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Analysis of (n, n) and (d, p) Reactions on the Same Target Using Phase-Equivalent Potentials

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The difference between (d, p) "Stripping into Continuum" and (n, n) Elastic Scattering on the same target has been investigated and discussed in terms of a non-local force and its local equivalent. In general our results indicate an influence of non local effects. In particular with the inclusion of the non locality a possible spin assignment for the levels at the excitation energies $E_x = 3.359$ MeV and 3.519 MeV in the reaction $^{15}\mathrm{N}(d, p)\mathrm{N}^{16}$ (unbound) has been indicated.

1. Introduction

Transfer reactions to unbound states have been treated with many theoretical methods. Huby and Mines [1] used the post interaction form of the DWBA but encountered difficulties in the slow convergence of the radial matrix elements. Vincent and Fortune [2] considered these difficulties by introducing the method of contour integration in the complex r-plane. A formulation in the momentum space has been given by Noble [3] while Gamov states have been used by Berggren [4] and Bang and Zimanyi [5] and deuteron break-up approaches have been proposed by Baur and Trautman [6] who have also made a study in terms of a coupled channels model [7].

In particular there has been a new development originated by Lipperheide and his colaborators outlined in [8] and references cited there, in which it is shown that in PWBA or DWBA approximations the cross-section for stripping to unbound (resonant) states is proportional to the half off-shell cross-section of the elastic scattering between the transferred particle and the target nucleus. Further it has been established [9] that especially in (d, p) stripping to unbound states, suppression of the three-body effects manifests itself as an off-shell dependence on the resonant state by introducing alterations in shape and width. This is true for transfer reactions to single-particle like resonance states as well as for those where the final states are of core excited configurations [7, 9, 10].

In both cases on- or off-shell resonances are associated with the presence of poles of the outgoing green function in the complex energy plane.

However, it is no longer obtained [9, 11] that the position of the poles remains unaltered by going from on - to off - the energy shell. This and the change of shape and width of the resonance can be related to the off-shell behavior.

The question now arises as to whether a non local potential which is equivalent on the energy shell to a local one but has a different off-shell behavior from it, introduces an additional off-shell effect in the stripping reaction and if it does in which way it will show up in the stripping cross-sections. In other words if on-shell scattering is related to the elastic scattering of the neutron on the target nucleus $A(n, n)A$ and off-shell scattering to an $A(d, p)B$ stripping reaction on the same target then the introduction of the non local force might give information about any variations into the corresponding continuum spectrum of $B = n + A$. This then could be used to interpret differences in the cross-sections and assist in the determination of resonance angular momenta. It should be noted here that one has to start with a Schroedinger equation off-the energy shell in order to put directly the scattered neutron off-shell and therefore the same study cannot be done by considering the off-shell behavior of the T-matrix for the two potentials.

The present work is initiated to study this question.

We have taken the reactions $^{15}N(n,n)^{15}N$ and $^{15}N(d,p)^{16}N$ for which the experimental results are known [13, 14] and we have selected two particular resonances at $E_x=3.359$ MeV and $E_x=3.519$ MeV because their spin and parity assignments have not been defined unambiguously. We find that in both cases the present work indicates a spin and parity assignment. Furthermore it shows that the non locality of the force does not alter the resonance behavior provided the two potentials are phase-equivalent on the energy shell and do not show large differences in their off-shell behavior, that is when their wave functions do not differ much at small distances (small Perey effect). In contrast if large off-shell differences, which are indicative of strong three body effects in the reaction, do manifest themselves in the resonance, allow for the determination of angular momentum. This discards the notion that the non locality introduces variations only in the off-shell background and shows the importance of the non locality of the nuclear force in order to predict accurate the cross-section in stripping reactions.

In Sect. 2 we present the general formalism. Then in Sect. 3 we apply it to the special case when background effects can be neglected as in the reaction $N^{15}(d,p)N^{16}$ leading to the unbound states at 3.359 and 3.519 MeV excitation energies. Some more general calculations are also reported in this section. Our conclusions are stated in Sect. 4.

2. Review of the Formalism

To make it certain that any differences are entirely related to additional off-shell effects produced by the non locality of the potential we have adopted the following procedure; we have considered a local force to fit a resonance state on the (n,n) system and then we have derived a non local force which was phase-equivalent on-the energy shell. This is obviously opposite to what one does usually, by first fitting the non local force and it is more involved in terms of numerical calculations. However, we had to proceed in this way *i)* in order to exclude any ambiguities as to whether non local effects were already present in the (n,n) system and *ii)* in order to separate the effects related to the non locality from those related to the energy dependence. Then we have used these two forces to calculate the differential cross-sections off-the energy shell for the corresponding (d,p) reaction. Here the equivalence was defined in terms of the Perey and Buck [12] transformation because of its simplicity.

Since for stripping to unbound states the differential cross-section is related to the half-off-shell scattering amplitude we consider this expression as the starting point of our investigation. In PWBA this is given by

$$\frac{d\sigma}{d\Omega\, dE}=\frac{2}{(2\pi\hbar)^2}\left(\frac{k_p}{k_d}\right) G|(\tfrac{1}{2}\vec{k}_d-\vec{k}_p)|^2\times \mathrm{Im} f_c(\vec{q}_n,\vec{q}_n,E) \tag{1}$$

where $G(\frac{1}{2}\vec{k}_d-\vec{k}_p)$ is the deutron form factor and $\mathrm{Im}f_c(\vec{q}_n,\vec{q}_n,E_n)$ is the imaginary part of the half-off-shell scattering amplitude of the neutron impinging on the target A. Here q_n and E_n are the momentum and the energy carried by the neutron for which in the present case holds;

$$E_n=\frac{\hbar^2}{2m_n}k_n^2+E'=\frac{\hbar^2}{2m_n}q_n^2.$$

The off-shell scattering amplitudes $f_c(\vec{q}_n,\vec{q}_n,E_n)$ are found by solving the Schroedinger equation off-shell inside the nucleus $(r<R)$ and by relating this solution to a scattered wave outside $(r\gg R)$.

We should mention here that the use of the PWBA is a crude approximation of the actual physical situation under consideration, since it does not take into account distortion effects which are usually a source of strong off-shell effects. However we decided to avoid here the use of the DWBA, to overcome numerical problems and because we are mainly interested in a comparative analysis namely in the difference of the off-shell behavior between two phase equivalent potentials. We should further emphasize that, although we tried to somehow compensate for the Butler-type cut-off's of the T-matrices by using Woods-Saxon type potentials instead of sharply varying potentials, our treatment serves only to obtain an estimate of the influence of the non locality and is by no means considered satisfactory.

The integro-differential Schroedinger equation off-the energy shell is given [9] by

$$\left(\frac{\hbar^2}{2m}\nabla^2+E\right)\psi(\vec{r})$$
$$=\int V(\vec{r},\vec{r}')\psi(\vec{r}')\,d^3r'+(E-E')\phi(\vec{r}'). \tag{2}$$

Where the difference $(E'-E)$ is called [8] the off-shell distance S, which is a function of the stripping angle θ and $\phi(\vec{r})$ represents an incoming wave. Obviously for (n,n) scattering $S=0$ and (2) reduces to its standard form. The Kernel $V(\vec{r},\vec{r}')$ is taken as:

$$V(\vec{r},\vec{r}')=\begin{cases}U(\vec{r})\,\delta(\vec{r}-\vec{r}') & \text{local case}\\ U(\frac{1}{2}|\vec{r}+\vec{r}'|)\,H(|\vec{r}-\vec{r}'|) & \text{non local case.}\end{cases} \tag{3}$$

The form chosen for $U(p)$ is similar to those employed in local optical model calculations without a surface absorption namely;

$$U(p)=(V+iW)f(p) \tag{4}$$

where $f(p)$ is the Woods-Saxon function and $H(|\vec{r}-\vec{r}'|)$ is the Gaussian function

$$H(|\vec{r}-\vec{r}'|)=\pi^{-3/2}\beta^{-3}\exp\left(-\left(-\frac{\vec{r}-\vec{r}'}{\beta}\right)\right) \tag{5}$$

of the original Perey and Buck [12] article. Partial wave expansion of (2) leads to an off-shell radial dif. equation of the form;

$$T_l u_{jl}(r)=\int_0^\infty g_l(r,r')u_{jl}(r')dr'+(k_n^2-q_n^2)j_{jl}(r). \tag{6}$$

Where $T_l=\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}+k_n^2$ and

$$g_l(r,r')=\begin{cases}U(r')\delta(r-r') & \text{local case}\\ 2\pi rr'\int_{-1}^{1}U(\vec{r},\vec{r}')P_l(x)dx & \text{non local case.}\end{cases} \tag{7}$$

For the potential of (3) the equivalence between local and non local forces leads [15] to the relation

$$U_L(r)\exp\left(\frac{m\beta^2}{2\hbar^2}(E-U_L(r))\right)=U_N(r) \tag{8}$$

where $U_L(r)$ and $U_N(r)$ have the functional dependence of (4).

The idea is to choose a set of local parameters which gives a good fit to the total (n,n) cross-section of the resonance under consideration for the scattering of neutrons from ^{15}N. Then to use (8) to derive $U_N(r)$. The range of the non-locality β is initially chosen arbitrarily and other parameters are adjusted until the total cross-section is again the same. Then these two phase-equivalent forces are used to calculate the differential cross-section of the above resonances for (d,p) stripping on ^{15}N.

3. Analysis of (n,n) and (d,p) Cross-Sections

We have chosen a light nucleus as target specifically the nucleus N^{15} because *i)* both the $N^{15}(d,p)N^{16}$ reaction is known from a single-proton spectrum of Hewka et al. [13] and Fuchs et al. [14] and the corresponding $N^{15}(n,n)$ total cross-section is measured by Zeitnitz et al. [15] and *ii)* only single and isolated resonances appear in the energy region 0.8–2.0 MeV which are interesting in term of analysing any variations that might arise due to the introduction of the non locality.

We have first tried to fit $U_L(r)$ and then define according to (8) and fit $U_N(r)$ in order to reproduce the total elastic cross-section of the $J^\pi=1^+$ state at $E_n=0.925$ MeV. This state is unbound to neutron decay and is also found in the $N^{15}(d,p)N^{16}$ reaction at an excitation energy $E_x=3.359$ MeV.

As in the ^{15}N(n,n) analysis of Zeitnitz et al. [15] our R-matrix analysis also permits both the $l=1$ and the $l=2$ transitions corresponding to $J^\pi=1^+$ or $J^\pi=(1,2,3)^-$ respectively. However, the ^{14}N$(t,p)^{16}$N reaction of Hewka et al. [13] indicates an $l=0$ transition and positive parity or $J^\pi=1^+$ in agreement with the T_{20} tensor analysing power measurement of Baxter et al. [16] for the ^{18}O(d,a) reaction which predicts unnatural parity thus permitting only odd l-values and again $J^\pi=1^+$. On the other hand for the ^{15}N(n,n) and the ^{15}N(d,p) reactions $J^\pi=1^+$ assignement would require an $l=1$ value thus excluding all even l-values for this state. We found it therefore instructive to investigate whether it is possible to fit this resonance with an $l=0$ or an $l=2$ wave.

We should strongly emphasize here that our investigation deals exclusively with resonances in a single particle potential and does not cover resonances formed through the coupling to a bound state embedded in the continuum. Some resonances of the above system may indeed be better described by the above mechanism but a more involved formalism for example a coupled channels model [7] seems more appropriate for their description. Under this assumption we have not been able to reproduce a resonant behavior at this energy with an $l=0$ wave despite a systematic search and the employment of several potential forms and a parameter-search routine. An $l=2$ fit with a sum of two Woods-Saxon potentials was somehow possible with both the local and the non local force but it was not entirely satisfactory. The discrepancy is even more contrasting in the (d,p) system where both forces are able to reproduce only background effects. This bears somehow on the experimental results where the resonance is "shadowed" by a strong background. However, the lack of any indication or even a resemblance of a resonance behavior, at least in the $l=0$ case, supports the exclusion of all even l-values for this state. This in conjunction with the $l=1$ fit of the R-matrix analysis is indicative of positive parity and therefore $J^\pi=1^+$ for this state. We shall defer however a definite conclusion, pending the results of an investigation similar to that of [7].

As next, we have chosen to apply our formalism to the 3.519 MeV level because despite the effort that has been devoted to determine its spin and parity there is a lack [16–19] of agreement from different experiments and different theoretical predictions. On the one hand our earlier [9] analysis of the (d,p) dif. cross-section is consistent with the results of the ^{14}N$(t,p)^{16}$N of Hewka et al. [13] and ^{15}N$(d,p)^{16}$N

and $^{15}N(n,n)^{16}N$ of Fuchs et al. [14] and Zeitnitz et al. [15] and Fortune et al. [17] in predicting an $l=2$ resonance or a $J^\pi=(1,2,3)^-$ assignment for this state. On the other, the reactions $^{18}O(\vec{d},a)^{16}N$ of Baxter et al. [16], $^{10}B(Li,p)^{16}N$ of Fortune and Medsker [18] and Fortune and Silverman [19], $^{17}O(d^3,He)^{16}N$ of Mairle et al. [20] and $^{14}C(^3He,d)^{16}N$ of Freiesleben and Weibezahn [21] indicate positive parity and a $J=2$ for this state.

In the present work our assumption of the reaction mechanism will independently show whether there is a correspondence between the (n,n) and the (d,p) cross-section and whether the same force could provide for a satisfactory fit for both systems. Despite a systematic search no satisfactory fit could be obtained with an $l=1$ or an $l=3$ wave for any of the two systems. In contrast, with an $l=2$ wave a perfect fit could been obtained with both the local and the non-local force for the (n,n) system. For the local force we have used a sum of two Woods-Saxon terms without surface absorption and with the following parameters: $V_1=36.83$ MeV, $R_1=3.4$ fm, and $\alpha_1=0.2$ fm and $V_2=68.93$, $R_2=0.61$ fm and $\alpha_2=0.45$ fm. The non-local force was constructed according to (8) with $\beta=0.5$ fm.

With the potentials defined in this way the (d,p) differential cross-sections were calculated using $A(14)$ of [9]. The $l=2$ partial wave was again used. The results are shown in Fig. 1. The predictions of both potentials are identical and apart from the background, they fit the (d,p) experimental data reasonably well. The R-matrix radius $r_m=4.69$ fm and the boundary condition $b=l$ were chosen in both cases.

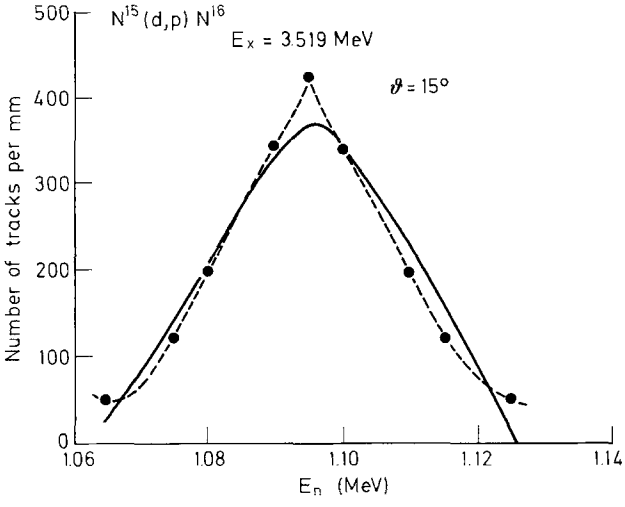

Fig. 1. The diff. cross-section as a function of the neutron energy En. The dotted line is freely drawn through the experimental points [13]. The full-line is the (d,p) calculation with the non local force normalized at $En=1.08$ to 200 tracks/per mm

A single R-matrix level was shown to be sufficient since more levels did not provide any essential variations, thus confirming a single and isolated resonance. We should mention here that the resonance appears exactly at $E_n=1.095$ MeV without the level shift reported in earlier works [9] and and this can be attributed to the elimination of sharp cut-offs through the Wood-Saxon form. The resonance functions $u_{jl}(r)$ of (6) were taken as expansions of the eigenstates of the potentials and they have been computed numerically. They were almost identical outside the nucleus but is was surprising to note that they were only slightly different inside (very small Perey effect). It appeared that the non locality did not effect the wave function strongly and therefore this fact would also manifest itself in the off-shell behavior. The almost exact fit to the on-shell and the reasonable fit to the off-shell data with an $l=2$ partial wave together with the failure of an $l=1$ or $l=3$ fit leads to a presumption that $J^\pi=1^-$ for this resonance. The only other possibility would appear to be a mixture of $l=1+3$ waves which of course would require $J^\pi=1^+$ but our results seem to contradict this possibility.

Since off-shell effects are related to contributions from the interior any variation of the off-shell behavior is a measure of the non locality. It is therefore surprising that no such variations are noticeable in the non-local case for small values of S (and hence θ). It appears that for a single and isolated resonance the introduction of the non locality does not produce unwanted effects in the resonance parameters.

It remained to be investigated whether this had to do with the off-shell behavior of the potentials, i.e. the type of the non locality or the method of equivalence. To answer this point we've calculated the Noyes-Kowalski [11] functions $f(p,k)$ defined by

$$f(p,k)=\langle p|T(E_k)|k\rangle/\langle k|T(E_k)|k\rangle \tag{9}$$

and the difference

$$\Delta f(p,k)=\langle p|T_L(E_k)-T_N(E_k)|k\rangle/\langle k|T_N(E_k)|k\rangle \tag{10}$$

which is easily shown to be

$$\Delta f(p,k)=(k^2-p^2)\int kr j_l(pr)(u_{jl}^L(r)-u_{jl}^N(r)dr/\sin\delta_l. \tag{11}$$

From (11) one sees that the difference in the off-shell behavior is directly related to the difference of the two wave functions (i.e. the damping function). Therefore the method of equivalence does not play any important role. Furthermore in the present example non local effects are very small (again due to the damping function) and this also results in very small off-shell differences.

In the example at hand it can be shown that despite the fact that the $\Delta f(p,k)$ increases almost linearly with the off-shell distance S it still remains below 10% for distances up to $S=6.4$ MeV which corresponds to a stripping angle of $\theta=30$ degrees. Our fits to angular distributions at such angles are also not satisfactory and though this is in line with our presumption that non-local effects in the (d,p) stripping reaction are responsible for this discrepancy it is disappointing to note that while we were able to find non-local forces which gave qualitatively good fits we were not able to find a non-local force which would exhibit strong non-local effects and which together with its local equivalent would fit the corresponding (n,n) and (d,p) data.

The fact that we have investigated here non local potentials of a particular class places a limitation on this conclusion. In general however our calculations leave no doubt that our assumption of the complete correspondence between (n,n) and (d,p) scattering from the one hand and local and non local force from the other is far too stringent. How one could relax this assumption and still be able to relate the non-locality to differences at large angles is an important question.

It might be argued that the suppression of the non locality is a result of the energy dependence of the non local force. In this respect it is interesting to consider whether a redefinition of the "on-shell" "off-shell" equivalence in the opposite direction would give better results and we are now investigating this point.

4. Conclusions

We have studied the relation of the (d,p) stripping reaction to unbound states and the (n,n) elastic scattering on the same target in terms of the relation between a non local force and its local equivalent.

Though our results suggest the influence of the non local effects in the (d,p) stripping we have been unable to relate exactly the non locality to the discrepancy between theoretical and experimental data since only equivalent non local forces with no too - strong non-localities were resulting from the (n,n) calculations. On the other hand the fact that both forces gave good fits in the corresponding (n,n) and (d,p) cross-section is indicative of the correctness of the underlying mechanism and it has been used to indicate resonance levels in the $^{15}N(d,p)^{15}N$ (unbound) system.

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