MODE MIXING IN FERROMAGNETIC RESONANCE IN MAGNETIC MICROSPHERES

P.A. VOLTAIRAS, D.I. FOTIADIS and C.V. MASSALAS

23-99

Preprint no. 23-99/1999

Department of Computer Science
University of Ioannina
451 10 Ioannina, Greece

Mode Mixing in Ferromagnetic Resonance in Magnetic Microspheres

P. A. Voltairas , D. I. Fotiadis

Department of Computer Science, University of Ioannina, P. O. Box 1186, Ioannina GR 451 10, Greece

C. V. Massalas*

Department of Mathematics, Division of Applied Mathematics and Mechanics, Rector's Office, University of Ioannina, P. O. Box 1186, Ioannina GR 451 10, Greece

(October 12, 1999)

Abstract

The problem of ferromagnetic resonance in magnetic microspheres is revisited due to related experiments in this size range. The eigenfrequency spectrum is examined more rationally compared to previous numerical computations, due to proper selection of the radial dependence of the solution. The cylindrically symmetric modes studied agree with experiments on Ni micrometer size particles for an exchange constant $A = 2 \times 10^{-7}\, erg/cm$, while the error is less than 9% for the well known value of $A = 3.4 \times 10^{-7}\, erg/cm$.

Typeset using REVTEX

I. INTRODUCTION

Understanding of the behavior of nano and micro-scale ferromagnetic particles in the presence of an externally applied magnetic field is of great importance due to variety of application in diverse fields. Nanoscale ferromagnetic particles have been detected in the human brain and can shed some light on controversial questions of whether weak electromagnetic fields might have biological effects, including cancer [1]. Furthermore, among the new methods of drug delivery is the use of vesicles that contain magnetic microspheres and can target drugs to specific location inside the human body via externally applied magnetic fields, in order to destroy cancer tumors [2,3]. Recently, magnetically generated gene transfer and DNA extraction with the aid of ferromagnetic nano- and microparticles was reported [4]. Colloidal suspensions of ferromagnetic nanoparticles in a liquid carrier, well known as magnetic fluids or ferrofluids [5], find a lot of technological (sealing, bearing, sensing, pumping, ink jet printing) [6] and medical applications (eye surgery [7], cancer therapy via magnetic hyperthermia [3,8–11]). The mathematical description of such phenomena in ferromagnetic solids is based on the phenomenological theory of micromagnetics [12]. Extended literature on the subject can be found in a recent monograph [13].

Magnetostatic resonance modes interpret experimental observations in spherical and ellipsoidal particles with diameters above 1mm [14]. In these modes magnetostatic forces dominate over exchange interactions. On the other extreme, for particles with diameters below 1μm, the resonance frequency depends on the size of the particles ($\propto R^{-2}$) as well as on the roots of the derivatives of spherical Bessel functions. These two originally theoretically predicted features of the modes could explain qualitatively [15–17] and in some cases quantitatively [18,19], related experiments. They were given the name *exchange resonance modes* [20], since the exchange energy in much larger than the magnetostatic one. Corrections to the theory in order to account for the experimentally observed size dependence were performed in Ref. [21], by introducing surface anisotropy effects. Similar considerations have also been proposed in Ref. [22] in order to explain resonance experiments in ferrite nanoparticles [23,24], but the presented solutions were limited to quasiuniform perturbation resonance modes, where the magnetostatic problem can be treated rather easily. When both magnetostatic and exchange interactions are of the same order of magnitude there is a mixing of the modes for particles above the size of about 1 μm. Such calculations have been performed for cylindrically symmetric modes [25], with the material parameters of magnetite and were extended to the case where damping effects are present [26]. In those calculations it was also possible to compute the modes of precession.

The present study focuses on the eigenfrequency spectrum of ferromagnetic microspheres ($d \gtrsim 1\mu m$). The proposed mathematical analysis is more rational compared to previous calculations [25,26], due to the proper selection of the radial dependence of the solution. Only cylindrically symmetric modes are concerned. Since ferromagnetic resonance has always been considered as the most accurate method of measuring the exchange constant A, quantitative agreement between numerical computations for Ni microspheres and experimental data for Ni microparticles [15–19] is obtained for $A = 2 \times 10^{-7}\, erg/cm$, while the error is less than 9% for the well known value $A = 3.4 \times 10^{-7}\, erg/cm$ [27].

II. PROBLEM FORMULATION

The motion of the magnetization in an external field is studied by the Landau-Lifshitz equation (in CGS units):

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \gamma_0 \boldsymbol{v} \times \boldsymbol{H}^{\mathrm{eff}}, \tag{1}$$

where

$$\boldsymbol{H}^{\mathrm{eff}} = \frac{C}{M_s}\nabla^2 \boldsymbol{v} - \frac{1}{M_s}\frac{\partial w_a}{\partial \boldsymbol{v}} + \boldsymbol{H} \tag{2}$$

is the effective field, $(\partial/\partial \boldsymbol{v})_i = \partial/\partial v_i$, $(i = x, y, z)$, $\boldsymbol{v}$ is a unit vector parallel to the magnetization, $C = 2A$ is the exchange constant, w_a is the anisotropy energy density, M_s is the saturation magnetization, t is time, γ_0 is the gyromagnetic ratio and $\boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{H}'$ is the magnetic field which is composed of the applied field $\boldsymbol{H}_0$ and that, $\boldsymbol{H}'$, created by the volume and surface charges of the magnetization distribution. The boundary conditions for the set of equations (1) are

$$\frac{\partial \boldsymbol{v}}{\partial n} = 0, \tag{3}$$

where $\partial/\partial n \equiv \boldsymbol{n} \cdot \nabla$ and $\boldsymbol{n}$ is the outward normal to the particle surface. The self-field $\boldsymbol{H}' = -\nabla V$, is determined from the potential problem [12]

$$\begin{aligned} &\nabla^2 V_{\text{in}} = 4\pi \nabla \cdot \boldsymbol{M} && \text{inside the particle} \\ &\nabla^2 V_{\text{out}} = 0 && \text{outside the particle.} \end{aligned} \tag{4}$$

with the following boundary conditions on the particle surface

$$\begin{aligned} V_{\text{in}} &= V_{\text{out}} \\ \frac{\partial V_{\text{in}}}{\partial n} &= \frac{\partial V_{\text{out}}}{\partial n} + 4\pi M_s \boldsymbol{n} \cdot \boldsymbol{v}. \end{aligned} \tag{5}$$

In experimental studies of resonance, a large dc field $\boldsymbol{H}_0$ is applied; its direction is identified here with the $z-$axis. The field $\boldsymbol{H}_0$ keeps the magnetization almost parallel to the $z-$axis so that v_x and v_y are small. To a first order in these small quantities, the differential equations (1) for steady-state solution, $(\,)e^{i\omega t}$, become

$$\left(\frac{C}{M_s}\nabla^2 - H_z\right) v_x - \frac{i\omega}{\gamma_o} v_y = \frac{\partial V_{\text{in}}}{\partial x} \tag{6a}$$

$$\left(\frac{C}{M_s}\nabla^2 - H_z\right) v_y + \frac{i\omega}{\gamma_o} v_x = \frac{\partial V_{\text{in}}}{\partial y}, \tag{6b}$$

where V_{in} is the potential due to the transverse magnetization $\boldsymbol{m} = M_s\left(v_x \boldsymbol{i} + v_y \boldsymbol{j}\right)$ and ω is the resonance frequency. We note here that we keep the same symbols for the time independent components of $\boldsymbol{v}$ and for the potentials to avoid confusion. The potential due to the z component is included in H_z, which for the case of the sphere studied here, has the form

$$H_z = H_0 - \frac{4\pi}{3} M_s + \frac{2K_1}{M_s}, \tag{7}$$

where K_1 is the anisotropy constant. For the linearized problem, cubic or uniaxial anisotropies lead to the same contribution in H_z, provided that the z is an easy axis.

III. PROBLEM SOLUTION - FREQUENCY EQUATION

We are interested only in cylindrically symmetric solutions. That is we assume that $\boldsymbol{v}$ does not depend on the coordinate ϕ. We use the components of $\boldsymbol{v}$ in a cylindrical coordinate system (ρ, ϕ, z) but express the spatial dependence in spherical coordinates (r, θ, ϕ). We introduce the dimensionless quantities

$$\begin{aligned} &\tau = \frac{r}{R}, \quad h = \frac{H_z}{2\pi M_s}, \quad h_r = \frac{\omega}{2\pi M_s \gamma_o}, \\ &u_{(\text{in,out})} = \frac{V_{(\text{in,out})}}{2\pi M_s R_o}, \quad S = \frac{R}{R_o} \\ &\nabla'^2 \equiv \frac{\partial^2}{\partial \tau^2} + \frac{1}{\tau}\frac{\partial}{\partial \tau} + \frac{1}{\tau^2}\frac{\partial^2}{\partial \theta^2} + \frac{1}{\tau^2}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial \theta}, \end{aligned} \tag{8}$$

where R is the radius of the sphere and

$$R_o = \frac{\sqrt{A}}{M_s}, \quad (A \equiv C/2)\,, \tag{9}$$

the exchange length.

Equations (6) and (4) are written as

$$\begin{aligned} &\left(\nabla'^2 - \frac{1}{\tau^2 \sin^2\theta} - \pi S^2 h\right) v_\phi + i\pi S^2 h_r v_\rho = 0, \\ &\left(\nabla'^2 - \frac{1}{\tau^2 \sin^2\theta} - \pi S^2 h\right) v_\rho - i\pi S^2 h_r v_\phi \\ &\qquad\qquad = \pi S \left(\sin\theta \frac{\partial}{\partial \tau} + \frac{\cos\theta}{\tau}\frac{\partial}{\partial \theta}\right) u_{\text{in}}, \\ &\nabla'^2 u_{\text{in}} = 2S\left(\frac{1}{\tau \sin\theta} + \sin\theta\frac{\partial}{\partial \tau} + \frac{\cos\theta}{\tau}\frac{\partial}{\partial \theta}\right) v_\rho, \end{aligned} \tag{10}$$

for $\tau \leq 1$ and

$$\nabla'^2 u_{\text{out}} = 0, \tag{11}$$

for $\tau \geq 1$.

The boundary conditions (3) and (5) reduce to:

$$\frac{\partial v_\rho}{\partial \tau} = 0, \tag{12a}$$

$$\frac{\partial v_\phi}{\partial \tau} = 0, \tag{12b}$$

$$u_{\text{in}} = u_{\text{out}}, \tag{12c}$$

$$\frac{\partial u_{\text{in}}}{\partial \tau} = \frac{\partial u_{\text{out}}}{\partial \tau} + 2S v_\rho, \tag{12d}$$

at $\tau = 1$.

We expand the solution of the BVP (10-12) in a double series of the form

$$v_\rho\,(\tau,\,\theta) = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} a_{n,k} j_n(\mu_{k,n}\tau) P_n^1(\cos\theta), \tag{13a}$$

$$v_\phi\,(\tau,\,\theta) = i\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} b_{n,k} j_n(\mu_{k,n}\tau) P_n^1(\cos\theta), \tag{13b}$$

$$u_{\text{in}}\,(\tau,\,\theta) = \sum_{k=1}^{\infty}\sum_{n=0}^{\infty} c_{n,k} j_n(\mu_{k,n}\tau) P_n(\cos\theta), \tag{13c}$$

$$u_{\text{out}}\,(\tau,\,\theta) = \sum_{k=1}^{\infty}\sum_{n=0}^{\infty} c_{n,k} \frac{j_n(\mu_{k,n})}{\tau^{n+1}} P_n(\cos\theta), \tag{13d}$$

where P_n^1 is the Legendre function, P_n is the Legendre polynomial, $j_n(x)$ is the $n-$th order spherical Bessel function and $a_{n,k}$, $b_{n,k}$, $c_{n,k}$ and $\mu_{k,n}$ are unknown coefficients. Note that (13d) is the most general solution of equation (11) that satisfies the boundary condition (12c). Substitution of the above trial solution into equations (10) leads to

$$\sum_{k=1}^{\infty}\left[\pi S^2 h_r a_{n,k} - \left(\mu_{k,n}^2 + \pi S^2 h\right) b_{n,k}\right] j_n\left(\mu_{k,n}\tau\right) = 0,$$

$$n \geq 1$$

$$\sum_{k=1}^{\infty}\Big[\pi S^2 h_r b_{n,k} - \left(\mu_{k,n}^2 + \pi S^2 h\right) a_{n,k}$$
$$+ \pi S \mu_{k,n}\left(\frac{c_{n-1,k}}{2n-1} + \frac{c_{n+1,k}}{2n+3}\right)\Big] j_n\left(\mu_{k,n}\tau\right) = 0, n \geq 1$$
$$\sum_{k=1}^{\infty}\Big[2S\mu_{k,n}\Big(\frac{(n+1)(n+2)}{(2n+3)} a_{n+1,k} \tag{14}$$
$$+ \frac{n(n-1)}{(2n-1)} a_{n-1,k}\Big) + \mu_{k,n}^2 c_{n,k}\Big] j_n\left(\mu_{k,n}\tau\right) = 0, n \geq 0.$$

Similarly substitution of the trial solution (13) into the boundary conditions (12) leads to

$$\sum_{k=1}^{\infty}\begin{Bmatrix} a_{n,k} \\ b_{n,k} \end{Bmatrix} \mu_{k,n}\, j_n'(\mu_{k,n}) = 0,\ n \geq 1$$
$$\sum_{k=1}^{\infty}\Big[2S\Big(\frac{(n+1)(n+2)}{(2n+3)} j_{n+1}\left(\mu_{k,n}\right) a_{n+1,k} \tag{15}$$
$$- \frac{n(n-1)}{(2n-1)} j_{n-1}\left(\mu_{k,n}\right) a_{n-1,k}\Big) - \mu_{k,n} j_{n-1}\left(\mu_{k,n}\right) c_{n,k}\Big]$$
$$= 0,\ n \geq 0$$

where the prime denotes differentiation with respect to the argument. So far the solution procedure is identical to that followed in previous studies [25,26]. The difference in the present solution procedure is on the proper selection of the $\mu_{k,n}$ and thus the radial dependence of the solution. The (arbitrary) definition used in Refs. [25,26] was the set of zeros of $j_1'(x) = 0$. Though from physical reasoning we would like to select the $\mu_{k,n}$ such that $j_n'(\mu_{k,n}) = 0$ in order to compare our results with the limit of exchange resonance modes [20], this is not feasible since in such a selection we can no longer make use of the orthogonality of spherical Bessel functions, which is of the form:

$$\int_0^1 \tau^2 j_n(\mu_{k,n}\tau) j_n(\mu_{\ell,n}\tau) d\tau = I_1\, \delta_{k\ell}, \tag{16}$$

where $I_1 = \frac{\mu_{k,n}}{2}\left[j_n'(\mu_{k,n})\right]^2$, for $\alpha_2 = 0$ and $I_1 = \frac{1}{2\mu_{k,n}}\left[\left(\frac{\alpha_1}{\alpha_2}\right)^2 + \mu_{k,n}^2 - \left(n+\frac{1}{2}\right)^2\right]\left[j_n(\mu_{k,n})\right]^2$, for $\alpha_2 \neq 0$, $n + \frac{1}{2} > -1$ and $\mu_{k,n}$ the k first positive roots of

$$\sqrt{x}\left[\left(\alpha_1 + \frac{\alpha_2}{2}\right) j_n(x) + \alpha_2 x j_n'(x)\right] = 0, \tag{17}$$

with α_1, α_2 real constants. In the following we will consider the case where $\alpha_2 = 0$ and thus for $x \neq 0$ equation (17) results in

$$j_n(\mu_{k,n}) = 0, \quad n \geq 0, \quad k \geq 1. \tag{18}$$

Due to (18) $j_n'(\mu_{k,n}) = j_{n-1}(\mu_{k,n}) = -j_{n+1}(\mu_{k,n})$, Eqs. (15) are written as

$$\sum_{k=1}^{\infty} a_{n,k}\,\mu_{k,n}\,j_{n+1}(\mu_{k,n}) = 0,\; n \geq 1 \tag{19a}$$

$$\sum_{k=1}^{\infty} b_{n,k}\,\mu_{k,n}\,j_{n+1}(\mu_{k,n}) = 0,\; n \geq 1 \tag{19b}$$

$$\sum_{k=1}^{\infty}\Big\{2S\Big[\frac{(n+1)(n+2)}{(2n+3)}a_{n+1,k} + \frac{n(n-1)}{(2n-1)}a_{n-1,k}\Big] + \mu_{k,n}\,c_{n,k}\Big\}j_{n+1}(\mu_{k,n}) = 0, \quad n \geq 0. \tag{19c}$$

Use of the orthogonality relation (16) with $\alpha_2 = 0$ in Eqs. (14) results into

$$\pi S^2 h_r a_{n,k} - \left(\mu_{k,n}^2 + \pi S^2 h\right) b_{n,k} = 0,\; n \geq 1,\; k \geq 1 \tag{20a}$$

$$\pi S^2 h_r b_{n,k} - \left(\mu_{k,n}^2 + \pi S^2 h\right) a_{n,k} + \pi S \mu_{k,n}\left(\frac{c_{n-1,k}}{2n-1} + \frac{c_{n+1,k}}{2n+3}\right) = 0, \quad n \geq 1,\; k \geq 1 \tag{20b}$$

$$2S\mu_{k,n}\left[\frac{(n+1)(n+2)}{(2n+3)}a_{n+1,k} + \frac{n(n-1)}{(2n-1)}a_{n-1,k}\right] + \mu_{k,n}^2 c_{n,k} = 0, n \geq 0, k \geq 1. \tag{20c}$$

Before proceeding to the further solution some elementary cases will be discussed.

Case 1 ($\mu_{k,n} = 0,\ \forall k, n$):

Then the BCs (15) are satisfied and (14) results in $h_r = \pm h$, for $a_{n,k}, h \neq 0$ and the magnetization precesses uniformly.

Case 2 ($\mu_{k,n}$ solutions of Eq. (18)):

For $n = 0$ Eq. (14.3) gives $c_{0,k} = -4S\,a_{1,k}/3\mu_{k,0}$ and (15.3) is automatically satisfied. For $n \geq 1$ and $k = 1$ $\mu_{1,k} = 0$, $\forall n \geq 1$ and (14) results in "uniform like" precession modes, with $h_r^{(1,n)} = \pm h$ for $a_{n,1}, h \neq 0$. But keeping only one term in the expansion is meaningless since it results in zero magnetization. Thus we just have to examine the case where $n \geq 1$ and $k \geq 2$. Then by solving (20a) for $b_{n,k}$ and (20c) for $c_{n,k}$ and substituting the results into (20b) we obtain the following recurrence relation

$$a_{n+2,k} = \beta_{n,k} a_{n,k} + \gamma_{n,k} a_{n-2,k}, \quad n \geq 1, \quad k \geq 2, \tag{21}$$

with

$$\begin{aligned} \beta_{n,k} = & \frac{(2n+5)}{(n+2)(n+3)} \Bigg\{ (2n+3) \\ & \times \frac{\mu_{k,n+1}}{\mu_{k,n}} \frac{\left(\pi S^2 h_r\right)^2 - \left(\mu_{k,n}^2 + \pi S^2 h\right)^2}{2\pi S^2 \left(\mu_{k,n}^2 + \pi S^2 h\right)} \\ & - \frac{n(n+1)}{(2n-1)} \left[\frac{\mu_{k,n+1}}{\mu_{k,n-1}} \frac{(2n+3)}{(2n+1)} + 1 \right] \Bigg\} \end{aligned} \tag{22}$$

$$\gamma_{n,k} = -\frac{\mu_{k,n+1}}{\mu_{k,n-1}} \frac{(n-1)(n-2)(2n+3)(2n+5)}{(2n-1)(2n-3)(n+2)(n+3)} \tag{23}$$

and $a_{-n,k} = 0, \quad \forall n \geq 1$. After solving (20c) for $c_{n,k}$ and substituting the result into (19c) we see that the later is automatically satisfied. Thus for this case we just have to satisfy (19a-19b), which are rewritten as

$$\begin{aligned} & \sum_{k=1}^{\infty} a_{n,k}\, \mu_{k,n}\, j_{n+1}\left(\mu_{k,n}\right) = 0, \\ & \sum_{k=1}^{\infty} \frac{\pi S^2 h_r}{\mu_{k,n}^2 + \pi S^2 h} a_{n,k}\, \mu_{k,n}\, j_{n+1}\left(\mu_{k,n}\right) = 0 \end{aligned} \tag{24}$$

From the recurrence relation (21) it is not difficult to show that

$$\begin{aligned} & a_{2\ell-1,k} = F_{2\ell-1,k}\, a_{1,k}, \quad \forall \ell \geq 1,\, k \geq 2 \\ & a_{2\ell,k} = F_{2\ell,k}\, a_{2,k}, \quad \forall \ell \geq 1,\, k \geq 2, \end{aligned} \tag{25}$$

with

$$F_{n,k} = \beta_{n-2,k} F_{n-2,k} + \gamma_{n-2,k} F_{n-4,k}, n \geq 3,\, k \geq 2, \tag{26}$$

and $F_{1,k} = F_{2,k} = 1$, $F_{0,k} = F_{-1,k} = 0$. Then substitution of (25) into the boundary conditions (24) results in the linear system:

$$\sum_{k=2}^{\infty} E_{\ell,k}^{(j)} a_{1,k} = 0, \quad \ell \geq 1,$$
$$\sum_{k=2}^{\infty} G_{\ell,k}^{(j)} a_{2,k} = 0, \quad \ell \geq 1, \tag{27}$$

$j = 1, 2$, with

$$E_{\ell,k}^{(1)} = F_{2\ell-1,k}\, \mu_{k,2\ell-1}\, j_{2\ell}\left(\mu_{k,2\ell-1}\right)$$
$$G_{\ell,k}^{(1)} = F_{2\ell,k}\, \mu_{k,2\ell}\, j_{2\ell+1}\left(\mu_{k,2\ell}\right)$$
$$E_{\ell,k}^{(2)} = \frac{\pi S^2 h_r}{\mu_{k,2\ell-1}^2 + \pi S^2 h} E_{\ell,k}^{(1)},$$
$$G_{\ell,k}^{(2)} = \frac{\pi S^2 h_r}{\mu_{k,2\ell}^2 + \pi S^2 h} G_{\ell,k}^{(1)}. \tag{28}$$

The infinite linear system (27) can be written in the form

$$\begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{29}$$

with $E_{2\ell-1,k} = E_{\ell,k}^{(1)}$, $E_{2\ell,k} = E_{\ell,k}^{(2)}$, $G_{2\ell-1,k} = G_{\ell,k}^{(1)}$, $G_{2\ell,k} = G_{\ell,k}^{(2)}$ $\ell,\ k \geq 1$. The fact that $k \geq 1$ does not mean that we take into account the first root of Eq. (18). In order for the system to have non-trivial solution the following condition has to be satisfied.

$$\det \begin{pmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{pmatrix} = 0. \tag{30}$$

IV. NUMERICAL RESULTS AND DISCUSSION

Equation (30) is the frequency equation. It is solved numerically using a matrix determinant computation routine along with a bisection method to refine steps close to its roots.

Since the infinite system is truncated into a finite one, an iterative convergence method is employed. The convergence criterion used is $\left|h_r^{(k,n)} - h_r^{(k-1,n)}\right| \leq 10^{-3}$. All the computations were performed for $2 \leq k \leq \ell$ and $1 \leq n \leq 2\ell$ with $\ell = 1, 2, 3, \ldots, 34$.

Computations have been performed for the material constants of Ni (see Table I), for varying sphere radius $R \in [0.5, 1]\mu$m. The gyromagnetic ratio was assigned the value $\gamma_0 = g \times 8.7939 \times 10^{-3} GHz/Oe$, with $g \simeq 2.2$. Calculations for Co-Ni microspheres will be presented in a future investigation. The first seventeen dimensionless eigenfrequencies, h_r, are cited in Table II, for the (k,n)=(34,68) mode. For Ni microparticles, with $R = 0.7\,\mu m$, the experimentally measured dimensionless resonance frequency $h_r^{exp} = 2.2468$ [15,17,19] differs less than 9% from the $11^{\rm th}$ eigenfrequency $h_r^{(34,68)} = 2.4636$ of Table II. Since, in general, the exchange constant is not very well defined, and usually is determined from resonance experiments, the present theory can be fitted to experimental data for that purpose. Thus assuming for Ni microspheres, an exchange constant of $A = 2 \times 10^{-7}\, erg/cm$ the disagreement with experiment is removed for the $12^{\rm th}$ eigenfrequency $h_r^{(24,48)} = 2.2496$ of Table III (error $\simeq 0.1\%$).

The size dependence of the resonance modes for Ni microspheres is plotted in Figure 1, for the eigenfrequencies (7-12) of Table II. The dot corresponds to the experiments of Ref. [15,17,19]. Though for *each* mode is not different from the size dependence of the exchange resonance modes ($h_r \propto S^{-2}$) it differs from that law due to the mixing of the modes. The problem of non-cylindrically symmetric modes is also under investigation.

REFERENCES

* Electronic Mail: cmasalas@cc.uoi.gr

[1] J. L. Kirschvink, Science **256**, May, 967 (1992).

[2] R. Langer, Science **249**, September, 1527 (1990).

[3] W. Andrä and H. Nowak (eds.) *Magnetism in Medicine* (Wiley-VCH, 1998).

[4] C. Bergemann, S. R. Julian, A. P. Mackenzie, A. W. Tyler, D. E. Farrell, Y. Maeno and S. NishiZaki, J. Mag. Mag. Mat. **194**, 45 (1999).

[5] R. E. Rosensweig, *Ferrohydrodynamics* (Cambridge University Press, 1985).

[6] K.. Raj and R. Moskowitz, J. Mag. Mag. Mat. **85**, 233 (1990).

[7] J. P. Dailey, J. P. Philips, C. Li, and J. S. Riffle, J. Mag. Mag. Mat. **194**, pp.140 (1999).

[8] A. Jordan, R. Scholz, P. Wust, H. Fähling and R. Felix, J. Mag. Mag. Mat. **201**, 413 (1999).

[9] A. Jordan, R. Scholz, P. Wust, H. Schirra, T. Schiestel, H. Schmidt and R. Felix, J. Mag. Mag. Mat. **194**, 185 (1999).

[10] R. Hiergeist, W. Andrä, N. Buske, R. Hergt, I. Hilger, U. Richter and W.Kaiser, J. Mag. Mag. Mat. **201**, 420 (1999).

[11] W. Andrä, C. G. d'Ambly, R. Hergt, I. Hilger and W. A. Kaiser, J. Mag. Mag. Mat. **194**, 197 (1999).

[12] W. F. Brown Jr, *Micromagnetics* (John Wiley and Sons, 1963).

[13] A. Aharoni, *Introduction to the Theory of Ferromagnetism* (Oxford Univ. Press, 1996).

[14] L. R. Walker, Phys. Rev. **105**, 390 (1959).

[15] G. Viau, F. Ravel, O. Acher, F. Fiévet-Vincent, and F. Fiévet, J. Appl. Phys. **76**, 6570

(1994).

[16] G. Viau, F. Ravel, O. Acher, F. Fiévet-Vincent, F. Fiévet, J. Mag. Mag. Mat. **140-144**, 377 (1995).

[17] G. Viau, F. Fiévet-Vincent, F. Fiévet, T. Toneguzzo, F. Ravel and O. Acher, J. Appl. Phys. **81**, 2749 (1997).

[18] P. Toneguzzo, O. Acher, G. Viau, F. Fiévet-Vincent and F. Fiévet, J. Appl. Phys. **81**, 5546 (1997).

[19] P. Toneguzzo, Thèse, Université Paris 7-Denis Diderot, Paris (1998).

[20] A. Aharoni, J. Appl. Phys. **69**, 7762 (1991).

[21] A. Aharoni, J. Appl. Phys. **81**, 830 (1997).

[22] V. P. Shilov, J. C. Bacri, F. Gazeau, F. Gendron, R. Perzynski and Yu. L. Raikher, J. Appl. Phys. **85**, 6642 (1999).

[23] F. Gazeau, J. C. Barci, F. Gendron, R. Perzynski, Yu. L. Raikher, V. I. Stepanov and E. Dubois, J. Mag. Mag. Mat. **186**, 175 (1998).

[24] E. Hasmonay, E. Dubois, J. C. Barci, R. Perzynski, Yu. L. Raikher and V. I. Stepanov, Eur. Phys. J. B **5**, 859 (1998).

[25] P. A. Voltairas, C. V. Massalas and I. E. Lagaris, Int. J. Engng. Sci. **31**, 1391 (1993).

[26] P. A. Voltairas and C. V. Massalas, J. Mag. Mag. Mat. **124**, 20 (1993).

[27] *Landolt-Börnstein, Series II/1, Magnetic Properties* (Springer, Berlin 1962), p. 113.

FIGURES

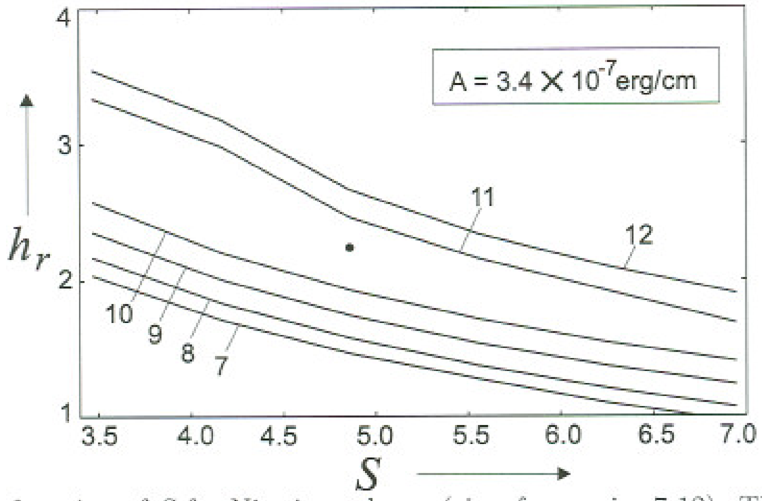

FIG. 1. h_r as a function of S for Ni microspheres (eigenfequencies 7-12). The dot corresponds to experiments of Refs. [15,17,19].

TABLES

TABLE I. Material constants for Ni [27].

$A\,(erg/cm)$	$4\pi M_s\,(Gauss)$	$K_1\,(erg/cm^3)$
3.4×10^{-7}	508.8	-4.26×10^4

TABLE II. Dimensionless eigenfrequencies h_r as a function of sphere radius R, for the (k,n)=(34,68) mode.

	$R = 0.5\,\mu$m	$R = 0.6\,\mu$m	$R = 0.7\,\mu$m	$R = 0.8\,\mu$m	$R = 0.9\,\mu$m	$R = 1.0\,\mu$m
1	1.4061	1.2310	0.9276	0.7152	0.4564	0.2736
2	1.4882	1.3174	1.0642	0.8524	0.5857	0.4233
3	1.7047	1.3973	1.1755	0.9857	0.7684	0.5317
4	1.7447	1.4631	1.2323	1.0942	0.8741	0.6782
5	1.8488	1.5475	1.2991	1.1754	0.9406	0.7978
6	1.9335	1.6186	1.3744	1.2142	1.0098	0.8695
7	2.0403	1.7123	1.4644	1.2750	1.0973	0.9573
8	2.1729	1.8356	1.5770	1.3721	1.2049	1.0660
9	2.3592	2.0094	1.7446	1.5373	1.3703	1.2332
10	2.5786	2.2091	1.9315	1.7156	1.5429	1.4017
11	3.3335	2.9814	2.4636	2.1621	1.9213	1.6830
12	3.5424	3.1764	2.6653	2.3419	2.1016	1.9004
13	3.6358	3.3217	2.8407	2.4739	2.1525	1.9511
14	3.7817	3.4332	2.9318	2.5326	2.2143	2.0019
15	3.9280	3.5792	3.0420	2.6160	2.2788	2.0650
16	4.1016	3.7806	3.1663	2.7119	2.3539	2.1314
17	4.3246	4.0113	3.3444	2.8309	2.4434	2.2176

TABLE III. The first seventeen dimensionless eigenfrequencies h_r for $R = 0.7\,\mu m$ and $A = 2 \times 10^{-7} erg/cm$ for the (k,n)=(24,48) mode.

1	0.6629
2	0.7506
3	0.8008
4	0.8543
5	0.9208
6	1.0782
7	1.1857
8	1.3514
9	2.0717
10	2.1253
11	2.1860
12	2.2496
13	2.3240
14	2.4126
15	2.5234
16	2.6902
17	2.8725