CONSTRUCTION OF A 3D PHYSICALLY-BASED MULTI-OBJECT DEFORMABLE MODEL

G. Bueno[†‡], *C. Nikou*[†‡], *F. Heitz*[†] *and J-P. Armspach*[‡]

[†]Laboratoire des Sciences de l'Image de l'Informatique et de la Télédétection
University of Strasbourg I (UPRES-A CNRS 7005)
4, Bd. Sébastien Brant, 67400 Illkirch, France
[‡]Institut de Physique Biologique, School of Medicine
University of Strasbourg I (UPRES-A CNRS 7004)
4, rue Kirschleger, 67085 Strasbourg Cedex, France

ABSTRACT

This paper addresses the problem of describing the significant *intra* and *inter* variability of 3D deformable structures within 3D image data sets. In pursuing it, a 3D probabilistic physically based deformable model is defined. The statistically learned deformable model captures the spatial relationships between the different objects surfaces, together with their shape variations. The structures of interest in each volume are parameterized by the amplitudes of the vibration modes of a deformable spherical mesh. For a given 3D image in the training set, a vector containing the largest vibration modes describing the desired object is created. This random vector is statistically constrained by retaining the most significant variation modes of its Karhunen-Loeve (KL) expansion on the considered population. The surfaces of the modeled structures thus deform according to the variability observed in the training set. A preliminary application of a 3D multi-object model for the segmentation of 3D brain structures from MR images is presented.

1. INTRODUCTION

Deformable models have shown to be an appealing approach for accommodating the *intra-individual/inter-individual* variability of 3D anatomical structures in medical image data sets [1, 2]. This paper presents results in the framework of brain imaging and forms part of an extension to the method described by Nikou *et al.* in [3]. The goal is the description of the spatial relationships between the structures of interest (corresponding here to brain anatomical structures) as well as the modeling of the shape variations observed over a representative image data set arising from different individuals.

A 3D physically-based statistical deformable model carrying information on multiple anatomical structures is presented. This model describes the deformations of a given surface as the ordered superimposition of vibrations of an initial mesh, at different frequencies [1, 4]. The vibration modes of the different anatomical structures are constrained by a statistical training, from a representative population. The remainder of the paper describes the construction and learning of the multi-object model, along with some preliminary results in the modeling of various brain anatomical structures.

2. 3D MULTI-OBJECT DEFORMABLE MODEL

In the proposed approach, the different structures have been extracted from a representative training set of fifty 3D MR images. The image data were first registered to a reference image, to compensate for rigid transformations [5]. In a second step, the 3D anatomical structures of interest were segmented from the registered data set, using supervised 3D-watershed segmentation methods [6], combined with the deformable matching of a reference segmentation map [7]. Modeled anatomical structures include the head, brain, ventricles, cerebellum, corpus callosum and right hippocampus surfaces (see fig. 4).

For a given 3D image in the training set, a vector containing the largest vibration modes describing the desired objects is created, [1, 4]. This random vector is statistically constrained by retaining the most significant variation modes of its KL expansion on the training population. The model consists of 3D points sampled on a spherical surface, following a quadrilateral cylinder topology [1]. The model nodes are stacked in vector:
$\mathbf{X}_0 = (x_1^0, y_1^0, z_1^0, ..., x_{N'N}^0, y_{N'N}^0, z_{N'N}^0)^T$, where N designates the number of points in the direction of the geographical longitude and N' is the number of points in the direction of the geographical latitude of the sphere. The physical model is characterized by its mass matrix $\mathbf{M}$, its stiffness matrix $\mathbf{K}$ and its dumping matrix $\mathbf{C}$. Its governing

0-7803-6297-7/00/$10.00 © 2000 IEEE

equation may be written as [1, 4]:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F} \qquad (1)$$

where $\mathbf{U}$ stands for the nodal displacements of the initial mesh $\mathbf{X}_0$. The image force vector $\mathbf{F}$ is based on the euclidean distance between the mesh nodes and their nearest contour points [8].

Since equation (1) is of order $3NN'$, where NN' is the total number of nodes of the spherical mesh, it is solved in a subspace corresponding to the truncated vibration modes of the deformable structure [1, 4], using the following change of basis:

$$\mathbf{U} = \Phi\tilde{\mathbf{U}} = \sum_i \tilde{u}_i \phi_i, \qquad (2)$$

where Φ is a matrix and $\tilde{\mathbf{U}}$ is a vector, ϕ_i is the i^{th} column of Φ and $\tilde{u}_i$ is the i^{th} scalar component of vector $\tilde{\mathbf{U}}$. By choosing Φ as the matrix whose columns are the eigenvectors of the eigenproblem:

$$\mathbf{K}\phi_i = \omega_i^2 \mathbf{M}\phi_i, \qquad (3)$$

and using the standard Rayleigh hypothesis [1], matrices $\mathbf{K}$, $\mathbf{M}$ and $\mathbf{C}$ are simultaneously diagonalized:

$$\begin{cases} \Phi^T\mathbf{M}\Phi = \mathbf{I} \\ \Phi^T\mathbf{K}\Phi = \Omega^2 \end{cases} \qquad (4)$$

Ω^2 is the diagonal matrix whose elements are the eigenvalues ω_i^2 and $\mathbf{I}$ is the identity matrix. Substituting (2) into (1) and premultiplying by Φ^T yields:

$$\ddot{\tilde{\mathbf{U}}} + \tilde{\mathbf{C}}\dot{\tilde{\mathbf{U}}} + \Omega^2\tilde{\mathbf{U}} = \tilde{\mathbf{F}} \qquad (5)$$

where $\tilde{\mathbf{C}} = \Phi^T\mathbf{C}\Phi$ and $\tilde{\mathbf{F}} = \Phi^T\mathbf{F}$.

In many computer vision applications [4], when the initial and the final state are known, it is assumed that a constant load $\mathbf{F}$ is applied to the object. Thus, equation (1) is called the equilibrium governing equation and corresponds to the static problem:

$$\mathbf{K}\mathbf{U} = \mathbf{F} \qquad (6)$$

In the new basis, equation (6) is simplified to $3NN'$ scalar equations:

$$\omega_i^2 \tilde{u}_i = \tilde{f}_i. \qquad (7)$$

In equation (7), ω_i designates the i^{th} eigenvalue, the scalar $\tilde{u}_i$ is the amplitude of the corresponding vibration mode (corresponding to eigenvector ϕ_i). Equation (7), indicates that instead of computing the displacements vector $\mathbf{U}$ from equation (6), its decomposition may be computed in terms of the vibration modes of the original mesh. The number of vibration modes retained in the object description, is chosen so as to obtain a compact but adequately accurate representation. A typical *a priori* value covering many types of standard deformations is the quarter of the number of degrees of freedom in the system [1] (i.e. 25% of the modes are kept). Figure 1 shows the parameterization of the head surface considered for a subject belonging to the training set, by the 25% lowest frequency modes. Although not providing a high resolution description of the surfaces, this truncated representation provides a satisfactory compromise between accuracy and complexity of the representation. The spherical model is initialized around the structures of interest (fig. 1(a)). The vibration amplitudes are explicitly computed by equation (7), where rigid body modes ($\omega_i = 0$) are discarded and the nodal displacements may be recovered using equation (2). The physical representation $\mathbf{X}(\tilde{\mathbf{U}})$ is finally given by applying the deformations to the initial spherical mesh:

$$\mathbf{X}(\tilde{\mathbf{U}}) = \mathbf{X}_0 + \Phi\tilde{\mathbf{U}} \qquad (8)$$

This parameterization is applied for the different segmented objects (fig. 4) in the training set and their statistical learning is performed. For each image $i = 1, ..., n$ ($n = 50$) in the training set, a vector $\mathbf{a}_i$ containing the M_s lowest frequency vibration modes describing the S different anatomical structures is created:

$$\mathbf{a}_i = (\tilde{\mathbf{U}}_i^1, \tilde{\mathbf{U}}_i^2 \ldots, \tilde{\mathbf{U}}_i^S)^T \qquad (9)$$

where:

$$\tilde{\mathbf{U}}_i^s = (\tilde{u}_1^s, \tilde{u}_2^s, \ldots, \tilde{u}_{M_s}^s)_i \qquad (10)$$

Random vector $\mathbf{a}$ is statistically constrained by retaining the most significant variation modes in its KL transform:

$$\mathbf{a} = \bar{\mathbf{a}} + \mathbf{P}\mathbf{b} \qquad (11)$$

where $\bar{\mathbf{a}}$ is the average vector of vibration amplitudes of the structures belonging to the training set, $\mathbf{P}$ is the matrix whose columns are the eigenvectors of the covariance matrix $\Gamma = [(\mathbf{a}-\bar{\mathbf{a}})^{\mathbf{T}}(\mathbf{a}-\bar{\mathbf{a}})]$ and $\mathbf{b}_i = \mathbf{P}^T(\mathbf{a}_i - \bar{\mathbf{a}})$ are the coordinates of $(\mathbf{a} - \bar{\mathbf{a}})$ in the eigenvector basis.

The deformable multi-object model is finally parameterized by the m most significant statistical deformation modes stacked in vector $\mathbf{b}$. By modifying $\mathbf{b}$, the different objects are deformed in conjunction (fig. 3), according to the anatomical variability observed in the training set.

Given a set of S initial spherical meshes, $\mathbf{X}_{INIT}$, corresponding to the structures described by the joint model:

$$\mathbf{X}_{INIT} = \begin{pmatrix} \mathbf{X}_0^1 \\ \vdots \\ \mathbf{X}_0^S \end{pmatrix}, \qquad (12)$$

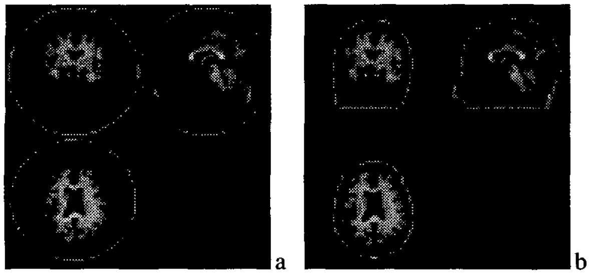


Fig. 1. *Brain structure parameterization from 3D MRI.* **(a)** *shows the initial spherical mesh superimposed to the structure to be parameterized.* **(b)** *presents the deformable model at equilibrium (25% of the modes).*

the statistical deformable model $\mathbf{X}(\mathbf{a})$ is thus represented by:

$$\mathbf{X}(\mathbf{a}) = \mathbf{X}_{INIT} + \underline{\overline{\Phi}}\mathbf{a} \tag{13}$$

Combining equations (11) and (13) we have:

$$\mathbf{X}(\mathbf{b}) = \mathbf{X}_{INIT} + \underline{\overline{\Phi}}\bar{\mathbf{a}} + \underline{\overline{\Phi}}\mathbf{P}\mathbf{b} \tag{14}$$

where:

$$\underline{\overline{\Phi}} = \begin{pmatrix} \Phi_1 & 0 & \dots & 0 \\ 0 & \Phi_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Phi_S \end{pmatrix}, \tag{15}$$

$$\mathbf{P} = \begin{pmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_S \end{pmatrix}, \quad \bar{\mathbf{a}} = \begin{pmatrix} \bar{\mathbf{a}}_1 \\ \vdots \\ \bar{\mathbf{a}}_S \end{pmatrix}$$

In equation (15), the columns of any $3NN' \times 3M_s$ matrix Φ_s are the eigenvectors of the spherical mesh describing surface s. Therefore, the spatial relation between the different structures, as well as the anatomical variability observed in the training set are compactly described by a limited number of parameters (typically $m \simeq 10$, corresponding to a compression ratio of about 10000:1).

The statistical multi-objet model may be used as a general purpose probabilistic atlas for the segmentation, labeling and interpretation of patient images. In fig. 2, we present a preliminary result corresponding to the segmentation of internal brain structures, for a 3D patient MR image not belonging to the training set. The segmentation method, described in [9], relies on two steps: the patient head surface is first segmented and parameterized by the model. The extracted head surface is then combined with the probabilistic model to provide a good initial prediction for the other internal anatomical structures. A standard matching algorithm is finally applied to adjust the predicted surfaces to the patient image data.

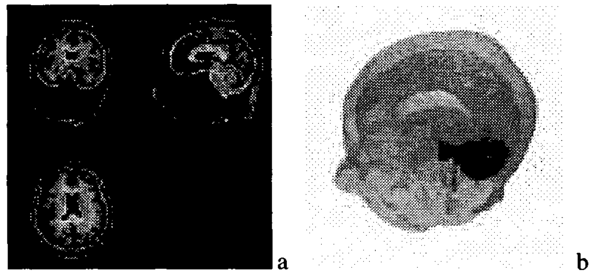


Fig. 2. **(a)** *Prediction of the different anatomical structures surfaces using the head surface and the probabilistic deformable model.* **(b)** *3D rendering of (a).*

3. CONCLUSION

The construction of a 3D multi-object statistical deformable model embedding information on the spatial relationships and anatomical variability of multiple anatomical structures in 3D medical images has been presented. This 3D multi-object deformable model is part of an on going project aiming at the development of a general purpose probabilistic atlas of the brain, to be used for the segmentation, registration, labeling and interpretation of 3D (MR/SPECT) images. While the results are emphasized in the medical domain, the approach may be applied to other modeling problems involving the description of the spatial, temporal or statistical variability of multiple interacting structures.

4. REFERENCES

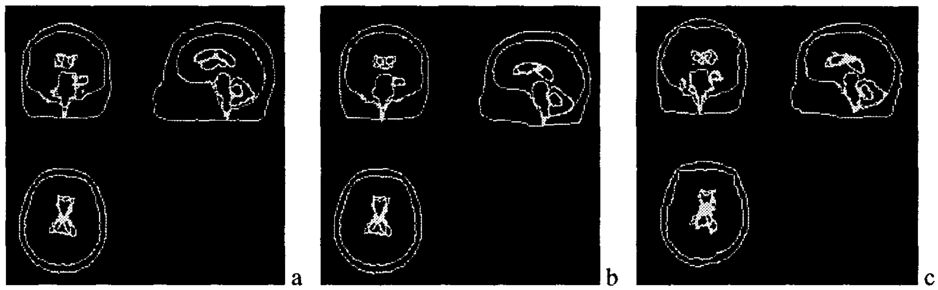
[1] C. Nastar and N. Ayache, "Frequency-based nonrigid motion analysis: Application to four dimensional medical images," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 18, no. 11, pp. 1069–1079, 1996.

[2] T. Mc Inerney and D. Terzopoulos, "Deformable models in medical image analysis: a survey," *Medical Image Analysis*, vol. 2, no. 1, pp. 91–108, 1996.

[3] C. Nikou, F. Heitz, J. P. Armspach, "Multimodal image registration using statistically constrained deformable multimodels," in *IEEE Int. Conf. Image Proc. ICIP 98*, Chicago, Oct. 1998.

[4] A. Pentland and S. Sclaroff, "Closed-form solutions for physically-based shape modeling and recognition," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 13, no. 7, pp. 730–742, 1991.

[5] C. Nikou, F. Heitz, and J. P. Armspach, "Robust registration of dissimilar single and multimodal images," in *Lecture Notes in Computer Science. Proc. of the* 5^{th}



$\mathbf{b}[1] = -3\sqrt{\lambda_1}$ $\mathbf{b}[1] = 0$ $\mathbf{b}[1] = 3\sqrt{\lambda_1}$

Fig. 3. *Deformations of the 3D joint model by varying the first statistical mode in vector* $\mathbf{b}$ *between* $-\sqrt{\lambda_1}$ *and* $\sqrt{\lambda_1}$. λ_i *designates the* i^{th} *eigenvalue of the covariance matrix* Γ. *Each image shows a multiplanar (sagittal, coronal, transversal) view of the 3D multi-object model.*

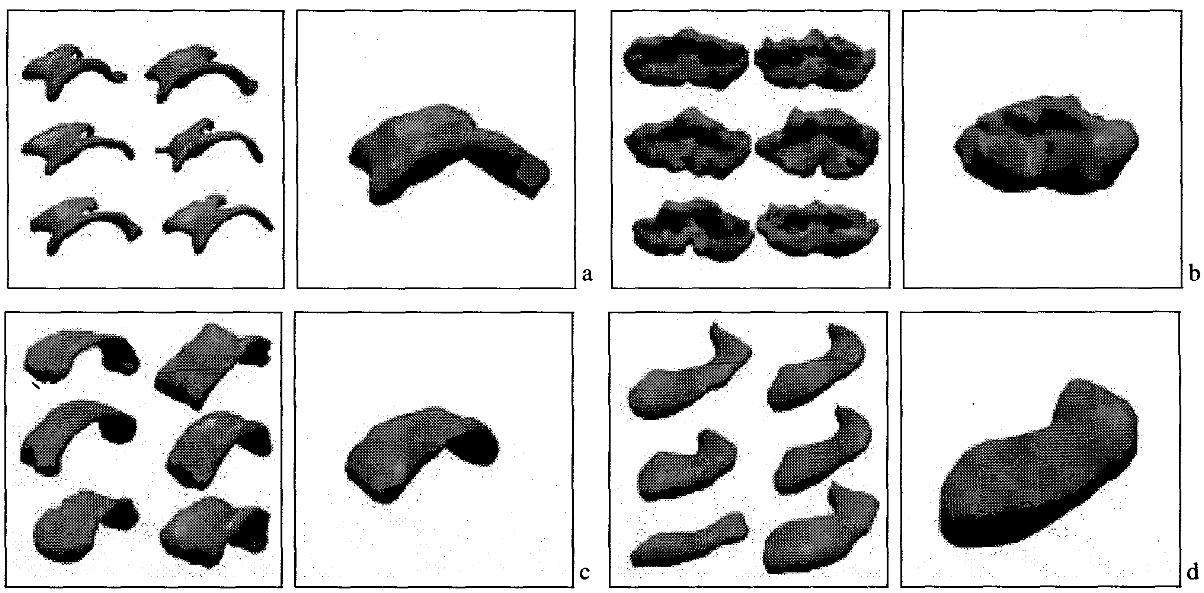


Fig. 4. *3D rendering of a subset of the different segmented anatomical structures and the corresponding average physically-based model. The structures shown are:* **(a)** *ventricles,* **(b)** *cerebellum,* **(c)** *corpus callosum and* **(d)** *right hippocampus.*

ECCV'98, Freiburg, Germany, 2-6 June 1998, vol. 2, pp. 51–65.

[6] G. Bueno, F. Heitz, and J. P. Armspach, "3D watershed-based segmentation on internal structures within MR brain images," *SPIE Proceedings*, vol. 3979, 2000, SPIE International Conference on Medical Imaging, San Diego, California, 12-19 Feb. 2000.

[7] O. Musse and F. Heitz and J. P. Armspach, "3D deformable image matching using multiscale minimization of global energy functions," in *Proc. of IEEE Int. Conf. Computer Vision and Pattern Recognition (CVPR'99), 23-25 June 1999, Fort Collins, Colorado, USA*, 1999, vol. 2, pp. 478–484.

[8] G. Borgefors, "On digital distance transforms in three dimensions," *Computer Vision and Image Understanding*, vol. 64, no. 3, pp. 368–376, 1996.

[9] C. Nikou, F. Heitz, J. P. Armspach, and G. Bueno, "A physically-based statistical deformable model for brain image analysis," in *Proc. of the* 6^{th} *European Conf. Computer Vision (ECCV'00)*, June 2000, Dublin Ireland.