

**INVERSE MAGNETOSTRICTION EFFECT IN THE
FERROMAGNETIC FILMS
(A MICROMAGNETIC MODEL)**

P.A. VOLTAIRAS, D.I. FOTIADIS and C.V. MASSALAS

18-99

Preprint no. 18-99/1999

**Department of Computer Science
University of Ioannina
451 10 Ioannina, Greece**

INVERSE MAGNETOSTRICTION EFFECT IN THIN FERROMAGNETIC FILMS (A MICROMAGNETIC MODEL)

P.A. Voltairas^{a,†}, D.I. Fotiadis^a, and C.V. Massalas^b

^a*Department of Computer Science, University of Ioannina, GR 451 10 Ioannina, Greece*

^b*Department of mathematics, Division of Applied Mathematics and Mechanics, University of Ioannina, GR 451 10 Ioannina, Greece*

The effect of stress on magnetic properties of ferromagnetic materials, well known as Inverse Magnetostriction Effect (IME), is studied for thin ferromagnetic films. The model used, is a micromagnetic one proposed in Ref. [1]. To calculate the stress dependence on coercivity, numerical non-uniform (NU) solutions, resulting from Finite Element Model (FEM) calculations, for the Brown's magnetoelastic equations are presented and compared with uniform Stoner-Wolfarth (SW) ones. We study only the case where the applied stresses are oriented parallel to the field's direction (Case 2 of Ref. [1]). Energy considerations confirm that the NU modes are unfavorable throughout the magnetization reversal.

PACS: 75.60.Ej; 75.70.-i; 75.80.+q.

Keywords: Micromagnetics; Thin Film; Magnetostriction; Magnetization Reversal.

1. Introduction

Magnetostrictive materials belong to the class of "smart material" since their hysteresis is controlled by mechanical stresses and their mechanical deformations by magnetic fields [2]. Thus they are very attractive as actuator and sensor devices, such as microrobots, micromotors, etc.[3]. The IME is one of the favorable research topics [2, 4]. A rigorous phenomenological theory of magnetoelastic interactions has been proposed [5], but due to its complexity only elementary solutions have been obtained [6]. A domain rotational model, that examines the role of material parameters and mechanical stresses on magnetic and magnetostrictive hysteresis has been proposed in an earlier work [7], but it ignored NU modes of magnetization reversal. Such NU modes have been investigated in the special case of pure magnetostrictive domain wall oscillations [8]. A one-dimensional micromagnetic model for studying the IME in thin ferromagnetic films, that embodies the non-uniformity in the magnetization distribution in its basic postulates and thus is self-consistent, was proposed in a previous work [1]. Uniform SW solutions as well as the nucleation modes for NU magnetization reversal were presented in Ref. [1] and NU solutions for the case where the applied field is perpendicular to the stress direction in Ref. [9].

In this work we discuss possible NU solutions to the previously proposed micromagnetic model that accounts for the IME in thin ferromagnetic films. We consider only the case where the applied field is parallel to the stress direction. Brown's micromagnetic equilibrium equations are solved numerically by the Galerkin Finite Element Method (GFEM). We look for typical Bloch wall solution to the problem under discussion [10]. The coercivity-stress ($h_c(\sigma)$) and remanence-stress ($m_r(\sigma)$) laws are obtained and compared with analytical [1] and experimental results [4]. Size effects are also discussed.

[†]Corresponding author. Fax: +30 (651) 97320; e-mail: pvolter@cs.uoi.gr

2. Brown's Micromagnetic Equations

We consider a thin-film of thickness $d = 2a$ along the x Cartesian axis. In its undeformed state the thin-film has its principal axes along the coordinate axis. The magnetization reverses in the zx -plane and the infinitesimal plane strains are confined to be uniform ($e_{xx} = -e_{zz} = e$, $e_{xz} = 0$). The applied uniform mechanical stresses are perpendicular to the film plane, along the direction of the uniform external magnetic field (Case 2 of Ref. [1]). The micromagnetic problem is then described by Eqs. (30) of Ref. [1]. We rescale them according to

$$\bar{S} = S \sqrt{\frac{|h_k|}{2}}, \quad (1)$$

where S is the reduced half thin-film thickness and h_k the reduced magnetocrystalline anisotropy constant. Due to (1), Eqs. (30) of Ref. [1] become,

$$\frac{d^2\theta}{d\bar{x}^2} + \left(\frac{\sin 4\theta}{4} - \left(4\bar{h}_{me}e + \frac{1}{|h_k|} \right) \frac{\sin 2\theta}{2} + \bar{h} \cos \theta \right) \bar{S}^2 = 0, \quad (2a)$$

$$\left. \frac{d\theta}{d\bar{x}} \right|_{\bar{x}=\pm\bar{S}} = 0, \quad (2b)$$

$$e(\bar{\sigma}, \bar{h}) = \frac{4\bar{\sigma} - 2 \left(2\bar{h}_{me} - \frac{1}{|h_k|} \right) \sin^2 \theta + \sin^2 2\theta}{4 \left(\bar{h}_e + 2\bar{h}_{me} \sin^2 \theta \right)} \quad (2c)$$

$$\bar{h}_{me}^* = \bar{h}_{me} e(\bar{\sigma}, \bar{h}) + \frac{\sin^2 \theta}{2} \quad (2d)$$

where

$$\begin{aligned} \bar{h}_{me} &= \frac{h_{me}}{|h_k|} = \frac{B_1}{2|K_1|}, & \bar{h}_{me}^* &= \frac{h_{me}^*}{|h_k|} = \frac{B_2}{2|K_1|}, & \bar{h} &= \frac{h}{|h_k|} = \frac{H_x^o}{|H_K|}, \\ \bar{\sigma} &= \frac{\sigma}{|h_k|} = \frac{T_x}{2|K_1|}, & \bar{h}_e &= \frac{h_e}{|h_k|} = \frac{c_{11} - c_{12}}{2|K_1|}, \end{aligned} \quad (3)$$

H_K is the anisotropy field ($H_K = 2K_1/\mu_0 M_s$) and M_s , K_1 , B_i , c_{ij} , $i, j = 1, 2$ are the saturation magnetization, the anisotropy, the magnetoelastic and the elastic constants, respectively. Here θ , \bar{h} , $\bar{\sigma}$, \bar{h}_e , \bar{h}_{me} and \bar{h}_{me}^* denote: the magnetization angle with respect to the applied field, the reduced applied magnetic field, the reduced applied mechanical stress, the reduced elastic and the reduced first and second magnetoelastic constants, respectively. The variation of B_2 due to the applied field H_x^o can be considered small compared to its tabulated value [11] and thus Eq. (2d) will be neglected in the following. We limit our discussion to materials with negative magnetocrystalline anisotropy ($h_k = -|h_k|$) and negative magnetostriction ($\bar{h}_{me}, \bar{h}_{me}^* \geq 0$), like Ni. Due to (1):

$$\bar{S} = \frac{a}{\delta}, \quad (4)$$

where $\delta = (C/2|K_1|)^{\frac{1}{2}}$ is the domain wall width, C denotes the exchange constant. The rescaled nucleation field (Eq. (61) of Ref. [1]) is expressed as

$$\bar{h}_n = 1 + \frac{1}{|h_k|} + \frac{2\bar{h}_{me} \left(\frac{1}{|h_k|} - 2\bar{h}_{me} + \bar{\sigma} \right)}{\bar{h}_e + 2\bar{h}_{me}}. \quad (5)$$

For convenience, in the following we ignore the bars in the definitions (3-4,6). The reduced magnetization $m = M_x/M_s$ along the field direction is computed as follows,

$$m \equiv \frac{1}{2S} \int_{-S}^{+S} \cos \theta dx. \quad (6)$$

Magnetostriction curves are computed according to the mean strain value $\langle e \rangle$ defined by:

$$\langle e \rangle \equiv \frac{1}{2S} \int_{-S}^{+S} e(\sigma, h) dx. \quad (7)$$

Since the elliptic integral obtained after integrating once Eq. (2a) cannot be computed analytically, a numerical solution is proposed.

3. Numerical Solution

In the present work we prefer to solve Brown's micromagnetic equations (2), with the GFEM. In this method the solution is expanded in quadratic elements

$$\theta = \sum_{i=1}^3 \theta_i \Phi_i(\xi), \quad (8)$$

where Φ_i is the quadratic basis function and θ_i is the unknown at the i^{th} node of the element. The GFEM calls for the weighted residuals R_i to vanish at each nodal position i :

$$R_i = \int_D L(\theta) \det(\mathbf{J}) d\xi, \quad (9)$$

where L is the nonlinear operator that acts on θ (see Eq. (2)) and \mathbf{J} is the Jacobian of the isoparametric mapping, with $D = [-S, S]$ the domain of solution. For the problem under discussion

$$R_i = - \int_0^1 \left[\frac{\theta' \Phi_i'}{(\Delta x)^2} - \left(\frac{\sin 4\theta}{4} - \left(4 h_{me} e + \frac{1}{|h_k|} \right) \frac{\sin 2\theta}{2} + h \cos \theta \right) S^2 \Phi_i \right] \Delta x d\xi, \quad (10)$$

where $(\cdot)' \equiv d/d\xi$ and $\Delta x \equiv 2S/N_e$ and N_e is the total number of elements used. All the numerical results obtained correspond to material constants of Ni [1]. In general each solution obtained, represents a point in the three dimensional space designated by (h, σ, S) . The results presented here are for varying h, σ and $S = 1.5$. For the material constants of Ni this corresponds to a film thickness $84.75nm$. In all our numerical computations, the degeneration of the solution to the SW limit represents more than a problem. This is because the nucleation mode for the presented problem is a coherent one [1]. Like in all nonlinear BVPs, and the present one (Eqs. (2)) possesses multiple solutions. We searched for typical Bloch domain walls excitations [10]. All the obtained solutions are summarized in the three modes shown in Fig. 1, for the same set of parameters: a compressive stress $\sigma = -5 \times 10^3$ and a reduced thickness $2S = 3$. Close to jumping fields in the hysteresis curve the numerical approach fails [14] but for very small systems with only few discretization points, the jumping fields can be computed with high accuracy [15], by refining the continuation step in the initial and the final solution branch. Magnetization profiles at various stages of the reversal are also included in Fig. 1. The departure from saturation for mode A is at $h_s = 0.1 < h_n = 0.8076$, and corresponds to typical perturbation of the SW mode B. At $h = -0.87$ the magnetization profile of mode A has the classical Bloch

wall structure, while at $h = -0.88$ a complete Bloch wall has been formed and at $h = -8$ two Bloch walls are formed. For the mode C, that is a physically non acceptable, since it corresponds to negative susceptibilities and passes from the origin ($m(h = 0) = 0$), continuation for $h_c = -0.343 \leq h \leq -0.31$ resulted into degeneration to the SW mode B. The nature of the singularity of this point, for mode C, cannot be determined and it is under investigation. Notice that mode A corresponds to higher coercivity and remanence with respect to the SW mode B. Typical solutions of the BVP for modes A and C are analogous to those presented elsewhere [9]. Energy estimations confirm that mode A is an unfavorable one throughout the magnetization reversal compared to mode B.

Without being able to prove it, we believe that the Brown-Shtrikman theorem for 1-D micromagnetic problems [12], derived for a rigid specimen, is also applicable and for an elastic ferromagnet that deforms uniformly and has no volumetric changes ($e_{ii} = 0$), confirming that the obtained NU solutions are unstable. This is because the only element that changes due to magnetostriction in such a deformation mode is the orientation of the magnetic easy axis, with respect to the crystallographic axes, and not its distribution within the material. Typical magnetization curves for mode A, for varying tensile and compressive stresses are plotted in Fig. 2a. Magnetostriction curves are plotted in Fig. 2b. Notice the shift of the magnetostriction curves along the strain axis due to the applied stress, as well as the change in their shape, with the departure from resultant strain observed at positive fields for tensile stresses. Though the remanence of mode A seems to decrease with applied stress, as it is the case for SW reversal [1], with a bit higher values, the coercivity has a minimum for compressive stresses and then increases. The $h_c(\sigma)$ law, for mode A, is given in Fig. 3 (the enclosed figure corresponds to $m_r(\sigma)$ law). For the particular case of Ni, the coercivities of Fig. 3 are as high as $H_c = 255 \text{ kA/m}$ for stresses $T_x = \pm 100 \text{ MPa}$. Thus the NU solutions of mode A are proved to be worst candidates for explaining the observed behavior in related experiments [4], than the SW coherent solutions of mode B [1]. Simulations have also been performed for varying size parameter $S > 1.5$ and all resulted in a decrease of the coercivity and remanence of mode A, as it is the case for experiments [4].

4. Conclusions

The one-dimensional micromagnetic model proposed in Ref. [1], that accounts for the IME in thin ferromagnetic films was solved numerically by the GFEM. Three branches in the magnetization curve were obtained: two NU modes that correspond to positive (mode A) and negative (mode C) susceptibilities, respectively, and the Stoner-Wolfarth (SW) mode (mode B) studied previously [1]. Mode A is energetically unfavorable throughout the magnetization reversal, compared to the SW coherent rotation. Mode C is physically non permissible one, since it corresponds to negative susceptibilities, and degenerates for large negative fields to the SW mode B. The coercivity and remanence of mode A are higher than those of the SW mode B and thus fail to explain the related experiments [4]. Thus, the NU magnetization distribution along the thin films thickness, is not capable to explain experimental results. Modification of the model after taking into account shearing and NU strains might be a step towards quantitative agreement with experiment. The general stability is not studied.

References

- [1] P. A. Voltairas, D. I. Fotiadis and C. V. Massalas, *Int. J. Eng. Sci.* (to appear).
- [2] R. M. Bozorth, *Ferromagnetism* (D. van Nostrand Company Inc., Princeton NY, 1951).
- [3] K. I. Arai and T. Honda, *Robotica* 14 (1996) 477; M. Takezawa et al., *J. Appl. Phys.* **83** (1998) 7303.
- [4] L. Callegaro et al. , *Appl. Phys. Lett.*, **68** (1996) 1279.
- [5] W. F. Brown, *Magnetoelastic Interactions* (Springer tract in natural Philosophy Vol. 9, Ed. by C. Truesdell, Berlin, 1966).
- [6] F. C. Moon, *Magnetosolid Mechanics* (John-Wiley and Sons, NY, 1984).
- [7] P. A. Voltairas, D. I. Fotiadis and C. V. Massalas, *J. Mag. Mag. Mat.* (to appear).
- [8] M. Kleman, *Phil. Mag.*, **19** (1969) 285.
- [9] P. A. Voltairas, D. I. Fotiadis and C. V. Massalas, *J. Mag. Mag. Mat.* (submitted).
- [10] A. Hubert and R. Schäfer, *Magnetic Domains (The Analysis of Magnetic Microstructures)* (Springer, Berlin, 1998).
- [11] Landolt-Börstein, *Magnetic Properties III* (Springer, Berlin, 1962).
- [12] W. Fr. Brown and S. Shtrikman, *Phys. Rev.* **125** (1962) 825.
- [13] A. Aharoni, *Introduction to the Theory of Ferromagnetism* (Oxford University Press., NY, 1996).
- [14] A. Hubert and W. Rave, *Phys. Stat. Sol.(b)* **211** (1999) 815.
- [15] J. Ehlert, F. K. Hübner and W. Sperber, *Phys. Stat. Sol. (a)* **116** (1989) 389.
- [16] A. Aharoni, E. H. Frei and S. Shtrikman, *J. Appl. Phys.* **30** (1959) 1956.

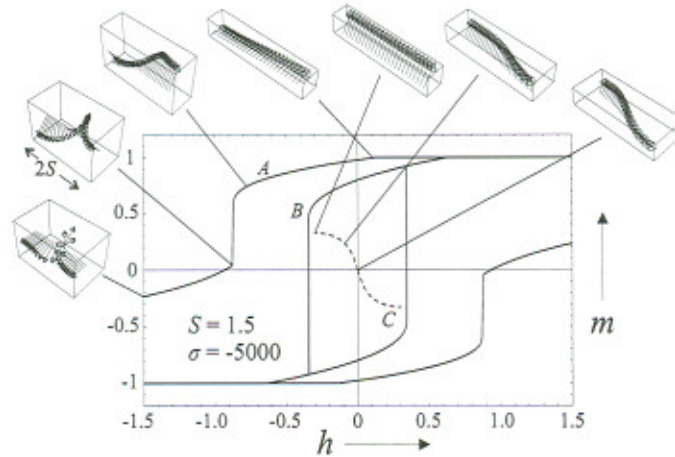


Figure 1: Magnetization curves for $S = 1.5$ and $\sigma = -5 \times 10^3$.

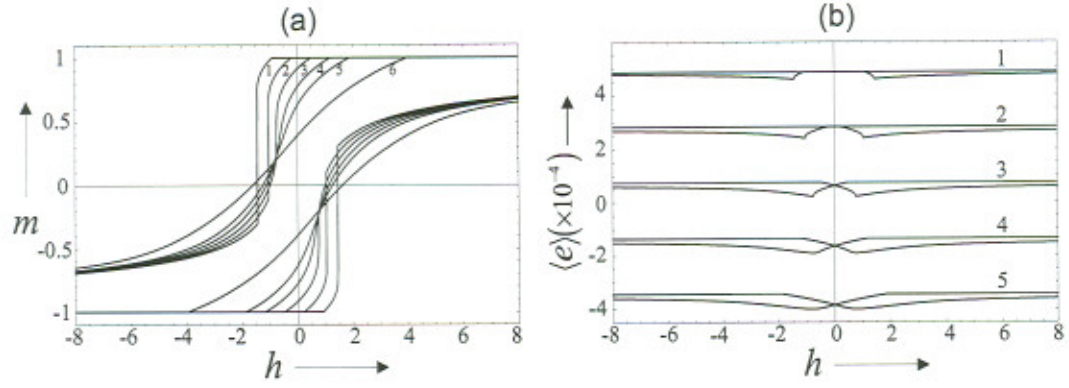


Figure 2: (a) Magnetization and (b) Magnetostriction curves of mode A for varying σ , with $S = 1.5$. The numbering from 1 to 6 corresponds to $\sigma = -2 \times 10^4$, -10^4 , 0 , 10^4 , 2×10^4 and 5×10^4 , respectively.

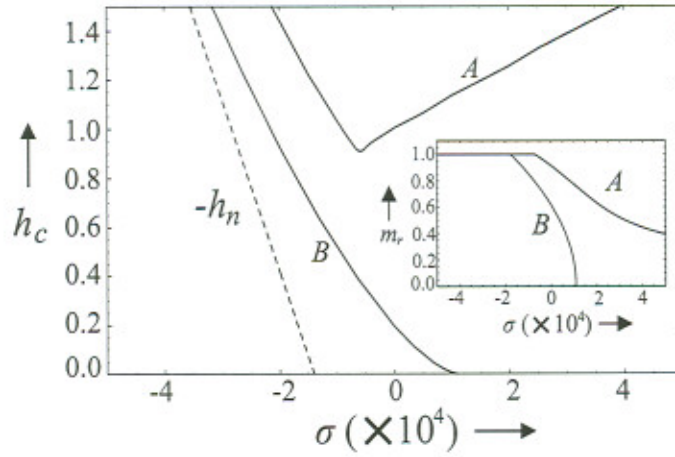


Figure 3: h_c vs. σ for $S = 1.5$ (the SW solution and h_n [1] are plotted for comparison).