

**EFFECT OF MAGNETOSTRICTION ON THE MAGNETIZATION
REVERSAL OF A FINE PARTICLE IN A
SOLID NON-MAGNETIC MATRIX**

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19-98

Preprint no. 19-98/1998

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1. ABSTRACT

Understanding micromagnetic processes in magnetic materials plays a crucial role for the design of new magnetic storage devices with improved characteristics. Among the phenomena that are present in ferromagnetic materials, magnetostriction is the one that has not been studied in extent, due to its complexity. In this paper a simple model is constructed to study the effect of magnetostriction on the magnetization reversal in ferromagnetic materials. The magnetization reversal mechanism is such that self-magnetostatic energy is minimized. In order to simplify the calculations the strains are assumed to be uniform and the magnetization reversal mechanism is coherent (Stoner-Wolfarth rotation). The equilibrium equations for the magnetization and strains are derived from the free energy functional. The dependence of magnetization and magnetostriction curves on the material parameters is discussed. The effect of stress on the magnetization and magnetostriction curves (well known as *inverse magnetostrictive effect*) is also included in the present analysis with proper selection of the boundary conditions.

2. INTRODUCTION

Magnetostrictive materials are very attractive for the production of micro-electromechanical systems (MEMS), such as microrobots, micromotors, etc. [1]. The magnetomechanical problem in its generality is extremely complicated due to its non-linear character [2-4]. Recently, new mathematical tools have utilized (Young measure) to explain the large magnetostriction observed in a class of ferromagnetic materials [5-7]. These theories are applied on large enough material geometries where the division of the crystal into domains is preferable, and thus are not capable of describing accurately the underlying microstructure. The main purpose of the present work is to present, through micromagnetic principles, the coupled magnetization and magnetostriction curves for ultra thin magnetoelastic films. Ultra thin films can be treated approximately as fine single domain particles [8]. In the literature [9-10] the effects of stress and magnetostriction on the magnetization are studied assuming additional terms in the magnetic anisotropy energy density. In order to examine the combined effects of stress and magnetostriction on the magnetization reversal of thin films a simple SW model is introduced that relies on the conventional free energy expansion of the

magnetocrystalline anisotropy energy density in the strains. We note that the model does not take into account the effect of internal stresses.

3. THE VARIATIONAL PRINCIPLE

Recently De -Hua et al. [12] studied the effect of stress on the magnetization reversal of NiFe/NiO thin films. Similar experiments for Ni thin films were also performed by Callegaro et al. [13]. In order to explain observations Callegaro and Puppini [13] and later Nowak, Puppini and Callegaro [14] developed a rotational hysteresis model that incorporates magnetoelastic (stress induced) energy into the magnetic anisotropy energy. This approach is usually followed in the literature by magneticians [15]. In the present study we will not adopt this approach but instead we will base our analysis directly on the energy functional proposed by mechanicians [2-4].

Following experimental observations, we suppose that the thin film extends infinitely in y and z directions and thus serves as a limiting case of an ellipsoid. In its undeformed state the thin film has its principal axes along the coordinate axes. We suppose that the magnetization vector per unit mass $\boldsymbol{\mu}$ rotates uniformly in the y, z plane, under the uniform external magnetic field H° , applied along the coordinate axis z . The uniform mechanical stresses T , are applied along the coordinate axis x . We further assume that the elastic constants do not depend on the magnetization. The crystal is supposed to be cubic with crystallographic axes identical with the coordinate axes. The present analysis relies on infinitesimal uniform plane strains ($u_i = e_{ij}x_j$, $e_{ij} = e_{ji} = \text{const.}$, u_i is the displacement vector and e_{ij} the strain tensor) described by:

$$e_{ij} = \begin{pmatrix} e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -e \end{pmatrix}, \quad (1)$$

with $e = \text{const.}$ This form of strains is indeed a major simplification of the problem, but in some experimental situation such type of strain distribution might be present and thus quantitative agreement is expected. Then $\nabla \cdot \mathbf{u} = 0$ and thus the spontaneous magnetization per unit volume M_s remains constant ($M_s \equiv \rho_0(1 - \nabla \cdot \mathbf{u})\mu_s$), where ρ_0 is the mass density in the undeformed configuration and μ_s is the spontaneous magnetization per unit mass (a characteristic constant of the material). Due to the assumptions made the magnetostatic-self energy is minimized. The Gibbs free energy functional [2-3], under the assumptions introduced, has the following form

$$G = \frac{K}{4} \sin^2 2\theta - B_1 e \cos^2 \theta + (c_{11} - c_{12}) e^2 - \mu_0 H_z^\circ M_s \cos \theta - T_x e, \quad (2)$$

where we denote with: θ the magnetization angle with respect to the applied field H_z° , K the magnetocrystalline anisotropy constant, B_1 the magnetoelastic constant, c_{11}, c_{12} the elastic constants, T_x the uniform stresses along the x direction, and μ_0 is the magnetic permeability of vacuum. Introducing the following dimensionless quantities

$$g = G/\mu_0 M_S^2, \quad m = \mu_z/\mu_s = \cos\theta, \quad \sigma = T_x/\mu_0 M_S^2, \quad h = H_z^0/M_S, \quad (3)$$

$$h_a = 2K/\mu_0 M_S^2, \quad h_{me} = B_1/\mu_0 M_S^2, \quad h_e = (c_{11} - c_{12})/\mu_0 M_S^2,$$

the dimensionless Gibbs free energy g reduces to

$$g = g(m, e) = \frac{1}{2} h_a m^2 (1 - m^2) - h_{me} e m^2 + h_e e^2 - h m - \sigma e. \quad (4)$$

The terms on the right hand side of (4) represent successively, the anisotropy energy, the magnetoelastic energy, the elastic energy, the Zeeman energy and the energy due to the reduced applied uniform mechanical stresses σ . The minima ($\delta g = 0$) of the energy functional (4) can now be obtained analytically.

Minimization of the Gibbs free energy (4) with respect to the magnetization m , under constant strain e , determines the effective field (h^{eff}) in the magnetic material:

$$h^{eff} = h = -m(h_a(2m^2 - 1) + 2h_{me}e). \quad (5)$$

The first term on the right hand side of (5) is a pure magnetic one, while the second accounts for magnetoelastic effects, whether stress or magnetostrictive in origin. Similarly, minimization of the energy functional (4) with respect to the strain e , under constant magnetization m , results in the following stress-strain constitutive law:

$$e = e(\sigma, m) = \frac{\sigma}{2h_e} + \frac{h_{me}m^2}{2h_e}. \quad (6)$$

As expected apart from the first term on the right hand side of (6) that represents the pure mechanical in origin strains, the second term accounts for the magnetostrictive strains (strains that result from the magnetoelastic character of the material). Substitution of equation (6) into equation (5) results in the following magnetic constitutive law

$$h = h(m, \sigma) = -\frac{m}{h_e} [(2h_a h_e + h_{me}^2)m^2 - h_a h_e + h_{me}\sigma]. \quad (7)$$

The stability conditions for the energy minima (6) and (7) are the usual ones

$$\left(\frac{\partial^2 g}{\partial e^2} \right)_m = 2h_e > 0, \quad (8.1)$$

$$\left(\frac{\partial^2 g}{\partial e^2} \right)_m = \frac{1}{h_e} [h_a h_e (1 - 6m^2) - h_{me}\sigma - h_{me}^2 m^2] > 0, \quad (8.2)$$

$$\left(\frac{\partial^2 g}{\partial e^2} \right)_m \left(\frac{\partial^2 g}{\partial m^2} \right)_e - \left(\frac{\partial^2 g}{\partial m \partial e} \right)^2 = h_a h_e (1 - 6m^2) - h_{me}\sigma - 3h_{me}^2 m^2 > 0. \quad (8.3)$$

The condition (8.1) is the well known stability criterion for ensuring the positive character of the pure elastic quadratic energy form. In general the magnetization is due to (7) $m = m(h, \sigma, h_a, h_{me}, h_e)$ with $m^2 \leq 1$.

For $h_a = 0$ (this corresponds to negligible crystalline anisotropy, amorphous materials belong to this class) the conditions (8.2-3) result in

$$h_{me} \sigma < -3h_{me}^2 m^2 < 0. \quad (9)$$

Thus the model is applicable only for materials with negative magnetostriction ($h_{me} > 0$) under compression ($\sigma < 0$), or for materials with positive magnetostriction ($h_{me} < 0$) under tension ($\sigma > 0$).

The effect of applied mechanical stresses is the subject of another study [16] where comparison with available experimental results is performed. Thus in the following we will consider only the case of zero mechanical stresses ($\sigma = 0$). Then due to (8.2-3)

$$h_a h_e - 3(2h_a h_e + h_{me}^2) m^2 > 0. \quad (10)$$

and since $m^2 \leq 1$ it is sufficient that

$$h_a < -\frac{3h_{me}^2}{5h_e} < 0. \quad (11)$$

Thus the model is applicable only for materials with negative magnetocrystalline anisotropy like Ni.

4. THE MAGNETIZATION AND MAGNETOSTRICTION CURVES

Due to the constitutive relations (6-7) we can obtain an analytical formula for the effect of magnetostriction on the coercive force. Solving Eq. (7) for the magnetization results in at most three real roots. Thus there is a single jump of the magnetization on the magnetization curve $m = m(h)$ that determines coercivity. This corresponds to

$$\frac{\partial h}{\partial m} = 0, \quad (12)$$

which due to (7) results in

$$m_C = \pm \sqrt{\frac{h_a h_e - h_{me} \sigma}{3(2h_a h_e + h_{me}^2)}}. \quad (13)$$

Substitution of (13) into (7) results in the coercive force

$$h_C = h_C(\sigma, h_{me}, h_a) = -\frac{2}{3h_e \sqrt{3}} \frac{(h_a h_e - h_{me} \sigma)^{3/2}}{\sqrt{2h_a h_e + h_{me}^2}}. \quad (14)$$

Similarly the present model can produce an analytical expression for the remanence which is defined as $m_R = m(h = 0)$. Then from (7) we obtain

$$m_R = \sqrt{3} m_C = \pm \sqrt{\frac{h_a h_e - h_{me} \sigma}{2h_a h_e + h_{me}^2}}. \quad (15)$$

Magnetization curves for materials with negative magnetostriction ($h_{me} > 0$) are shown in Fig. 1 for material parameters that correspond to Ni, for varying magnetostrictive constants and $\sigma = 0$. The material constants for Ni are [16]: $M_s = 40 \text{ kA/m}$, $c_{11} - c_{12} = 9.5 \times 10^{10} \text{ N/m}^2$, $\lambda_{100} = -5 \times 10^{-5}$, $K = -4.26 \times 10^3 \text{ J/m}^3$. The selection of the magnetoelastic parameters is such that the stability condition (11) is fulfilled.

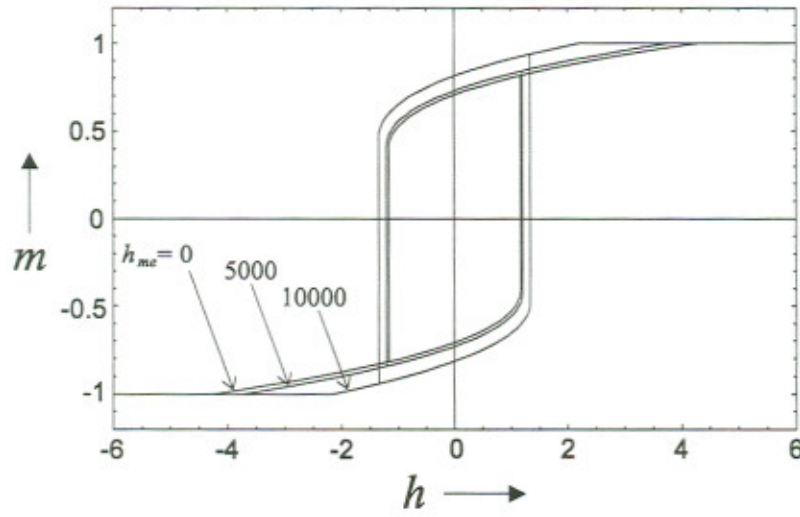


Figure 1: Magnetization curves for $h_a = -4.28$, $h_e = 4775208$ and $h_{me} \in [0, 10^4]$ with step 5×10^4 .

The magnetostrictive effect is strongly exaggerated in Fig. 1 in order to observe the change of the coercive force and of the remanence (for Ni, $h_{me} = 3581$). The increment in the coercive force h_C and in remanence m_R is of the order of 10^{-2} for magnetoelastic constants as large as $h_{me} = 4 \times 10^3$. This fact is illustrated in Fig. 2 for the coercivity.

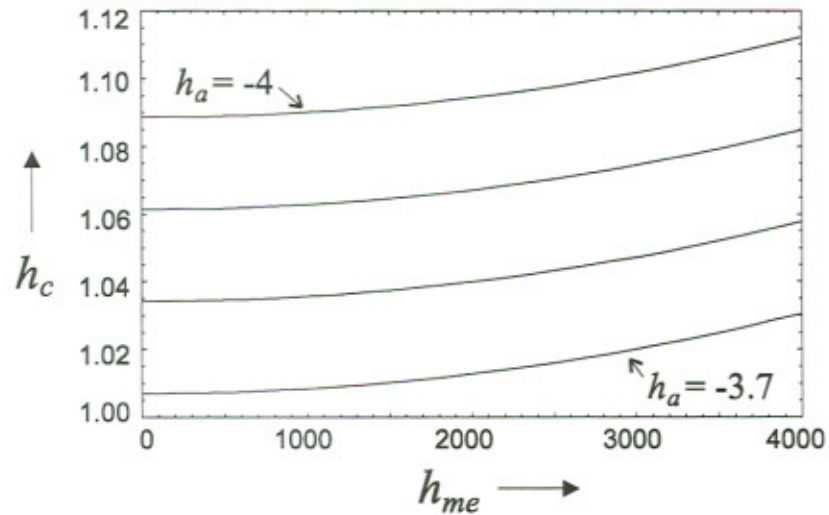


Figure 2: Variation of coercivity h_c with magnetoelastic constant h_{me} for material constants of Ni ($h_e = 4775208$ and $h_a \in [-4, -3.7]$ with step 0.1).

The satisfaction of the stability condition (11) results in a hysteretic magnetostrictive curve (well known as butterfly strain-field loop) that is shown in Fig. 3.

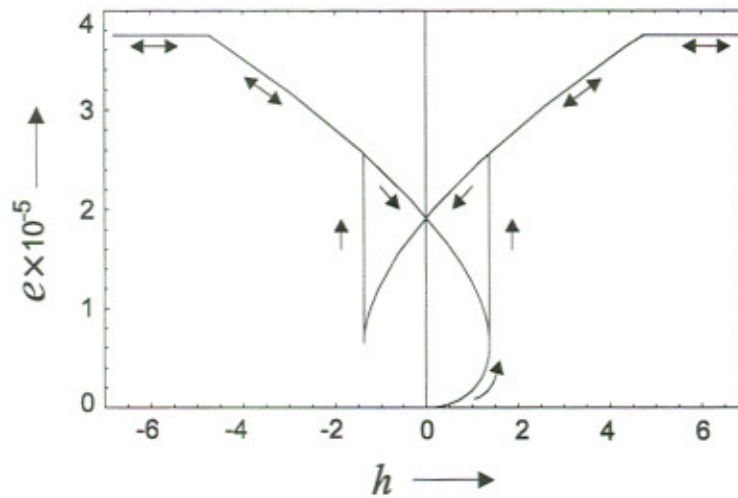


Figure 3: Strain e vs applied field h for material constants of Ni ($h_e = 4775208$ and $h_a = -5$, $h_{me} = 3581$).

Due to the assumptions made the initial curve is not smooth and exhibits an irreversible jump at $h = h_c$. The role of anisotropy on the magnetostrive hysteresis is evident in Fig. 4.

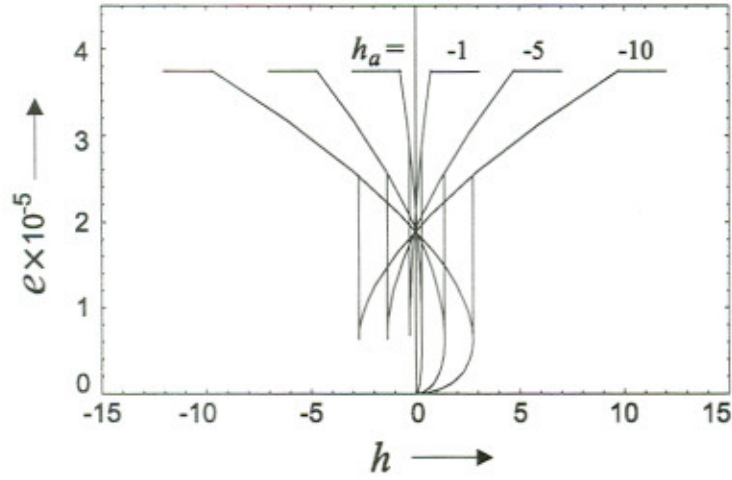


Figure 4: Strain e vs applied field h for material constants of Ni ($h_e = 4775208$, $h_{me} = 3581$), with varying h_a .

It is obvious that for small anisotropy only small applied magnetic fields are capable to produce the resultant length change (strain) in the material. Efficient performance of actuator devices require small anisotropies to produce the strain in the material. The higher the magnetoelastic constant the higher the resultant strain as it is deduced from Fig. 5.

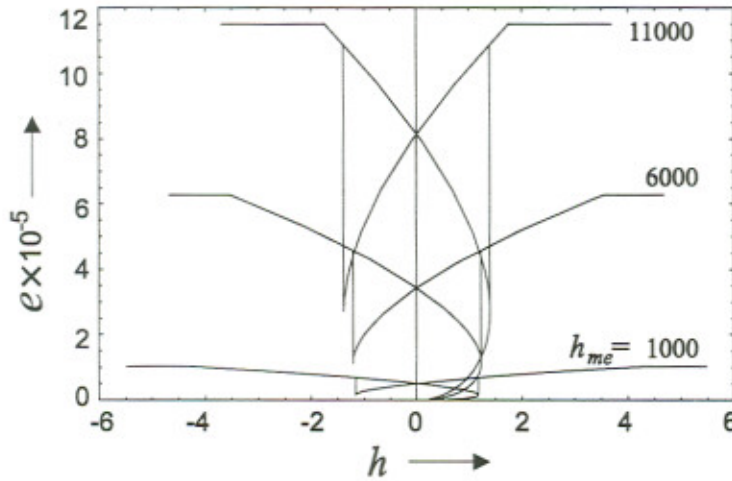


Figure 5: Strain e vs applied field h for material constants of Ni ($h_e = 4775208$, $h_a = -4.28$), with varying $h_{me} \in [1, 11] \times 10^3$ with step 5×10^3 .

5. DISCUSSION

In the present study a simple SW model is presented that accounts for both ferromagnetic and magnetostrictive hysteresis. The model accounts for all type of interactions, in contrast to the usual models that incorporate the applied stresses and magnetostrictive strains as additional terms in the magnetocrystalline anisotropy energies. Stability criteria are also introduced for the presented solutions. Analytical $h_c = h_c(\sigma, h_{me}, h_a)$ and $m_R = m_R(\sigma, h_{me}, h_a)$ relations were derived. The well known inverse magnetostrictive effect that accounts for the effect of stress on the magnetization curve is also included. It is confirmed that the change in the coercivity and remanence due to magnetostriction is a second order effect. The effect of anisotropy on

the magnetostrictive hysteresis dictates the region of efficient performance of actuator devices. However, the model does not take into account internal stresses, as well as non-uniform reversal.

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