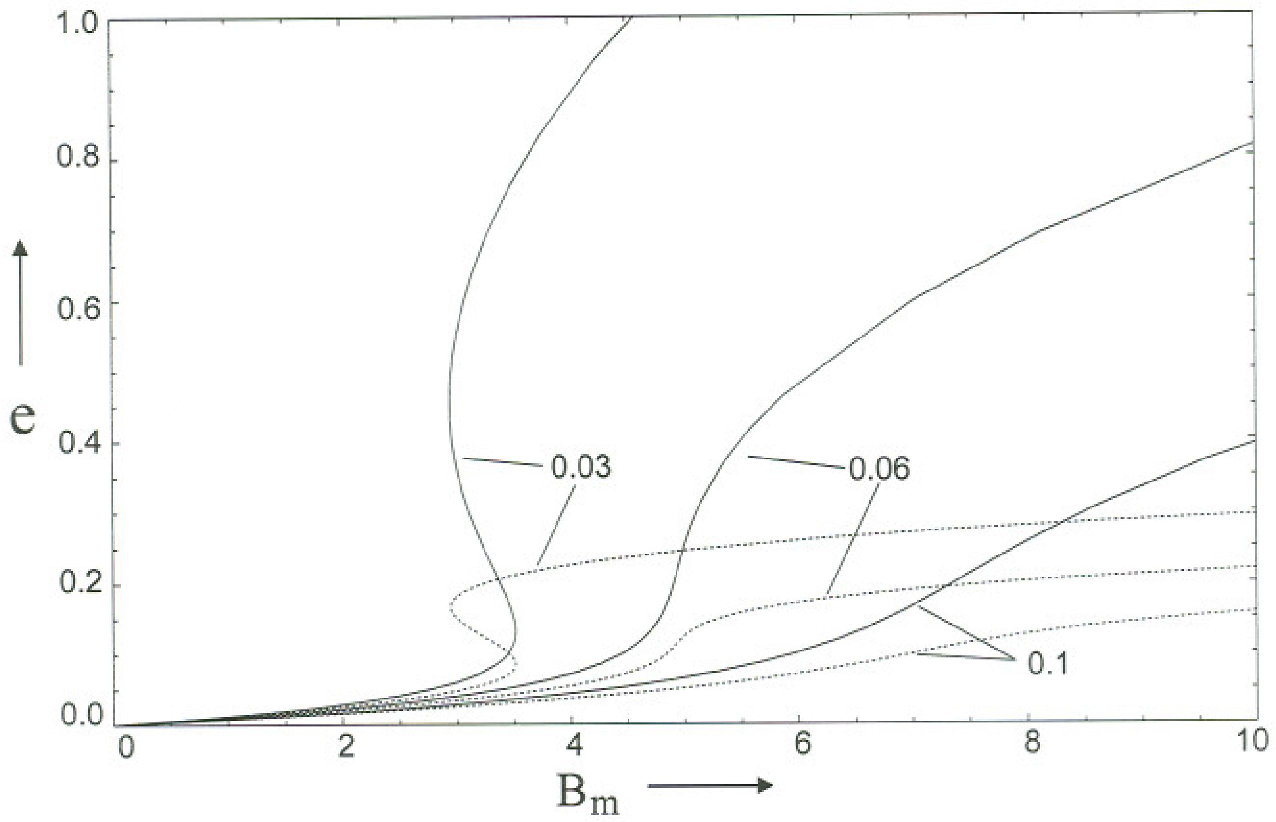


Figure 5