A 2-D PREISACH MODEL FOR INHOMOGENEOUS MAGNETS

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Abstract

Previous work on magnetic hysteresis modeling using the Preisach formalism forms the basis of this work which proposes a 2-D Preisach-type model for anisotropic inhomogeneous RE-TM magnets. The model deals with the two phases in a statistical sense and is not constrained by the specific geometry or the number of soft inclusions. The probability density function of the coercivities is taken to be the weighted sum of two densities, one for each phase. The effect of interactions is accounted for by a third probability density function the shape of which depends on the nature of interactions, exchange or magnetostatic. The effect of misaligned grains is modeled by a fourth distribution. The vector operator used is based on the Stoner-Wohlfarth model of coherent rotation that is rather inadequate but the only one available at the moment. The material properties are related to the parameters of the statistical distributions used by the model and an attempt has been made to make the identification procedure more systematic. The model is identified for a single phase $Sm_2Fe_{14}Ga_3C_2$ sample and a two-phase exchange-coupled $Sm_2Fe_{14}Ga_3C_2/\alpha$-Fe sample. Calculated major and first order minor loops are compared against existing experimental data for the two samples.

I. INTRODUCTION

Modeling of the non-linear behavior is always a challenging task. This is particularly true in the case of hysteresis, in general, and magnetic hysteresis more specifically. In systems exhibiting hysteresis, the present output is a function not only of the present input but also of the sequence of inputs the system has accepted so far. This is evident in the magnetization/field relationship, the stress/strain relationship, or the output/input relationships of economic systems even. Therefore, a model of hysteresis should first of all keep track of the history of the system and adjust the output accordingly. In the case of magnetic hysteresis, the model must also exhibit good vector properties and reproduce both the reversible and the irreversible part of the magnetization response (output) to an applied field sequence (input). It should account for the fact that the reversible component is due to rotation while the irreversible component is due to the switching of the magnetization vector. Both rotation and switching depend on the intrinsic properties and microstructure of the magnet as well as the interactions developed in it. The identification process of such a model must link these material properties to the model parameters. In the case of inhomogeneous systems/magnets, the model must also account for the different properties of and interaction between the two phases.

Using the above as rough guidelines, the Preisach formalism[1] stands out as a good vehicle for the design of a hysteresis model for inhomogeneous magnets. Models using the Preisach formalism record and take into account the history of the material in an elegant and simple manner. Scalar in nature, they are able to reproduce only irreversible processes in which case the reversible component of the magnetization has to be added on. Vector Preisach-type models for homogeneous magnetic materials have also been designed[2] that are able to account for both rotation and switching of the magnetization. The identification of these models is based on bulk measurements of macroscopic material properties. The identification process consists of establishing a well-defined method of linking the material properties to the model parameters and it is usually a challenge.

In this work, we first give a brief overview of the Preisach formalism and proceed with the description of the new model. In an attempt to make the identification process more systematic and test the properties of the new model, the latter has been identified for both a single-phase $Sm_2Fe_{14}Ga_3C_2$ and a two-phase exchange-coupled $Sm_2Fe_{14}Ga_3C_2/\alpha\text{-}Fe$ magnet.

II. DESCRIPTION OF THE MODEL

According to the Preisach formalism (scalar case), a system exhibiting hysteresis can be described by a characteristic probability density function $\rho(H_+, H_-)$ defined over a triangular plane, called the *Preisach plane* (see Fig. 1a). H_+ and H_- are the upper and lower turning points of elementary square loop operators, $\gamma(H_+, H_-) = \pm 1$, like the one shown on Fig. 1b. In the case of a magnetic material, we can think of an elementary loop as the switching characteristic of a collection of grains or particles that for convenience will be referred to as a "pseudoparticle". The total magnetization contained in this pseudoparticle is given by the value of the density function $\rho(H_+, H_-)$ at the point (H_+, H_-) on the plane. When the sum of interactions the elementary loop "experiences" is zero, the loop is centered at $H = 0$ and $H_+ = H_- = H_c$. H_c in this case is not the coercivity of the magnet but the "coercivity" of the "pseudoparticle" represented by the elementary square loop. When the sum of interactions, H_i, is not zero, the loop is shifted to the left or to the right according to the direction of the resultant interaction force, thus making it easier or harder to switch. The plane is bounded by $H_c = 0$, $H_+ = H_{sat}$ and $H_- = -H_{sat}$ where H_{sat} is the field needed for saturation. Note that $\rho(H_+, H_-) = \rho(H_c, H_i)$. On the assumption that the two variables H_c and H_i are independent, the bivariate density function can be written as a product of two probability density functions $\rho(H_c)\rho(H_i)$. The effect of any sequence of fields on the magnetization of the material is retained in the form of a "staircase" boundary separating the regions of positive and negative magnetization (see Fig. 1a). The magnetization as a function of the applied field $\boldsymbol{M}(\boldsymbol{H})$ is then given by the integral of the characteristic material density over the Preisach plane.

The scalar Preisach model keeps track of the history of the medium in a rather elegant manner and can be implemented by fast and rather simple algorithms. A major drawback is its inability to reproduce reversible processes because of the switching characteristic adopted through the scalar operator $\gamma(H_+, H_-)$. Vector Preisach-type models that replace the scalar operator $\gamma(H_+, H_-)$ by a vector one $\boldsymbol{\Gamma}(H_+, H_-)$ have been able to account for the reversible component without further processing. Such an operator is the 2D Stoner-Wohlfarth model[3] (see Fig. 1c) which is also used in this work. In the vector case, the boundary, serving as the "memory" of the system, no longer consists of vertical and horizontal segments only. Rotation makes switching easier and the boundary is curved in such a way as to allow for more switching.

Since the inhomogeneous permanent magnets are RE-TM alloys consisting of a hard and a soft magnetic phase interacting with each other, the new model, in the spirit of the Preisach formalism, takes into account the different characteristics of the two phases in a statistical sense. The density of coercive fields $\rho(H_c)$ is taken to be the weighted sum of two densities $\rho_1(H_{soft})$ and $\rho_2(H_{hard})$ for the soft and hard phases respectively. The weight attached to each function depends on the % content of the soft phase, w. The characteristic density is then written as:

$$\rho(H_+,H_-)=\rho(H_c,H_i)=\rho(H_c)\rho(H_i)=\left[w\rho_1(H_{soft})+(1-w)\rho_2(H_{hard})\right]\rho_3(H_i) \tag{1}$$

The angular dispersion of the misaligned grains is accounted for by the superposition of the response of Preisach planes dispersed according to a fourth distribution $\rho_4(a)$ where a is the angle a plane forms with the easy axis[3].

For an appropriate vector switching mechanism, $\Gamma(H_+,H_-)$, the magnetization response of the magnet to a sequence of applied fields $H_t, t=0,1,2,\ldots$ is then given by:

$$\boldsymbol{M(H_t)}=\int_{-\pi/2}^{\pi/2}\int_{-H_{sat}}^{H_{sat}}\int_{H_t}^{H_{sat}}\rho_4(a)\rho_3(H_i)\left[w\rho_1(H_{c,soft})+(1-w)\rho_2(H_{c,hard})\right]\boldsymbol{\Gamma}(H_+,H_-)dH_+dH_-da \tag{2}$$

The identification of such a model is not a straightforward process. The parameters of the four probability density functions ρ_j, with $j=1,2,3,4$, must be related to macroscopic properties of the material, such as the coercivity H_c, the saturation magnetization M_s, the squareness, S, and coercivity squareness, S^*, of the alloy as well as and the percentage content of each phase.

III. RESULTS AND DISCUSSION

In order to test the model, all four probability density functions were assumed to be normal $N(\mu_i,\sigma_i^2)$, and the effect of the densities' parameters on the shape of the M-H curve was first studied.

To take care of the angular dispersion, Preisach planes were dispersed around the easy axis in steps of 5° $(-90^o<\alpha<+90^o)$. Each Preisach plane was discretized into 50 cells that were assigned weights according to the characteristic density. Each cell corresponds to a "pseudoparticle" obeying to the switching and rotation mechanism of the Stoner-Wohlfarth model. The orientation of the magnetization vector is calculated for each cell. The sum of the weighted contributions of each cell yields the magnetization response of each Preisach plane to the applied field. The superposition of the responses of all the Preisach planes yields the total magnetization.

Since all the probability density functions are normal, eight parameters need to be determined for the identification of the model: four mean values, μ_i and four standard deviations, σ_i, $i = 1,2,3,4$. Since the misaligned grains are taken to be symmetrically dispersed around the material easy axis, the function of the angular dispersion $\rho_4(\alpha)$ is centered at 0 degrees, $\mu_4 = 0$. The squareness S of the major loop is controlled solely by the standard deviation[4], σ_4. The function of interactions $\rho_3(H_i)$ is also centered at zero field on the assumption that the *average* interaction field experienced is zero, $(\mu_3 = 0)$. The standard deviation σ_3 is controlling the slope of the major loop around the coercivity: the wider the distribution is the lower is the slope. The parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ of the two coercivity functions affect mainly the coercivity of the loop and are the most difficult to determine.

Experimental data of single-phase $Sm_2Fe_{14}Ga_3C_2$ and two-phase exchange-coupled $Sm_2Fe_{14}Ga_3C_2/\alpha$-Fe, as reported by *Feutrill et. al.* [5], was used to test the hysteresis behavior of the model. The two sets of model parameters used for the two samples are shown in Table I. The percentage content of the soft phase, w, is 40%.

Fig. 2 shows major loop curves calculated by the model for both sets of parameters. In the single-phase case, the coercivity is roughly 4 times larger and can be empirically expressed as $H_c \approx 3 \times \mu_2$ while in the two-phase case, $H_c \approx 3 \times (w \times \mu_1 + (1-w) \times \mu_2)$. The standard deviation of the interactions density in the exchanged-coupled case is 2.5 times larger than that in the single-phase which is reflected in the coercivity squareness of the two loops. This suggests a way to distinguish between exchange and magnetostatic interactions since exchange-coupled magnets exhibit higher coercivity squareness.

A comparison between measured and calculated major and minor loops curves is shown in Figs. 3-4. The agreement between theoretical and experimental curves is better in the single-phase case. It has been found that the reversible susceptibility of the two-phase magnet has two peaks, one for each phase while in the single-phase case there is only one peak, at the coercivity[5]. This kind of behavior was not possible to be reproduced by the model. There are two possible explanations for that. First, the S-W model assumes coherent rotation of the magnetization of a magnetically isolated ellipsoid with uniaxial anisotropy. However, it has been found that inhomogeneous magnets do not rotate coherently[6]. A vector operator more appropriate for inhomogeneous materials is currently the subject of work in progress. Second, when the magnetization becomes negative in the descending branch of the loop the model predicts "less switching" than in the measured curve. On the assumption that as the applied field becomes more negative the effect of the stray fields becomes more prominent and assists the switching process, the probability density function of the interactions need not symmetrical. Current work also involves a study of alternative choices of probability density functions and the development of an algorithm for the identification process.

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TABLE I: Model parameters for the single-phase $Sm_2Fe_{14}Ga_3C_2$ and two-phase exchange-coupled $Sm_2Fe_{14}Ga_3C_2$/α-Fe samples.

	Single-phase	Two-phase
μ_1 (kOe)	n/a	800
σ_1 (kOe)	n/a	500
μ_2 (kOe)	5500	1200
σ_2 (kOe)	2000	700
μ_3 (kOe)	0	0
σ_3 (kOe)	800	2000
μ_4 (degs)	0	0
σ_4 (degs)	40	60

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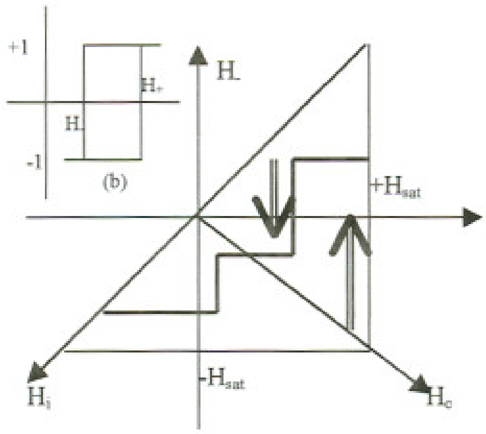
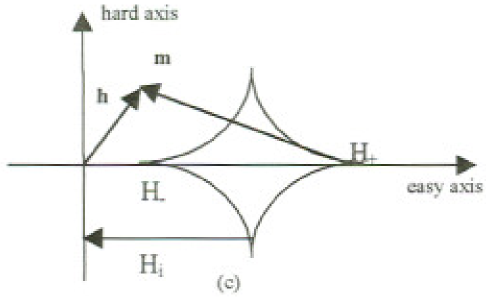


(α)



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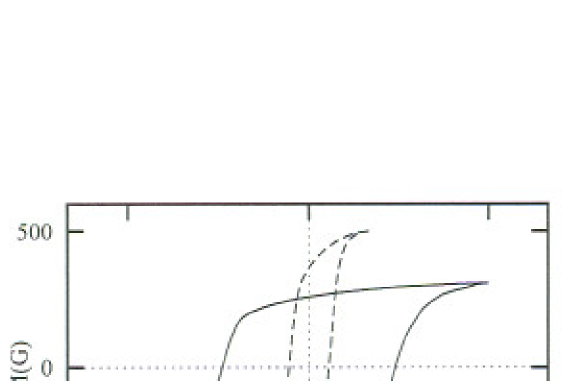
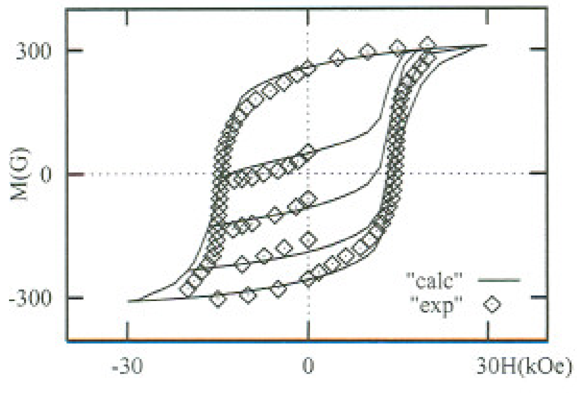
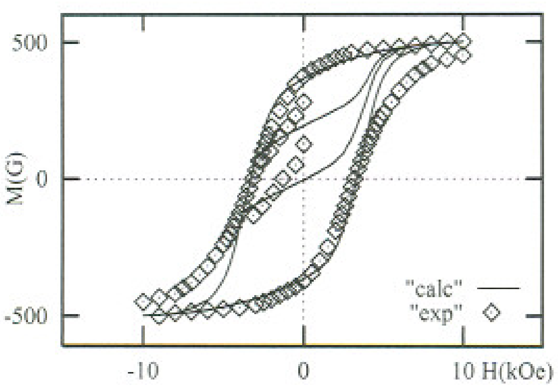


500
0
-500
M(G)
-30
0
30H(kOe)
"single-phase"
"two-phase"



300
0
-300
M(G)
-30
0
30H(kOe)
"calc"
"exp"



500
0
-500
M(G)
-10
0
10 H(kOe)
"calc"
"exp"