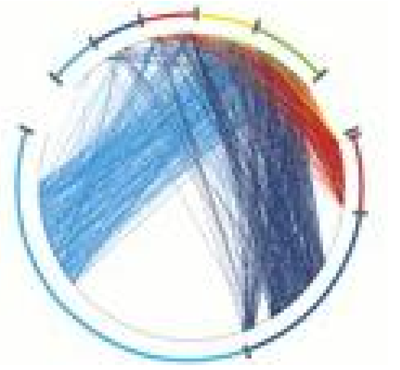
Models and Algorithms for Complex Networks

Searching in Small World Networks

Lecture 7



Small world phenomena

- Small worlds: networks with short paths

Stanley Milgram (1933-1984):
“The man who shocked the world”

Obedience to authority (1963)

Small world experiment (1967)

Small world experiment

- Letters were handed out to people in Nebraska to be sent to a target in Boston
- People were instructed to pass on the letters to someone they knew on first-name basis
- The letters that reached the destination followed paths of length around 6
- Six degrees of separation: (play of John Guare)

- Small world project: http://smallworld.columbia.edu/index.html

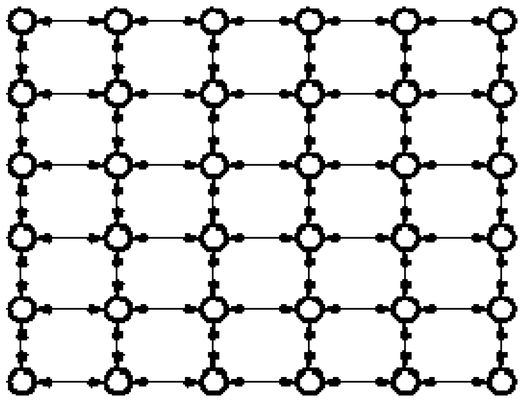
Milgram's experiment revisited

- What did Milgram's experiment show?
  - (a) There are short paths in large networks that connect individuals
  - (b) People are able to find these short paths using a simple, greedy, decentralized algorithm
- Small world models take care of (a)
- Kleinberg: what about (b)?

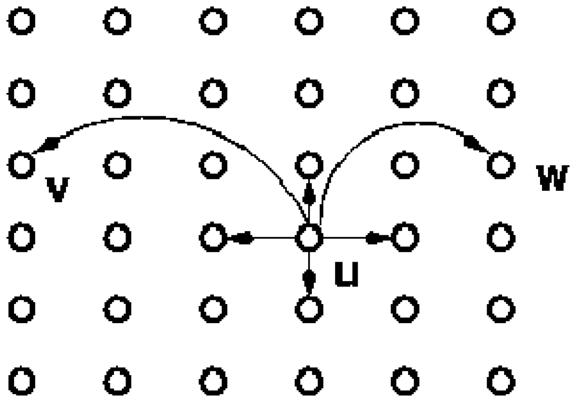
Kleinberg's model

- Consider a directed 2-dimensional lattice
- For each vertex u add q shortcuts
  - choose vertex v as the destination of the shortcut with probability proportional to $[d(u,v)]^{-r}$
  - when $r = 0$, we have uniform probabilities

A)



B)



Searching in a small world

- Given a source s and a destination t, the search algorithm
  1. knows the positions of the nodes on the grid (geography information)
  2. knows the neighbors and shortcuts of the current node (local information)
  3. operates greedily, each time moving as close to t as possible (greedy operation)
  4. knows the neighbors and shortcuts of all nodes seen so far (history information)

- Kleinberg proved the following
  - When $r=2$, an algorithm that uses only local information at each node (not 4) can reach the destination in expected time $O(\log^2 n)$.
  - When $r<2$ a local greedy algorithm (1-4) needs expected time $\Omega(n^{(2-r)/3})$.
  - When $r>2$ a local greedy algorithm (1-4) needs expected time $\Omega(n^{(r-2)/(r-1)})$.
  - Generalizes for a d-dimensional lattice, when $r=d$ (query time is independent of the lattice dimension)
    - $d = 1$, the Watts-Strogatz model

The decentralized search algorithm

- Given a source s and a destination t, the search algorithm
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  2. knows the neighbors and shortcuts of the current node (local information)
  3. operates greedily, each time moving as close to t as possible (greedy operation)
  4. knows the neighbors and shortcuts of all nodes seen so far (history information)

Kleinberg results

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  2. knows the neighbors and shortcuts of the current node (local information)
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Kleinberg's results

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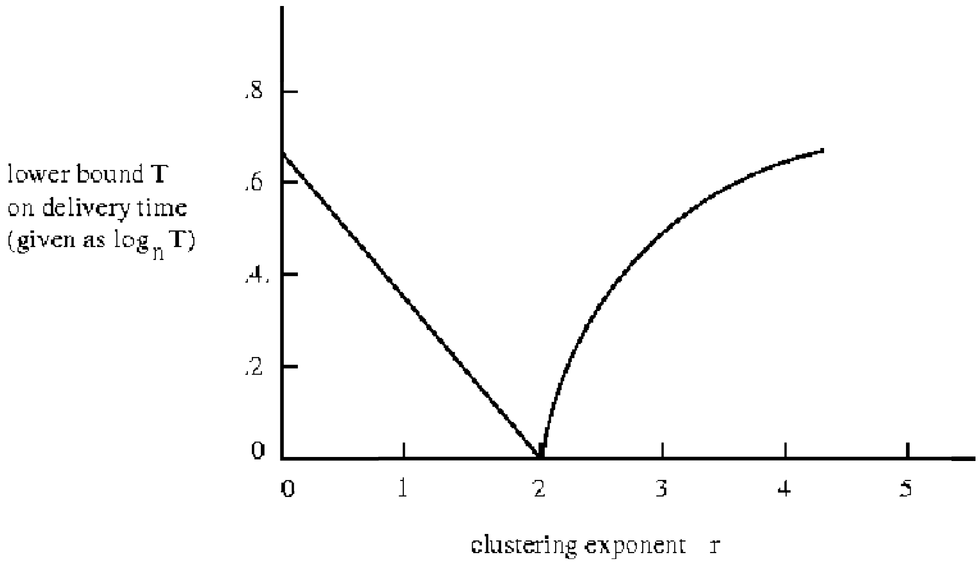
- When $r<2$ a local greedy algorithm (1-4) needs expected time $\Omega(n^{(2-r)/3})$.
- When $r>2$ a local greedy algorithm (1-4) needs expected time $\Omega(n^{(r-2)/(r-1)})$.

Searching in a small world

- For $r < 2$, the graph has paths of logarithmic length (small world), but a greedy algorithm cannot find them
- For $r > 2$, the graph does not have short paths
- For $r = 2$ is the only case where there are short paths, and the greedy algorithm is able to find them



Generalization

- When $r=2$, an algorithm that uses only local information at each node (not 4) can reach the destination in expected time $O(\log^2 n)$.
- When $r<2$ a local greedy algorithm (1-4) needs expected time $\Omega(n^{(2-r)/3})$.
- When $r>2$ a local greedy algorithm (1-4) needs expected time $\Omega(n^{(r-2)/(r-1)})$.

- The results generalize for a d-dimensional grid. The algorithm works in expected $O(\log^2 n)$ time, when $r=d$

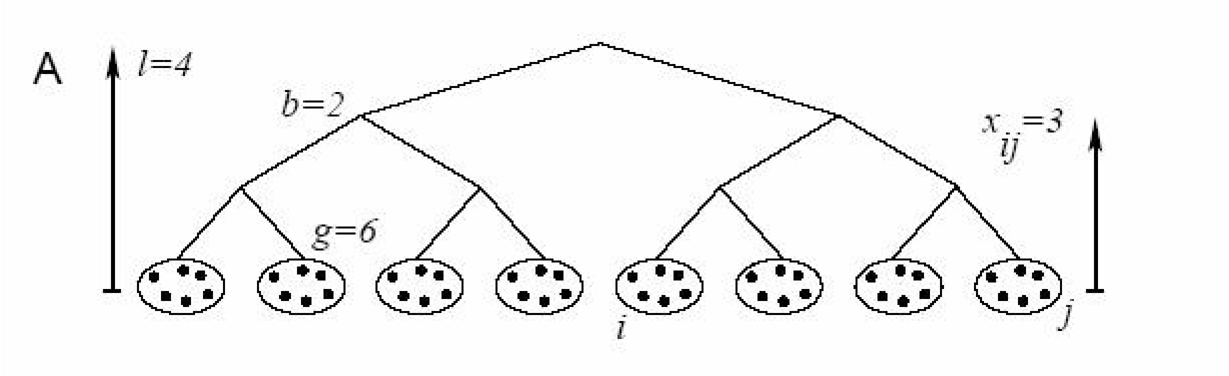
Extensions

- If there are logn shortcuts, then the search time is O(logn)
  - we save the time required for finding the shortcut

- If we know the shortcuts of logn neighbors the time becomes $O(\log^{1+1/d} n)$

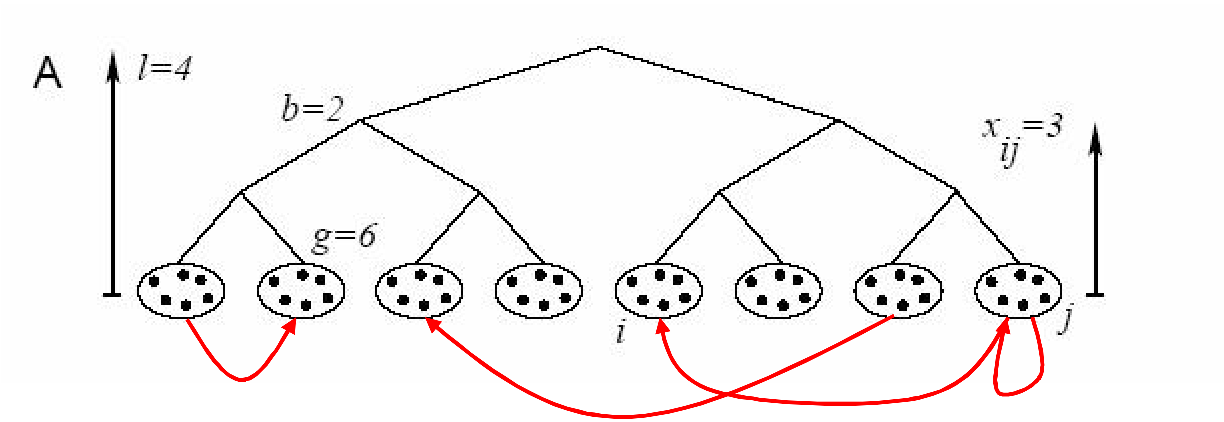
Other models

- Lattice captures geographic distance. How do we capture social distance (e.g. occupation)?
- Hierarchical organization of groups
  - distance h(i,j) = height of Least Common Ancestor



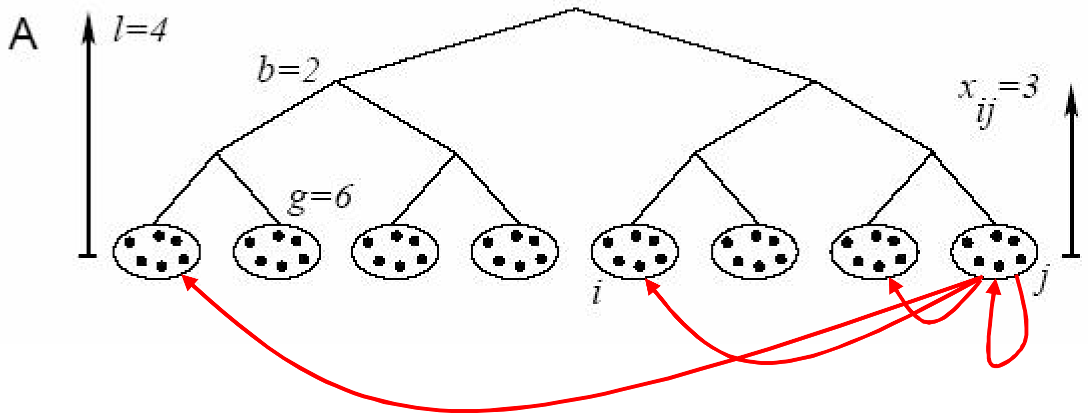
Other models

- Generate links between leaves with probability proportional to $b^{-\alpha h(i,j)}$
  - $b=2$ the branching factor



Other models

- Theorem: For α=1 there is a polylogarithimic search algorithm. For α≠1 there is no decentralized algorithm with poly-log time
  - note that α=1 and the exponential dependency results in uniform probability of linking to the subtrees



Searching Power-law networks

- Kleinberg considered the case that you can fix your network as you wish. What if you cannot?
- [Adamic et al.] Instead of performing simple BFS flooding, pass the message to the neighbor with the highest degree
- Reduces the number of messages to $O(n^{(a-2)/(a-1)})$

References

- J. Kleinberg. The small-world phenomenon: An algorithmic perspective. Proc. 32nd ACM Symposium on Theory of Computing, 2000
- J. Kleinberg. Small-World Phenomena and the Dynamics of Information. Advances in Neural Information Processing Systems (NIPS) 14, 2001.
- Renormalization group analysis of the small-world network model, M. E. J. Newman and D. J. Watts, Phys. Lett. A 263, 341-346 (1999).
- Identity and search in social networks, D. J. Watts, P. S. Dodds, and M. E. J. Newman, Science 296, 1302-1305 (2002).
- Search in power-law networks, Lada A. Adamic, Rajan M. Lukose, Amit R. Puniyani, and Bernardo A. Huberman, Phys. Rev. E 64, 046135 (2001)