Πανεπιστήμιο Κρήτης, Τμήμα Επιστήμης Υπολογιστών
Άνοιξη 2009

ΗΥ463 - Συστήματα Ανάκτησης Πληροφοριών
Information Retrieval (IR) Systems

Ευρετηρίαση, Αποθήκευση και Οργάνωση Αρχείων
(Indexing, Storage and File Organization)

Κεφάλαιο 8

Δομές Ευρετηρίου: Διάρθρωση Διάλεξης

- Εισαγωγή - κίνητρο
- Ανεστραμμένα Αρχεία (Inverted files)
- Δένδρα Καταλήξεων (Suffix trees)
- Αρχεία Υπογραφών (Signature files)
- Σειριακή Αναζήτηση σε Κείμενο (Sequential Text Searching)
- Απάντηση Επερωτήσεων "Ταιριάσματος Προτύπου" (Answering Pattern-Matching Queries)

Σειριακή Αναζήτηση Κειμένου: Το πρόβλημα

Το πρόβλημα:

find the first occurrence (or all occurrences) of a string (or pattern) ***p*** *(*of length *m)* in a string ***s*** *(*of length *n)*

Commonly, *n* is much larger than *m*.

Χρήσεις:

- Για εύρεση των εγγράφων που περιέχουν μια λέξη (αν δεν έχουμε ευρετήριο).
- Στην περίπτωση που έχουμε ανεστραμμένο ευρετήριο με block addressing.
- Στην περίπτωση που έχουμε αρχείο υπογραφών για να βεβαιωθούμε ότι ένα match δεν είναι false drop.

Sequential Text Searching Algorithms

- Brute-Force Algorithm
- Knuth-Morris-Pratt
- Boyer-Moore family

Brute-Force Algorithm

***Brute-Force* (BF), or *sequential* text searching:**

Try all possible positions in the text. For each position verify whether the pattern matches at that position.

Since there are *O(n)* text positions and each one is examined at *O(m)* worst-case cost, the worst-case of brute-force searching is ***O(nm)***.

Brute-Force Algorithm

```
Naive-String-Matcher(S,P)
n := length(S)
m := length(P)
for i = 0 to n-m do
        if P[1..m] = S[i+1 .. i+m] then
                return "Pattern occurs at position i"
        fi
od
```

The naive string matcher needs worst case running time O((n-m+1) m)

For n = 2m this is $O(n^2)$

Its average case is O(n) (since on random text a mismatch is found after O(1) comparisons on average)

The naive string matcher is not optimal, since string matching can be done in time O(m + n)

Knuth-Morris-Pratt & Boyer-Moore

- Πιο γρήγοροι αλγόριθμοι που βασίζονται σε **μετακινούμενο (ολισθαίνον) παράθυρο (sliding window)**
- Γενική ιδέα:
  - They employ a ***window*** of length *m* which is slid over the text.
  - It is *checked* whether the text in the window is equal to the pattern (if it is, the window position is reported as a match).
  - Then, the window is *shifted* forward.
- Οι αλγόριθμοι διαφέρουν στον τρόπο με τον οποίο ελέγχουν και ολισθαίνουν (μετακινούν) το παράθυρο.

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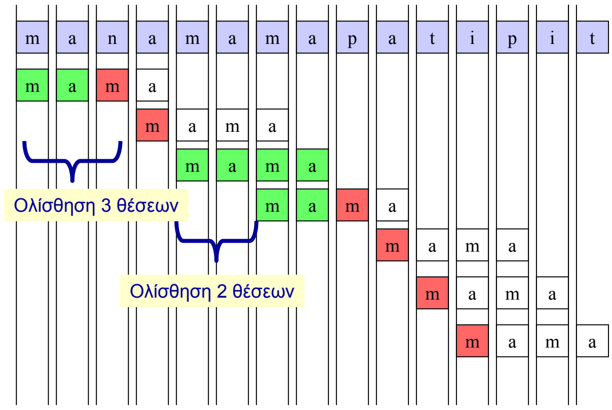
Ολίσθηση Παραθύρου: Η γενική ιδέα

p="mama"

s



Ιδέα: να αποφύγουμε να δοκιμάσουμε όλες τις θέσεις για ένα παράθυρο

Πως; Χρησιμοποιώντας την πληροφορία από προηγούμενους ελέγχους παραθύρου

Αφού ελέγξουμε ένα παράθυρο, ανεξάρτητα αν ταιριάζει με το pattern ή όχι, ήδη έχουμε ταιριάξει μια ακολουθία χαρακτήρων και μπορεί όλοι να ταίριαζαν εκτός πιθανόν από τον τελευταίο από αυτούς

Πρέπει να κοιτάξουμε τα prefixes (προθέματα) του pattern και να δούμε αν ταιριάζει με κάποιο suffix του παραθύρου

Το μεγαλύτερο δυνατό suffix που ταιριάζει με κάποιο πρόθεμα => μετακίνηση του παραθύρου στην αρχή του

Εξαρτάται μόνο από το pattern

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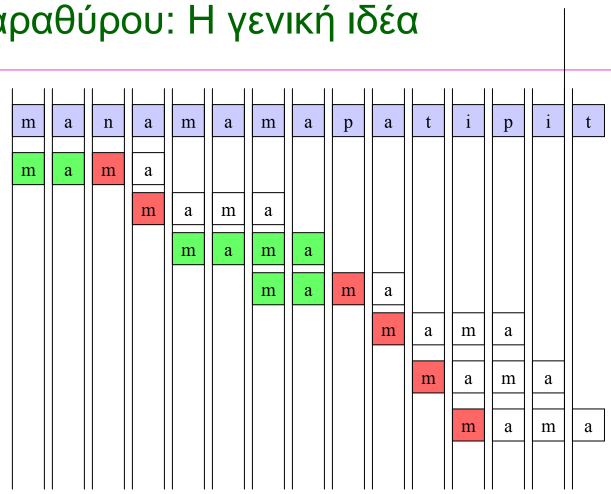
Ολίσθηση Παραθύρου: Η γενική ιδέα

p=“mana”

s



Knuth-Morris-Pratt & Boyer-Moore

Knuth-Morris-Pratt (KMP) [1970]

- The pattern *p* is preprocessed to build a table called *next*.
- The *next* table at position *j* says which is the longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different.
- Hence *j-next[j]-1* window positions can be safely skipped if the characters up to *j-1* matched and the *j*-th did not.

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11
p[j]	a	b	r	a	c	a	d	a	b	r	a
next[j]	0	0	0	0	1	0	1	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11
p[j]	a	b	r	a	c	a	d	a	b	r	a
next[j]	0	0	0	0	1	0	1	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11
p[j]	a	b	r	a	c	a	d	a	b	r	a
next[j]	0	0	0	0	1	0	1	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11
p[j]	a	b	r	a	c	a	d	a	b	r	a
next[j]	0	0	0	0	1	0	1	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11
p[j]	a	b	r	a	c	a	d	a	b	r	a
next[j]	0	0	0	0	1	$\boxed{0}$	1	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11
p[j]	a	b	r	a	c	a	d	a	b	r	a
next[j]	0	0	0	0	1	0	$\boxed{1}$	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4

KMP: the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4

Σε κάθε θέση κοιτάμε τους χαρακτήρες στα αριστερά μας (που προηγούνται)

Θέλουμε το «μεγαλύτερο» δηλαδή το στοιχείο που ακολουθεί να είναι διαφορετικό

Exploiting the next table

next[j] = longest proper prefix of *p[1..j-1]* which is also a suffix and the characters following prefix and suffix are different

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4
j-next[j]-1	0	1	2	3	3	5	5	7	8	9	10	7

***j-next[j]-1* window positions can be safely skipped if the characters up to *j-1* matched and the *j*-th did not.**

Knuth-Morris-Pratt (KMP) [1970]

The algorithm moves a window over a text and a pointer inside the window

- Each time a character <u>matches</u>, the pointer is advanced
- Each time a character <u>does not match</u>, the window is shifted forward in the text to the position given by next (the pointer position in the text does not)

Example: match until 2nd char

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4
j-next[j]-1	0	<u>1</u>	2	3	3	5	5	7	8	9	10	7

s	a	a	r	i	c	a	b	r	a	c	a		
p	a	b	r	a	c	a	d	a	b	r	a		
		a	b	r	a	c	a	d	a	b	r	a	

1

Example: match until 3rd char

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4
j-next[j]-1	0	1	<u>**2**</u>	3	3	5	5	7	8	9	10	7

s	a	b	a	i	c	a	b	r	a	c	a		
p	a	b	r	a	c	a	d	a	b	r	a		
			a	b	r	a	c	a	d	a	b	r	a

2

Example: match until 7th char

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4
j-next[j]-1	0	1	2	3	3	5	<u>**5**</u>	7	8	9	10	7

s	a	b	r	a	c	a	b	r	a	c	a					
p	a	b	r	a	c	a	d	a	b	r	a					
						a	b	r	a	c	a	d	a	b	r	a

5

Example: pattern matched

j	1	2	3	4	5	6	7	8	9	10	11	
p[j]	a	b	r	a	c	a	d	a	b	r	a	
next[j]	0	0	0	0	1	0	1	0	0	0	0	4
j-next[j]-1	0	1	2	3	3	5	5	7	8	9	10	7

s	a	b	r	a	c	a	d	a	b	r	a	c							
p	a	b	r	a	c	a	d	a	b	r	a								
								a	b	r	a	c	a	d	a	b	r	a	

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KMP: Complexity

- Since at each text comparison the window or the text pointer advance by at least one position, the algorithm performs at most *2n* comparisons (for the case where m=n), and at least *n*.
- The overall complexity is O(m+n)
  - The worst case is exactly n+m for finding the 1st occurrence
- Remarks:
  - We shouldn't however forget the cost for building the *next* table.
  - On average is it not much faster than BF

Finite-Automaton-Matcher

Finite-Automaton-Matcher

- For every pattern of length m there exists an automaton with m+1 states that solves the pattern matching problem.

- *KMP is actually a Finite-Automaton-Matcher*

Finite Automata (επανάληψη)

A deterministic finite automaton M is a 5-tuple $(Q, q_0, A, \Sigma, \delta)$, where

- Q is a finite set of **states**
- $q_0 \in Q$ is the **start state**
- $A \subseteq Q$ is a distinguished set of **accepting states**
- Σ, is a finite **input alphabet**,
- $\delta: Q \times \Sigma \to Q$ is called the **transition function** of M

Let $\varphi : \Sigma \to Q$ be the final-state function defined as:

For the empty string ε we have: $\varphi(\varepsilon) := q_0$

For all $a \in \Sigma$, $w \in \Sigma^*$ define $\varphi(wa) := \delta(\varphi(w), a)$

M accepts w if and only if: $\varphi(w) \in A$

Example (I)

p=«abba»

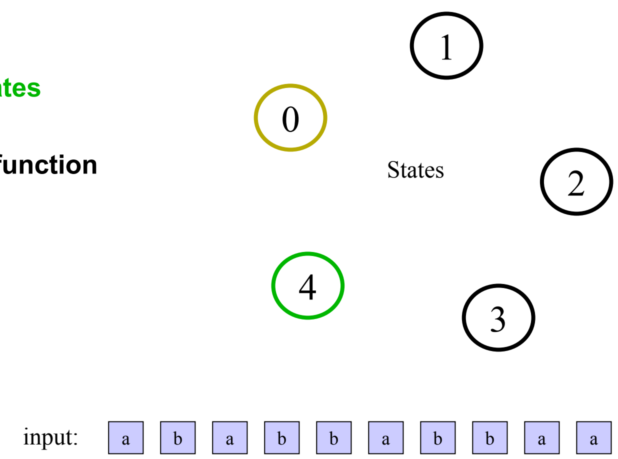
Q is a finite set of states

$q_0 \in Q$ is the start state

Q is a set of accepting sates

Σ: input alphabet

$\delta: Q \times \Sigma \to Q$: transition function



Example (II)

Q is a finite set of states

$q_0 \in Q$ is the start state

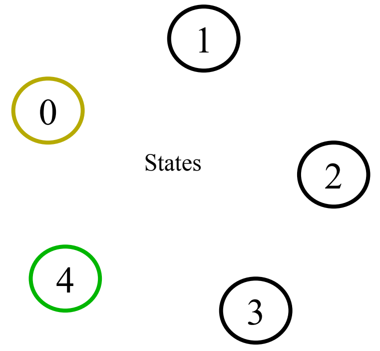
Q is a set of accepting states

Σ: input alphabet

δ: Q × Σ → Q: transition function

input / state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2

p=«abba»



Example (III)

Q is a finite set of states

$q_0 \in Q$ is the start state

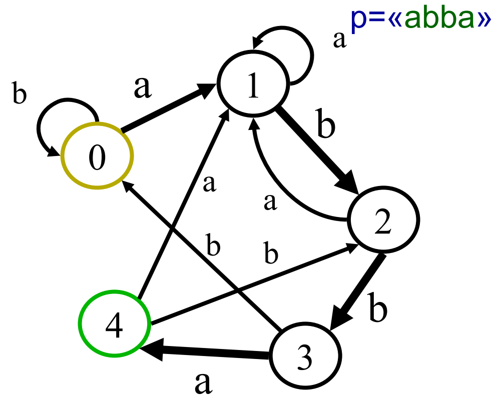
Q is a set of accepting sates

Σ: input alphabet

δ: Q × Σ → Q: transition function

input / state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2

p=«abba»



Example (IV)

Q is a finite set of states

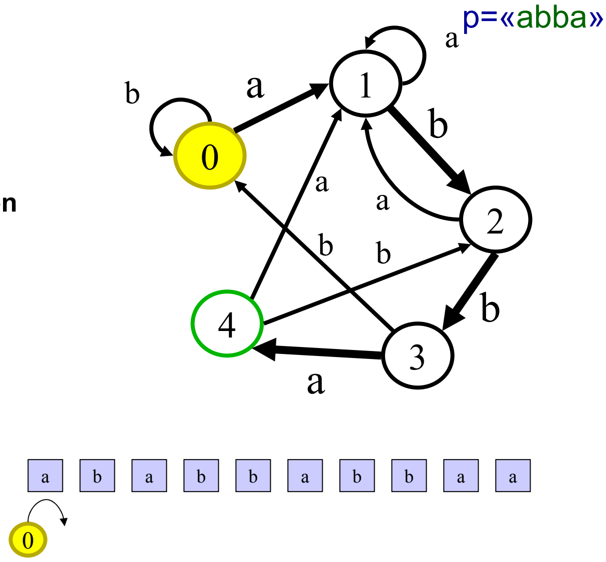
$q_0 \in Q$ is the start state

Q is a set of accepting sates

Σ: input alphabet

δ: Q × Σ → Q: transition function

input \ state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2



Example (V)

Q is a finite set of states

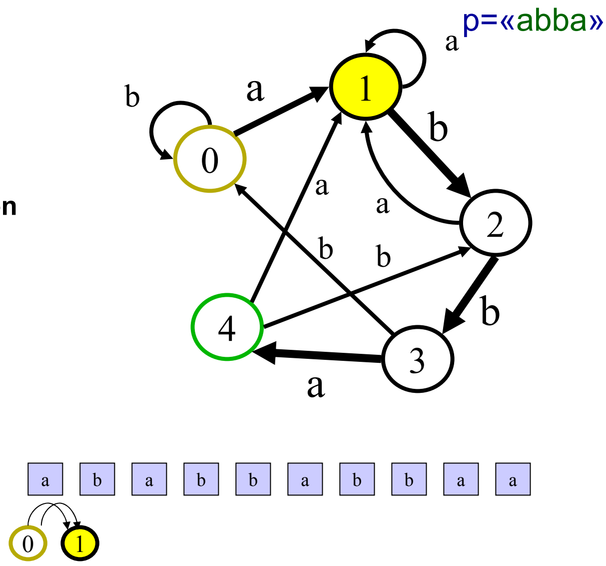
$q_0 \in Q$ is the start state

Q is a set of accepting sates

Σ: input alphabet

δ: Q × Σ → Q: transition function

input \ state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2



Example (VI)

Q is a finite set of states

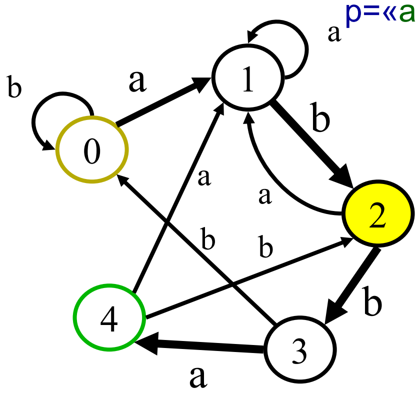
$q_0 \in$ Q is the start state

Q is a set of accepting sates

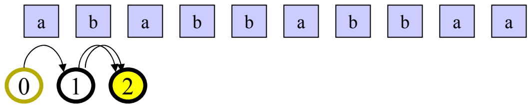
Σ: input alphabet

δ: Q × Σ → Q: transition function

p=«abba»



input / state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2



Example (VII)

Q is a finite set of states

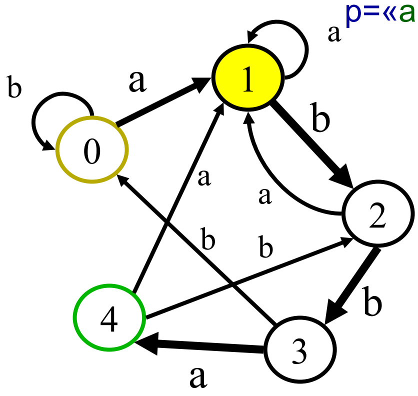
$q_0 \in$ Q is the start state

Q is a set of accepting sates

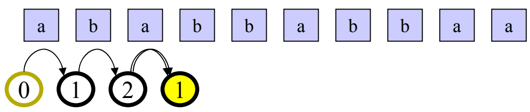
Σ: input alphabet

δ: Q × Σ → Q: transition function

p=«abba»



input / state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2



Example (VIII)

Q is a finite set of states

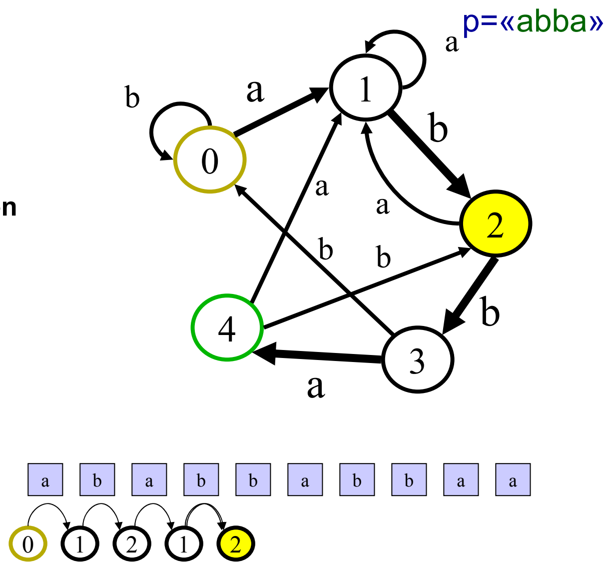
$q_0 \in$ Q is the start state

Q is a set of accepting sates

Σ: input alphabet

δ: Q × Σ → Q: transition function

input / state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2



Example (IX)

Q is a finite set of states

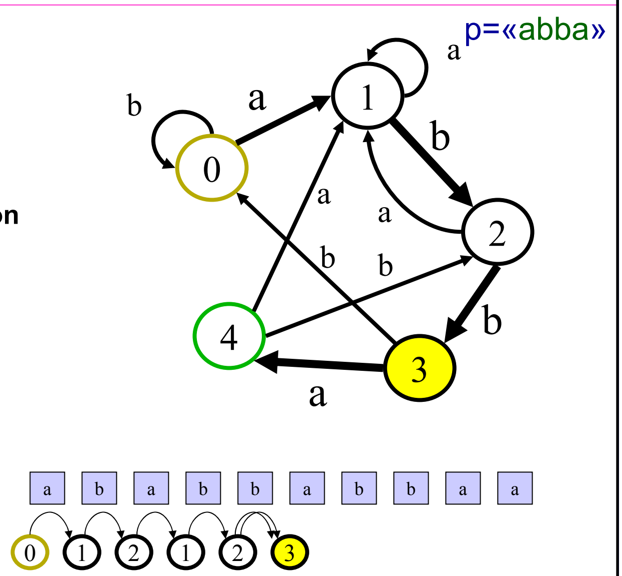
$q_0 \in$ Q is the start state

Q is a set of accepting sates

Σ: input alphabet

δ: Q × Σ → Q: transition function

input / state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2



Example (X)

Q is a finite set of states

$q_0 \in Q$ is the start state

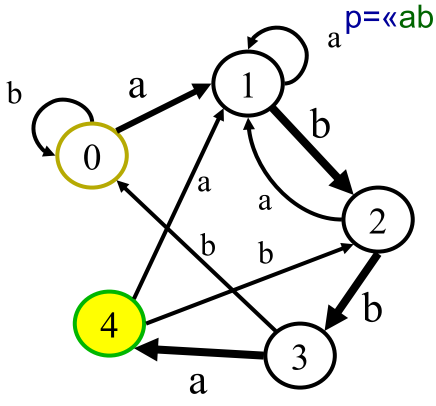
Q is a set of accepting sates

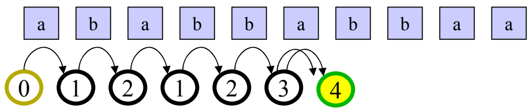
Σ: input alphabet

δ: Q × Σ → Q: transition function

input / state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2

p=«abba»





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Example (XI)

Q is a finite set of states

$q_0 \in Q$ is the start state

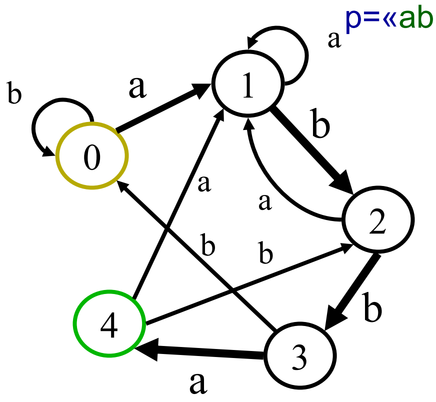
Q is a set of accepting sates

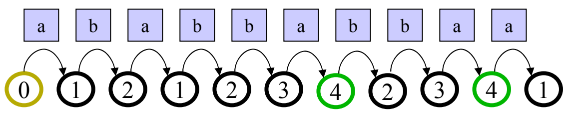
Σ: input alphabet

δ: Q × Σ → Q: transition function

input / state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2

p=«abba»





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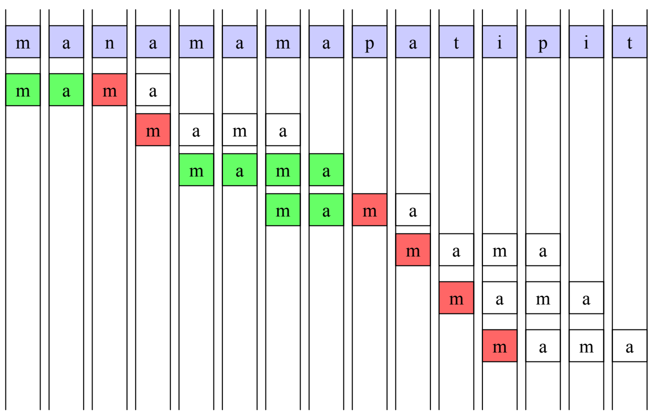
Finite-Automaton-Matcher

For every pattern P of length m there exists an automaton with m+1 states that solves the pattern matching problem with the following algorithm:

```
Finite-Automaton-Matcher(T,δ,P)
n := length(T)
q := 0    // initial state
for i =1 to n do
    q := δ(q,T[i])  // transition to the next state
    if q = m then  // if we reached the state m (which is the final)
         return "Pattern occurs at position "  i-m
    fi
od
```


Computing the Transition Function: It is actually the idea of KMP



How to Compute the Transition Function?

Let P_k denote the first k letter string of P (i.e. the prefix of P with length k)

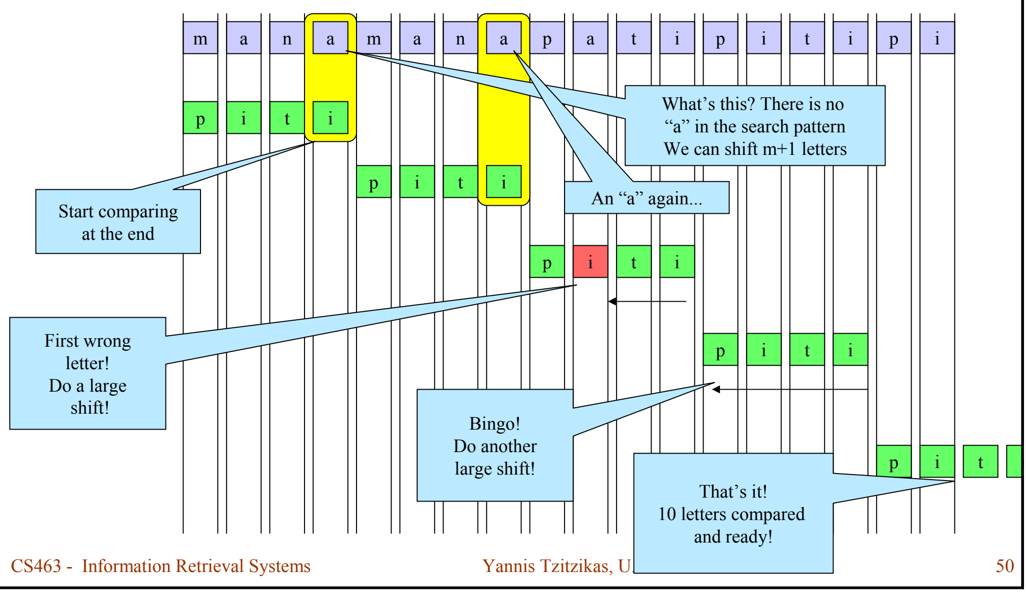
```
Compute-Transition-Function(P, Σ)
    m :=  length(P)
    for q = 0 to m do
        for each character a ∈ Σ do
          k := 1+min(m,q+1)
          repeat
              k := k-1
          until P_k is a suffix of P_q a
           δ(q,a) := k
        od
    od
```


Boyer-Moore (BM)

Boyer-Moore (BM) [1975]

- Motivation
  - KMP yields genuine benefits only if a mismatch as preceded by a partial match of some length
    - only in this case the pattern slides more than one position
  - Unfortunately, this is the exception rather than the rule
    - mathes occur much more seldom than mismatches
- The idea
  - start comparing characters at the **end of the pattern** rather than at the beginning
  - like in KMP, a pattern is pre-processed

Boyer-Moore: The idea by an example



Sequential Text Searching
Synopsis

Find the first occurrence (or all occurrences) of a string (or pattern) ***p*** *(of length m)* in a string ***s*** *(of length n)*

- Brute Force Algorithm
  - $O(n^2)$ running time (worst case)
- KMP ~ Finite Automaton Matcher
  - Let a (finite) automaton do the job
    - Cost: cost to construct the automaton plus the cost to "consume" the string s
  - O(m+n) running time (worst case)
    - m: for constructing the next table
    - n: for searching the text
- BM Algorithm
  - Bad letters allow us to jump through the text
  - Faster in practice
  - O(nm) running time (worst case)
  - O(n log(m)/m) average time

Other string searching algorithms

- Rabin-Karp
- Shift-Or (it is sketched in the Modern Information Retrieval Book)
- *...and many others ..*
- *..*

For more

Algorithm	Preprocessing time	Matching time[1]
Naïve string search algorithm	0 (no preprocessing)	Θ(n m)
Rabin-Karp string search algorithm	Θ(m)	average Θ(n+m), worst Θ(n m)
Finite state automaton based search	Θ(m |Σ|)	Θ(n)
Knuth-Morris-Pratt algorithm	Θ(m)	Θ(n)
Boyer-Moore string search algorithm	Θ(m + |Σ|)	Ω(n/m), O(n)
Bitap algorithm (*shift-or, shift-and, Baeza-Yates-Gonnet*)	Θ(m + |Σ|)	Θ(n)

For more see

- **String searching algorithm**
  http://en.wikipedia.org/wiki/String_searching_algorithm
  - To remember what Theta/Omega is, see
    - http://delivery.acm.org/10.1145/1010000/1008329/p18-knuth.pdf
- **EXACT STRING MATCHING ALGORITHMS,** Christian Charras - Thierry Lecroq,
  http://www-igm.univ-mlv.fr/~lecroq/string/index.html (it includes animations in Java)

Δομές Ευρετηρίου: Διάρθρωση Διάλεξης

- Εισαγωγή - κίνητρο
- Inverted files (ανεστραμμένα αρχεία)
- Suffix trees (δένδρα καταλήξεων)
- Signature files (αρχεία υπογραφών)
- Sequential Text Searching
- **Answering Pattern-Matching Queries**

Answering Pattern Matching Queries

- Searching Allowing Errors (**Levenshtein** distance)
- Searching using Regular Expressions

Searching Allowing Errors

- Δεδομένα:
  - **Ένα κείμενο (string) T, μήκους n**
  - **Ένα pattern P μήκους m**
  - **k επιτρεπόμενα σφάλματα**
- Ζητούμενο:
  - **Βρες όλες τις θέσεις του κειμένου όπου το pattern P εμφανίζεται με το πολύ k σφάλματα**

Remember: Edit (Levenstein) Distance:

Minimum number of character *deletions*, *additions,* or *replacements* needed to make two strings equivalent.

"misspell" to "mispell" is distance 1

"misspell" to "mistell" is distance 2

"misspell" to "misspelling" is distance 3

Searching Allowing Errors

Naïve algorithm

– Produce all possible strings that could match P (assuming k errors) and search each one of them on T

Searching Allowing Errors: Solution using **Dynamic Programming**

- Dynamic Programming is the class of algorithms, which includes the most commonly used algorithms in speech and language processing.
- Among them the **minimum edit distance algorithm for spelling error correction.**
- Intuition:
  – *a large problem can be solved by properly combining the solutions to various subproblems.*

Searching Allowing Errors: Solution using **Dynamic Programming**

Έναν m x n πίνακα C

Γραμμές θέσεις του pattern

Στήλες θέσεις του text

C[i, j]: ο ελάχιστος αριθμός λαθών για να ταιριάξουμε το $P_{1..i}$ με ένα suffix του $T_{1..j}$

C[0, j] = 0

C[i, 0] = i /* delete i characters

Η ιδέα είναι ο υπολογισμός μιας τιμής του πίνακα με βάση τις προηγούμενες (δηλαδή, ήδη υπολογισμένες) γειτονικές της

Searching Allowing Errors: Solution using **Dynamic Programming**

C[i, j]: ο ελάχιστος αριθμός λαθών για να ταιριάξουμε το $P_{1..i}$ με ένα suffix του $T_{1..j}$

C[i, j] =

αν $P_i = T_i$

τότε C[i-1, j-1]

Αλλιώς ο καλύτερος τρόπος από τα παρακάτω

replace P_i με T_i (η το συμμετρικό) κόστος 1+C[i-1,j-1]

delete P_i κόστος 1+ C[i-I, j]

delete T_i 1 + C[i,j-1]

add ??

Searching Allowing Errors:
Solution using **Dynamic Programming (II)**

Problem Statement: T[n] text string, P[m] pattern, k errors

Example: T = “surgery”, P = “survey”, k=**2**

To explain the algorithm we will use a m x n matrix C

one row for each char of P, one column for each char of T

(latter on we shall see that we need less space)

T

P

		s	u	r	g	e	r	y
s								
u								
r								
v								
e								
y								

Searching Allowing Errors:
Solution using **Dynamic Programming (III)**

T = “surgery”, P = “survey”, k=**2**

οι γραμμές του C εκφράζουν πόσα γράμματα του pattern έχουμε ήδη καταναλώσει (στη 0-γραμμή τίποτα, στη m-γραμμή ολόκληρο το pattern)

C[0,j] := 0 for every column j

(no letter of P has been consumed)

C[i,0] := i for every row i

(i chars of P have been consumed, pointer of T at 0. So i errors (insertions) so far)

T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1							
u	2							
r	3							
v	4							
e	5							
y	6							

Searching Allowing Errors: Solution using **Dynamic Programming (IV)**

if P[i]=T[j] **THEN** C[i,j] := C[i-1,j-1]

// εγινε match άρα τα "λάθη" ήταν όσα και πριν

Else C[i,j] := **1** + *min* of:

- C[i-1,j]
  - // i-1 chars consumed P, j chars consumed of T
  - // ~delete a char from T
- C[i,j-1]
  - // i chars consumed P, j-1 chars consumed of T
  - // ~ delete a char from P
- C[i-1,j-1]
  - // i-1 chars consumed P, j-1 chars consumed of T
  - // ~ character replacement

Searching Allowing Errors: Solution using **Dynamic Programming: Example**

- T = "surgery", P = "survey", k=**2**

T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1	0	1	1	1	1	1	1
u	2	1	0	1	2	2	2	2
r	3	2	1	0	1	2	2	3
v	4	3	2	1	1	2	3	3
e	5	4	3	2	2	1	2	3
y	6	5	4	3	3	**2**	**2**	**2**

Solution using **Dynamic Programming: Example**

- T = “surgery”, P = “survey”, k=**2**

T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1	0	1	1	1	1	1	1
u	2	1	0	1	2	2	2	2
r	3	2	1	0	1	2	2	3
v	4	3	2	1	1	2	3	3
e	5	4	3	2	2	1	2	3
y	6	5	4	3	3	2	2	2

Solution using **Dynamic Programming: Example**

- T = “surgery”, P = “survey”, k=**2**

T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1	0	1	1	1	1	1	1
u	2	1	0	1	2	2	2	2
r	3	2	1	0	1	2	2	3
v	4	3	2	1	1	2	3	3
e	5	4	3	2	2	1	2	3
y	6	5	4	3	3	2	2	2

Solution using **Dynamic Programming: Example**

- T = “surgery”, P = “survey”, k=**2**

T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1	0	1	1	1	1	1	1
u	2	1	0	1	2	2	2	2
r	3	2	1	0	1	2	2	3
v	4	3	2	1	1	2	3	3
e	5	4	3	2	2	1	2	3
y	6	5	4	3	3	2	2	2

Solution using **Dynamic Programming: Example**

- T = “surgery”, P = “survey”, k=**2**

T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1	0	1	1	1	1	1	1
u	2	1	0	1	2	2	2	2
r	3	2	1	0	1	2	2	3
v	4	3	2	1	1	2	3	3
e	5	4	3	2	2	1	2	3
y	6	5	4	3	3	2	2	2

1 + $\boxed{}$

Solution using **Dynamic Programming: Example**

- T = “surgery”, P = “survey”, k=**2**

T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1	0	1	1	1	1	1	1
u	2	1	0	1	2	2	2	2
r	3	2	1	0	1	2	2	3
v	4	3	2	1	1	2	3	3
e	5	4	3	2	2	1	2	3
y	6	5	4	3	3	2	2	2

1 + $\boxed{}$

Solution using **Dynamic Programming: Example**

- T = “surgery”, P = “survey”, k=**2**

T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1	0	1	1	1	1	1	1
u	2	1	0	1	2	2	2	2
r	3	2	1	0	1	2	2	3
v	4	3	2	1	1	2	3	3
e	5	4	3	2	2	1	2	3
y	6	5	4	3	3	2	2	2

1 + $\boxed{}$

Solution using **Dynamic Programming: Example**

- T = “surgery”, P = “survey”, k=**2**

T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1	0	1	1	1	1	1	1
u	2	1	0	1	2	2	2	2
r	3	2	1	0	1	2	2	3
v	4	3	2	1	1	2	3	3
e	5	4	3	2	2	1	2	3
y	6	5	4	3	3	**2**	**2**	**2**

Solution using **Dynamic Programming: Example**

- T = “surgery”, P = “survey”, k=**2**

T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1	0	1	1	1	1	1	1
u	2	1	0	1	2	2	2	2
r	3	2	1	0	1	2	2	3
v	4	3	2	1	1	2	3	3
e	5	4	3	2	2	1	2	3
y	6	5	4	3	3	**2**	**2**	**2**

Bold entries indicate matching positions.

- Cost: O(*mn*) time where *m* and *n* are the lengths of the two strings being compared.
- Παρατήρηση: <u>η πολυπλοκότητα είναι ανεξάρτητη του κ</u>

Solution using **Dynamic Programming: Example**

- T = “surgery”, P = “survey”, k=**2**

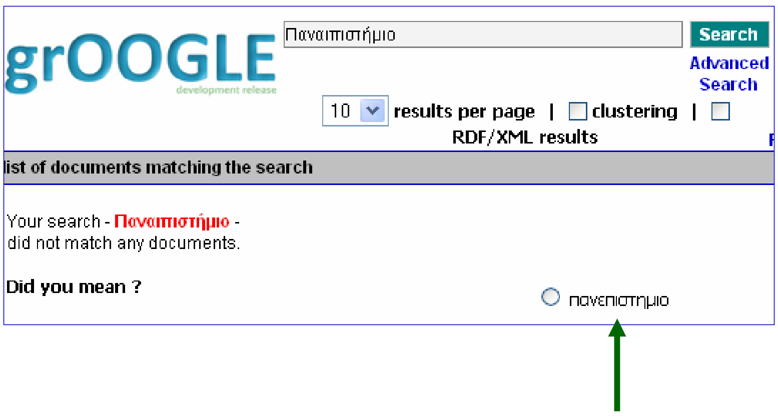
T

P

		s	u	r	g	e	r	y
	0	0	0	0	0	0	0	0
s	1	0	1	1	1	1	1	1
u	2	1	0	1	2	2	2	2
r	3	2	1	0	1	2	2	3
v	4	3	2	1	1	2	3	3
e	5	4	3	2	2	1	2	3
y	6	5	4	3	3	2	2	2

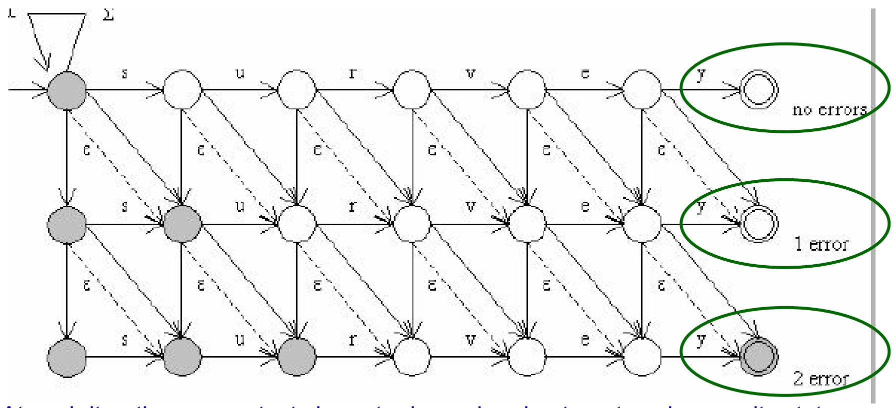
- Cost: O(*mn*) time where *m* and *n* are the lengths of the two strings being compared.
- **O(m) space** as we need to keep only the previous column stored
  - So we donnot have to keep a mxn matrix

Εφαρμογή στο groogle



Searching Allowing Errors
Solution with a Nondeterministic Automaton

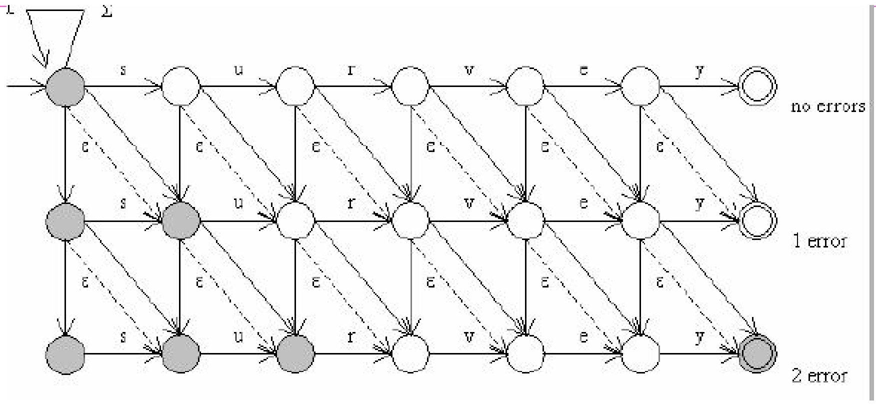
Searching Allowing Errors: Solution with a Nondeterministic Automaton



- At each iteration, a new text character is read and automaton changes its state.

- Every row denotes the number of errors seen
  - (0 for the first row, 1 for the second, and so on)
- Every column represents matching to pattern up to a given position.

Searching Allowing Errors: Solution with a Nondeterministic Automaton



- **Horizontal** arrows represents matching a document.
- **Vertical** arrows represent insertions into pattern
- **Solid diagonal** arrows represent replacements (they are unlabelled: this means that they match any character)
- **Dashed diagonal** arrows represent deletion in the pattern (ε: empty).

Searching Allowing Errors: Solution with a Nondeterministic Automaton

- Search time is O(n)
  - άρα η μέθοδος αυτή είναι πιο αποδοτική από την τεχνική με δυναμικό προγραμματισμό (που ήταν O(*mn*))
- However, if we convert NDFA into a DFA then it will be huge in size

Searching using Regular Expressions

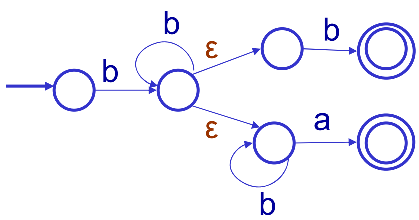
Searching using Regular Expressions

Classical Approach
(a) Build a ND Automaton
(b) Convert this automaton to deterministic form

(a) Build a ND Automaton

Size O(m) where m the size of the regular expression

Π.χ. regex = b b* (b | b* a)

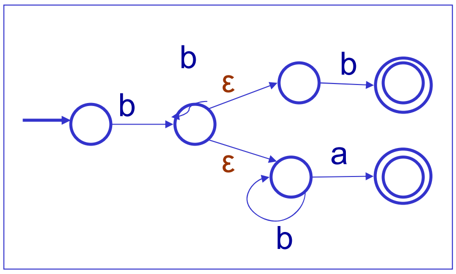


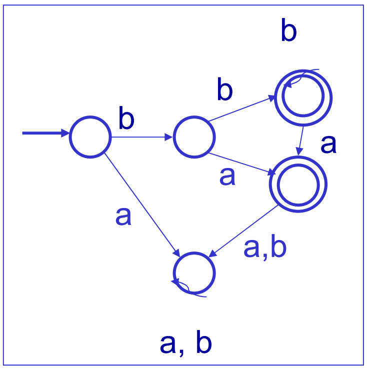
Searching using Regular Expressions (II)

(b) Convert this automaton to deterministic form

– It can search any regular expression in O(n) time where n the size of text
– However, its size and construction time can be exponential in m, i.e. O(m 2^m).

b b* (b | b* a) = (b b* b | b b* b* a) = (b b b* | b b*a)



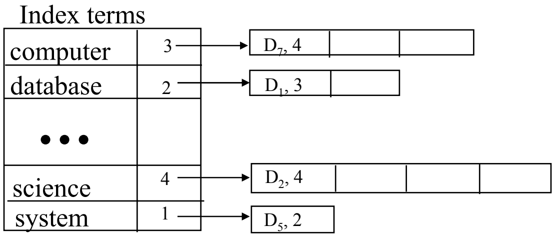


Bit-Parallelism to avoid constructing the deterministic automaton (NFA Simulation)

Pattern Marching Queries and Index Structures

Pattern Matching Using **Inverted Files**

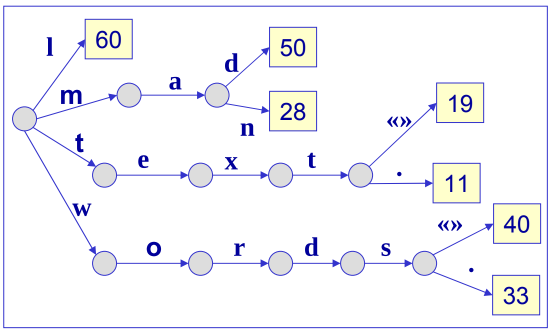
- Προηγουμένως είδαμε πως μπορούμε να αποτιμήσουμε επερωτήσεις με κριτήρια τύπου Edit Distance, RegExpr, ανατρέχοντας στα κείμενα.
- Τι κάνουμε αν έχουμε ήδη ένα Inverted File ?
  - Ψάχνουμε το Λεξιλόγιο αντί των κειμένων (αρκετά μικρότερο σε μέγεθος)
  - Βρίσκουμε τις λέξεις που ταιριάζουν
  - Συγχωνεύουμε τις λίστες εμφανίσεων (occurrence lists) των λέξεων που ταίριαξαν.
- If **block addressing** is used, the search must be completed with a sequential search over the blocks.
- Technique of inverted files is not able to efficiently find approximate matches or regular expressions that span many words.



Pattern Matching Using **Suffix Trees**

- Τι κάνουμε αν έχουμε ήδη ένα Suffix Tree?
- Μπορούμε να αποτιμήσουμε τις επερωτήσεις εκεί, αντί στα κείμενα;

Suffix Trie

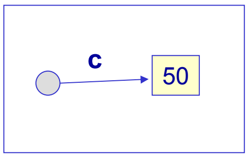


Pattern Matching Using **Suffix Trees (II)**

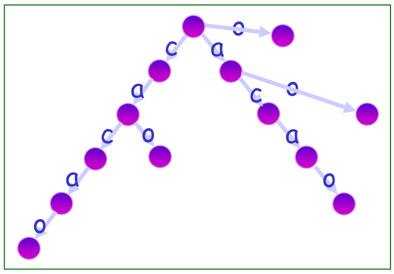
If the suffix trees index **all text positions** (not just word beginnings) it can search for words, prefixes, suffixes and sub-stings with the same search algorithm and cost described for word search.

Indexing all text positions normally makes the suffix array size **10 times or more the text size**.

cacao



versus



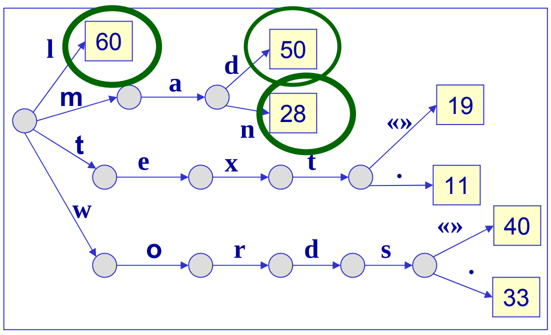
if word beginnings

if all text positions

Pattern Matching Using **Suffix Trees (III)**

- **Range queries** are easily solved by just searching both extremes in the trie and then collecting all the leaves lie in the middle.

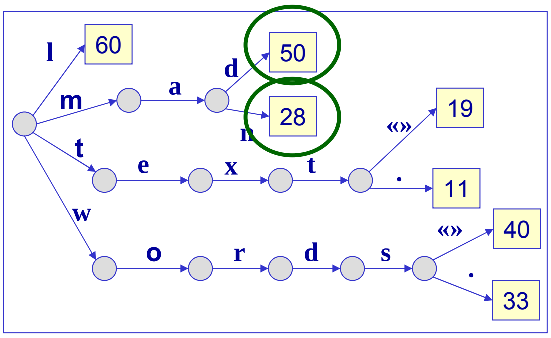
Consider the query: **"letter" < q < "many"** where < has **a lexicographic meaning**



Pattern Matching Using **Suffix Trees (IV)**

- **Regular expressions** can be searched in the suffix tree. The algorithm simply simulates sequential searching of the regular expression

q=ma*



Some Software Packages for String Searching

- A recent software package that implements several recently emerged string matching algorithms (code available in C++) is available at:
  - http://flamingo.ics.uci.edu/releases/1.0/
- StringSearch: Searching algorithms written in Java (includes implementations of the Boyer-Moore and the Shift-Or (bit-parallel) algorithms). These algorithms are easily five to ten times faster than the naïve implementation found in java.lang.String. Available at
  - http://johannburkard.de/software/stringsearch/
- Algorithm FJC in Java
  - http://www.sfu.ca/~cjenning/fjs/index.html

References

- Some slides were based on the slides of
  - Christian Schindelhauer (University of Paderborn)